True SYK or (con)sequences

D. V. Khveshchenko
UNC-Chapel Hill, Physics & Astronomy

IIP 07/31/19

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'Truth...or consequences!'

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Outline

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1. Holographic conjecture and condensed matter physics

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Cond. Mat. v.3, p.40 (2018)

Sci. Post. Phys. v.5, p.012 (2018)

Lith. J. Phys. v.59, p ??? (2019), v. 55, p. 208 (2015), v. 56, p.125 (2016)

EPL, v. 109, p. 61001 (2015), v. 111, p.17003 (2015) , v. 104, p. 47002 (2013)

Phys. Rev. B 86, 115115 (2012)

Standard model of condensed matter

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$$H = T_e + T_i + U_{ee} + U_{ei} + U_{ii}$$

Long-ranged Coulomb

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Interaction effects:

- uninteresting (Fermi liquid)
- interesting, yet already known 2-particle (e-e, e-h) instabilities
- interesting and unknown: 'non-Fermi liquids',..

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- uninteresting (Fermi liquid)
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Purely electronic: $T_{i,}\ U_{ei}, U_{ii} \rightarrow 0,$

(Super)strongly interacting: $T_e \longrightarrow 0$ ('Flat band')

Quest for elusive NFL

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- new (still untested) tool: **holography** ('AdS/C**M**T')

Holography primer

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- **Boundary** (quantum) theory → **Bulk** (semi) classical gravity (+ other fields)

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S &= \frac{1}{2\kappa_N^2}\int d^4x\sqrt{-g}\,[\mathcal{R}-2\Lambda+\mathcal{L}_m+\mathcal{L}_{cs}]\,,\\
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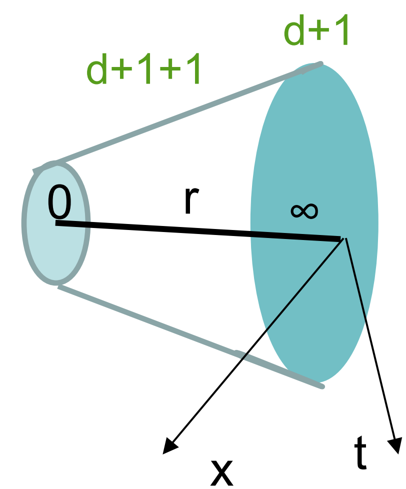
Feynman diagrams

Classical Einstein-type eqs



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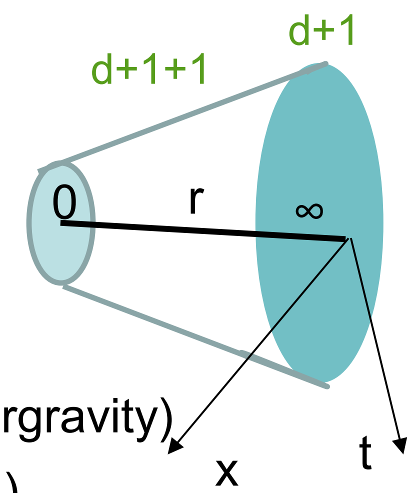
Feynman diagrams | Classical Einstein-type eqs



- d=4 Q=4 SU(N) SYM <-> type-IIB superstrings (d=5 supergravity)

  (t'Hooft, Suskind, Maldacena, Witten, Gubser, Klebanov, Polyakov,…)

- SUSY,
- multi-component (focusing on N>>1),
- Lorentz and scale-invariant,
- boundary theory: very strongly interacting

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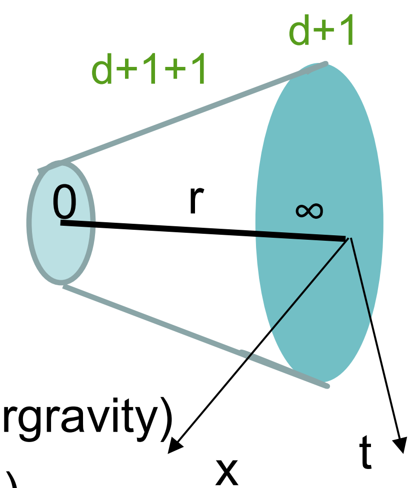

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- SUSY,
- multi-component (focusing on N>>1),
- Lorentz and scale-invariant,
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- How much of that can be relevant to condensed matter systems?

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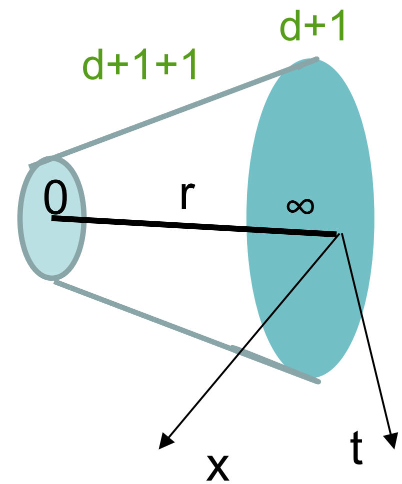

- Why it would not work:

- **non-SUSY**,
- only a few components (**N~1**),
- Lorentz , scale, translationally, and/or rotationally **non-invariant**,
- boundary theory: only moderately interacting (**T~U**),...

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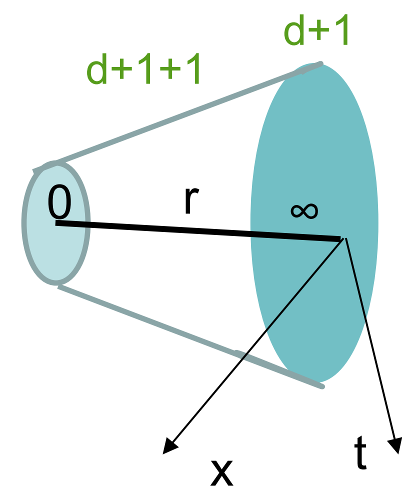

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- **Why it might still work**:
- emergent effective (local) **geometry**,
- perturbation theory/RG in d+1 dimensions --> **classical EOMs** in d+2,
- **tensor networks**,...

Holographic correspondence: evidence (?)

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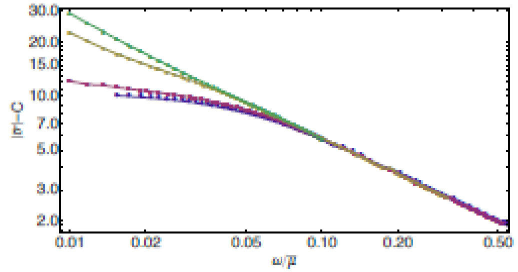
- **Data fitting**:

Optical conductivity in cuprates

(non-SUSY, N~1, T~U)



G,Horowitz and J.Santos, 1302.6586

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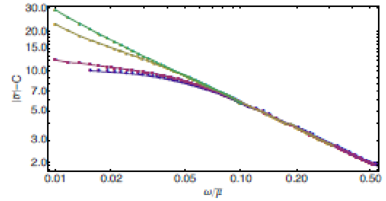

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- **Experiment**:

  $\frac{\eta}{s}$ ratio (>1/4π)

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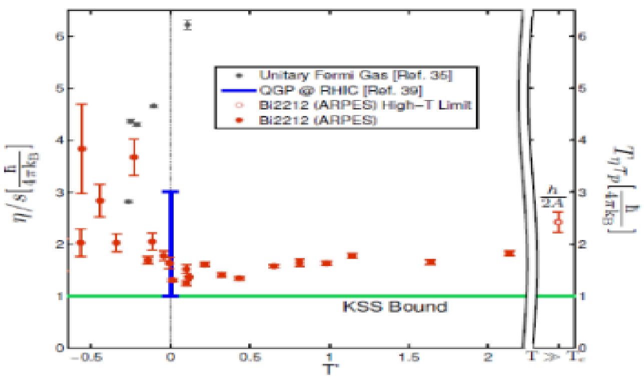

J.Rameau et al, 1409.5820

Indirect (Im G) ?

Universal KSS bound?

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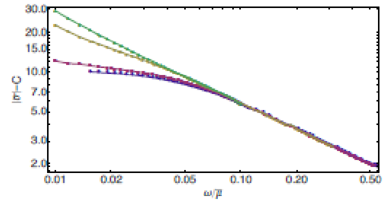

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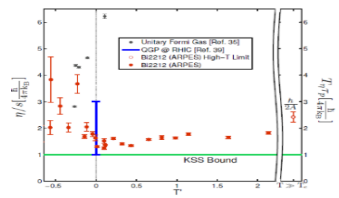

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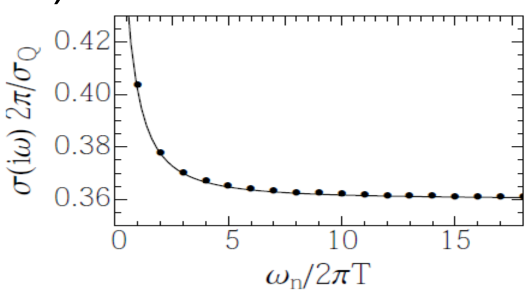
- (Almost) **exact methods** (MC):

  2d Bose-Hubbard model



E.Katz et al, 1409.3841

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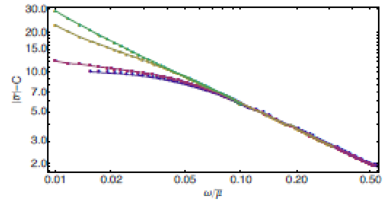

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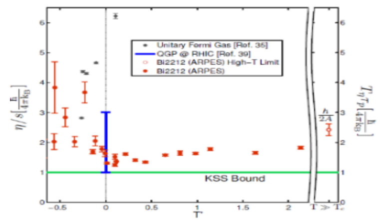

J.Rameau et al, 1409.5820

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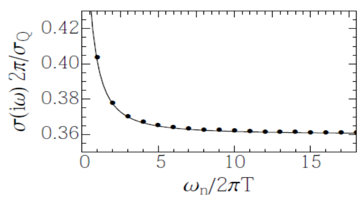

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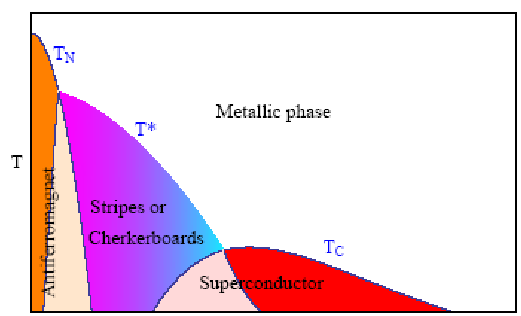

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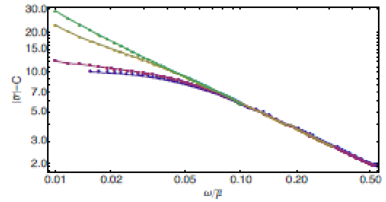

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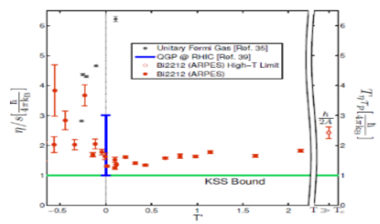

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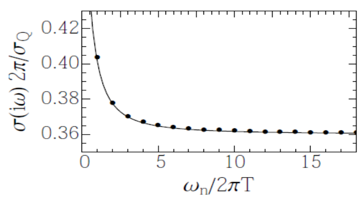

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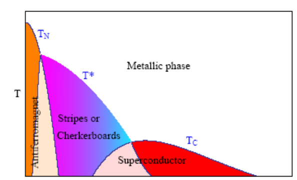

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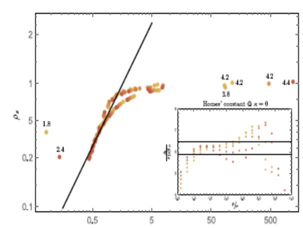
but **quantitative** (**sic**!) agreement:



J.Erdmenger et al,1501.07615

$$\rho_s(T=0) = C\sigma_{DC}(T_c)T_c.$$

Status of AdS/CMT (a.k.a. non-AdS/non-CFT)

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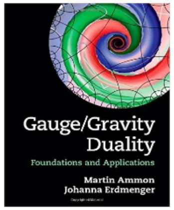
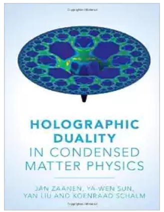
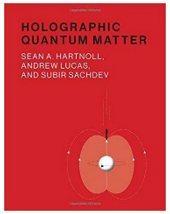
- **Textbooks**: 





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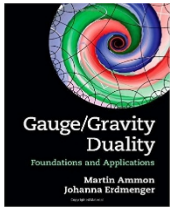
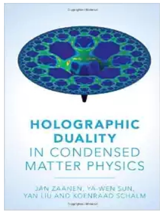
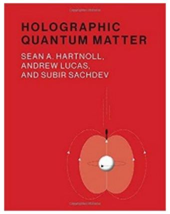
- Textbooks:    ,...

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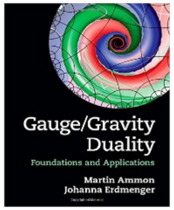
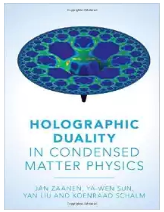
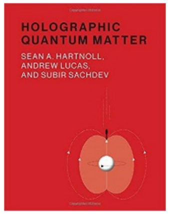
- Textbooks: 





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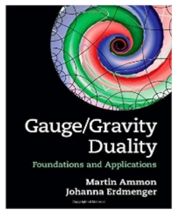

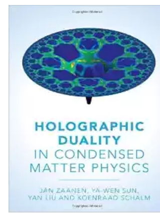

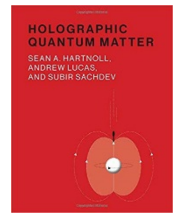

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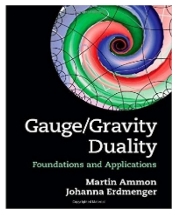

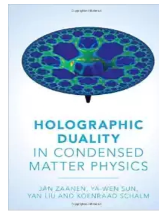

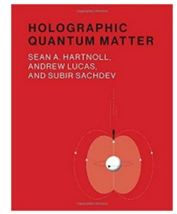

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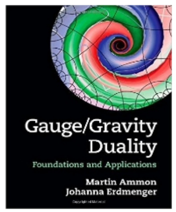

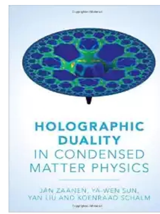

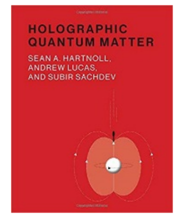

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- **New directions**:
• strong coupling hydrodynamics,
• quantum chaos and information scrambling,
• SYK and beyond,...

Holography: physical origin?

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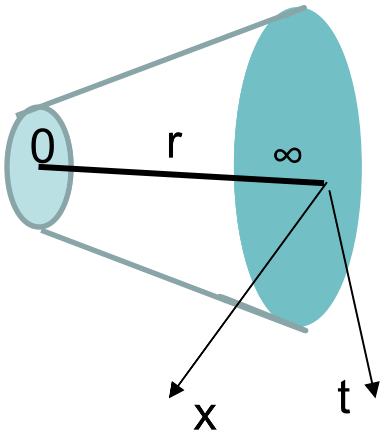
- Emergent extra dimension:

- Dynamical renormalization (energy/length/information) scale, ´RG=GR´



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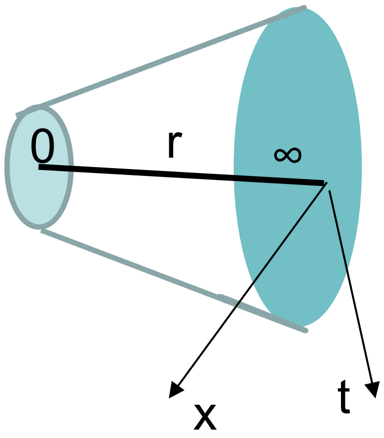
- Thermodynamics of phase transitions (Fisher/Ruppeiner),
- Quantum information theory, tensor networks (Bures),
- Bloch bands, dynamical time evolution (Berry),
- Quantum Hall and other topological states (Fubini/Study),...



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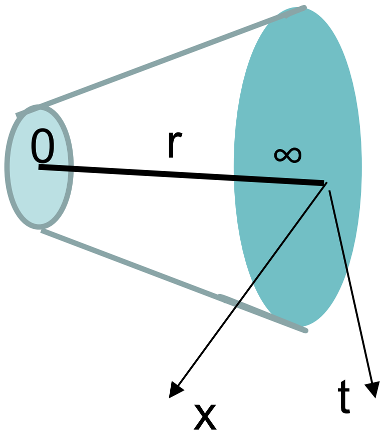

- Geometric nature of certain physical observables:

- Hall conductance =1st Chern class (Niu-Thouless,...),
- Entanglement entropy =Area of extremal surface (Ryu-Takayanagi) ,
- What else?

Holography light

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- **Fixed** classical metric,
- Non-SUSY and **N-irrelevant** (equiv. to 0^{th} order in $1/N$),
- The bulk 'dual' is **not dynamical** ('boundary problem')

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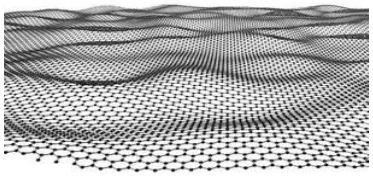

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Potentially problematic:
- Curved 3d space
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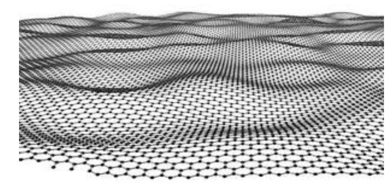

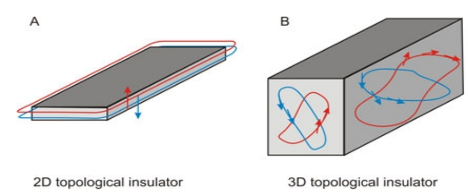

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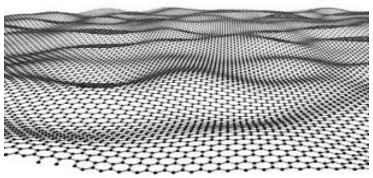

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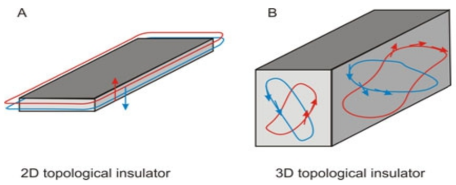

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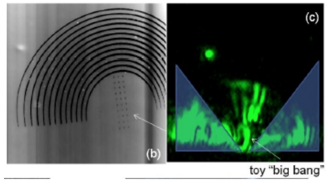
- **Hyperbolic metamaterials** (optical/IR)



Graphene: scotch tape-induced relativity

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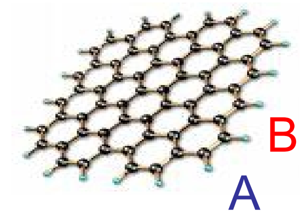

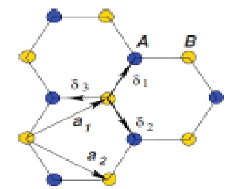

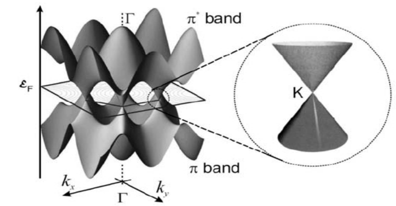

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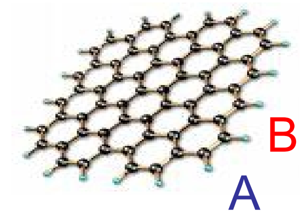

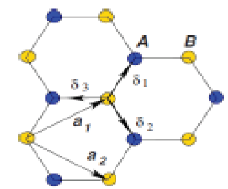

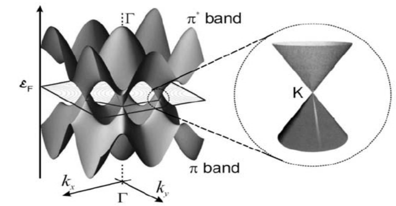

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- Klein tunneling,
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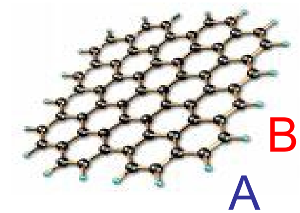

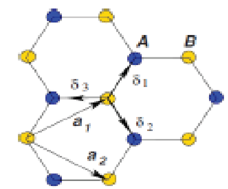

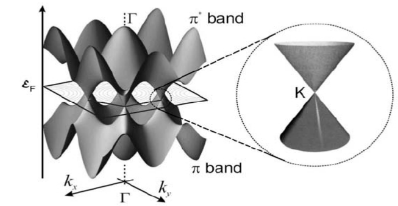

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- (non-) abelian gauge fields and solitons,
- Mimicking gravity and cosmology,
- **Analogue holographic correspondence**

DVK 1305.6651

Elastic strain in graphene

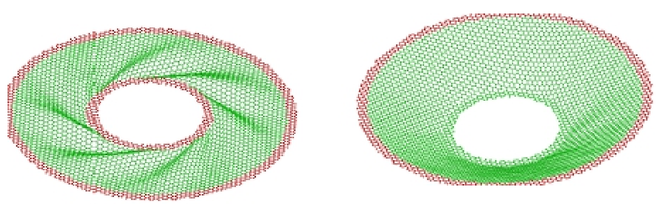
- **Hopping** Hamiltonian $$H = -\sum_{\mathbf{i},\mathbf{n}} t(\mathbf{r_i}, \mathbf{r_i} + \mathbf{n})\, a^{\dagger}_{\mathbf{r_i}} b_{\mathbf{r_i}+\mathbf{n}} + \mathrm{H.\ c.}$$

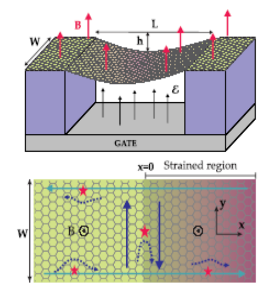
- **Strain** tensor $$\epsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial X_j} + \frac{\partial u_j}{\partial X_i}\right) + \frac{1}{2}\left(\frac{\partial u_k}{\partial X_i}\frac{\partial u_k}{\partial X_j}\right).$$

- **Elastic energy**

$$\mathcal{H}_{elastic} = \frac{\kappa}{2}\int d^2\vec{r}\,\left[\nabla^2 h(\vec{r})\right]^2 + \int d^2\vec{r}\left\{\frac{\lambda}{2}\left[\sum_i u_{ii}(\vec{r})\right]^2 + \mu\sum_{ij}\left[u_{ij}(\vec{r})\right]^2\right\}$$

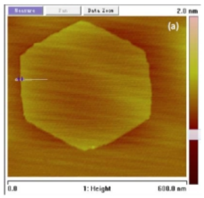
- **Stress** engineering



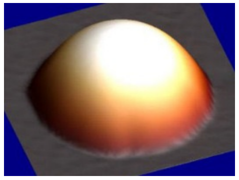


F.Guinea et al, '11

- Induced **fermion mass**
via hybridization with substrate



S.Tang et al, '13



N.Levy et al,'10

Emergent pseudo-(gravi)magnetic field

- **Vector** potential $A_x(\mathbf{R}) - iA_y(\mathbf{R}) = \frac{1}{qv_F}\sum_{n}\delta t(\mathbf{r},\mathbf{r}+\mathbf{n})e^{i\boldsymbol{K}\cdot\boldsymbol{n}} \simeq \frac{\hbar\beta}{2qa}(\epsilon_{xx}-\epsilon_{yy}+2i\,\epsilon_{xy})$

- **Higher order** terms
$$A_x^{(c)} = -\frac{3a^2V_{pp\pi}^0}{8qv_F}\left[\left(\frac{\partial^2 h}{\partial y^2}\right)^2 - \left(\frac{\partial^2 h}{\partial x^2}\right)^2\right],$$
$$A_y^{(c)} = -\frac{3a^2V_{pp\pi}^0}{4qv_F}\left[\frac{\partial^2 h}{\partial x\partial y}\left(\frac{\partial^2 h}{\partial y^2}+\frac{\partial^2 h}{\partial x^2}\right)\right]$$

$$\beta = -\partial\log t(r)/\partial\log r\big|_{r=a}$$

M.A.H.Vozmediano et al, ‘10;
A.L.Kitt et al,’12;
F. de Juan et al,’12

Position-dependent Fermi velocity (?)

- **Emergent gravity**: Weitzenbock geometry

G.Volovik and M.Zubkov, ‘13
A. Iorio and P.Pais, '15

$$\mathbf{H}_- = -\sigma^3\,\mathrm{f}_a^k\sigma^a[\partial_k + i\mathbf{A}_k], \quad a=1,2;k=1,2$$
$$\mathbf{H}_+ = -\sigma^2\Big(\sigma^3\,\mathrm{f}_a^k\sigma^a[\partial_k - i\mathbf{A}_k]\Big)\sigma^2.$$

$$\mathcal{H} = i\sigma^3\mathbf{H}_- = -ie\,\mathrm{e}_a^k\sigma^a\circ[\partial_k + i\mathbf{A}_k]$$

$$g_{\mu\nu} = e_a^\mu e_b^\nu\eta^{ab}$$

$$\mathrm{e}_a^i = \mathrm{f}_a^i/e, \quad e = [\det \mathrm{f}]^{1/2} = v_F(1-\frac{1}{3}(\Delta_2+\Delta_3+\Delta_1))$$
$$\mathrm{f}_a^i = v_F\left(\delta_a^i - \begin{bmatrix}\Delta_1 & \frac{(\Delta_2-\Delta_3)}{\sqrt{3}}\\ \frac{(\Delta_2-\Delta_3)}{\sqrt{3}} & \frac{2}{3}(-\frac{1}{2}\Delta_1+\Delta_2+\Delta_3)\end{bmatrix}\right)$$
$$\mathbf{A}_1 = \frac{1}{2v_Fa}(\mathrm{e}_2^1+\mathrm{e}_1^2), \quad \mathbf{A}_2 = \frac{1}{2v_Fa}(\mathrm{e}_1^1-\mathrm{e}_2^2)$$

$$g_{\mu\nu}^{\mathrm{graphene}} = \begin{pmatrix} 1 & 0 & 0\\ 0 & & \\ 0 & & g_{ij}\end{pmatrix}$$

Holographic boundary propagator

- Fermion action: $$S = \int drdtd^dx\sqrt{|det\hat{g}|}\bar{\psi}\gamma_a e^a_\mu(i\partial_\mu+\frac{i}{8}\omega^{bc}_\mu[\gamma_b,\gamma_c]+A_\mu-m)\psi$$
- Background metric: $$ds^2 = -f(z)dt^2 + g(z)dz^2 + h(z)d\vec{x}^2$$
- Radial Schroedinger's eq.: $$\frac{\partial^2\psi}{\partial r^2} = V(r)\psi$$
- WKB solutions: $$\psi_\pm(r,\omega,k) \sim \frac{1}{V^{1/4}(r)}e^{\mp\int_r^R dr'\sqrt{V}(r')}$$
- Asymptotic behavior: $$G(\tau,x) \sim \exp(-S_0(\tau,x))$$
- Extremal action: (geodesic) $$S(\tau,x) = L\omega\int du\sqrt{g_{uu}+g_{\tau\tau}(\frac{d\tau}{du})^2+g_{xx}(\frac{dx}{du})^2}$$

$$S(\tau,x) = L\omega^2\int_{u_0}^{u_t} du\sqrt{\frac{g_{uu}}{r(u)}} \qquad mR \gg 1$$

$$r(u) = \omega^2 - k_x^2/g_{xx}(u) - k_\tau^2/g_{\tau\tau}(u)$$

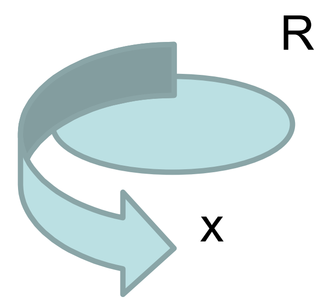
$$\tau = Lk_\tau\int_{u_0}^{u_t}\frac{du}{g_{\tau\tau}}\sqrt{\frac{g_{uu}}{r(u)}}, \quad x = Lk_x\int_{u_0}^{u_t}\frac{du}{g_{xx}}\sqrt{\frac{g_{uu}}{r(u)}}$$

$$u_t = (\omega/\sqrt{k_\tau^2+k_x^2})^{1/\alpha}$$

Bulk-edge correspondence

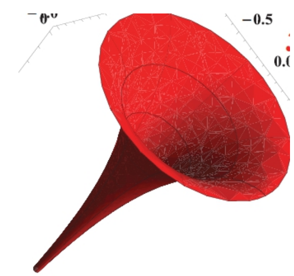
- Flat metric $dl^2_{flat} = dr^2 + r^2 d\phi^2$ $\qquad ds^2 = d\tau^2 + dl^2$

$$S_{flat}(\tau, x) = m\sqrt{\tau^2 + 4R^2 \sin^2(x/2R)} \qquad G(\tau, x) \sim \exp(-S_0(\tau, x))$$



- Surface of rotation $dl^2_{sor} = dr^2[1 + (\frac{\partial h(r)}{\partial r})^2] + r^2 d\phi^2$

$$S_{sor}(\tau, x) = m\sqrt{\tau^2 + (Rx^\eta)^{2/(\eta+1)}}$$



$$h(r) \sim (R/r)^\eta$$

- Boundary propagator: 1d bosonization

$$G^{\pm}_{bos}(\tau, x) \sim \exp[-\int \frac{dk}{2\pi} \frac{2 + U_k}{\epsilon_k} (1 - e^{\pm ikx - \epsilon_k t})]$$

$$\epsilon_k = k\sqrt{1 + U_k} \qquad U(x) \sim 1/x^\sigma$$

- Matching x-asymptotics: $\eta = (1 - \sigma)/(1 + \sigma)$

(time-of-flight, tunneling, noise power spectrum, etc).

Bulk-edge correspondence: more examples

- Generalized Beltrami trumpet: $dl^2_{log} = dr^2 + R^2 \exp(-2(r/R)^\lambda) d\phi^2$

$$dl^2 = d\rho^2/\rho^2 + \rho^2 d\phi^2$$

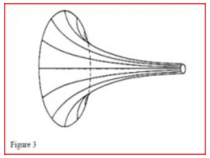


Figure 3

$$S_{log}(\tau, x) = m\sqrt{\tau^2 + R^2(\ln x/a)^{2/\lambda}}$$

Cf., semi-local regime:

$$S_{s-l}(\tau, x) = \sqrt{(1-\nu_0)^2(\ln \tau/a)^2 + m^2x^2} \qquad AdS_2 \times R^d.$$

- $\lambda = 1$ Luttinger: $G(0,x) \sim 1/x^{mR}$

$\lambda = 2/3$ Coulomb interaction in 1d: $G(0,x) \sim \exp(-const \ln^{3/2} x)$

Underlying physics: another manifestation of the **equivalence principle**?

"Curvature in the bulk = Phantom force at the boundary"

String holography meets its optical namesake

String holography meets its optical namesake

- **Artificial metric** in electrically and/or magnetically active media

W.Lu et al,'10,
T.Mackay and A.Lakhtakia,'10

$$\gamma_{ij} = g_{ij}/|g_{\tau\tau}| = \epsilon_{ij}/det\hat{\epsilon} = \mu_{ij}/det\hat{\mu}$$

$$\epsilon_{ij} = \mu_{ij} = \sqrt{-\hat{g}}g_{ij}/|g_{\tau\tau}|$$

$$\frac{\omega^2}{c^2}\vec{D}_\omega = \vec{\nabla}\times\vec{\nabla}\times\vec{E}_\omega \text{ and } \vec{D}_\omega = \overleftrightarrow{\varepsilon}_\omega\vec{E}_\omega$$

$$\frac{\omega^2}{c^2} = \frac{k_z^2}{\varepsilon_1} + \frac{k_x^2 + k_y^2}{\varepsilon_2}$$

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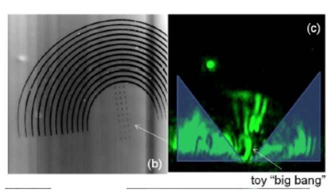
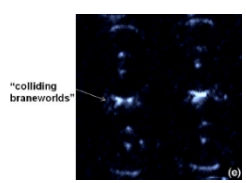
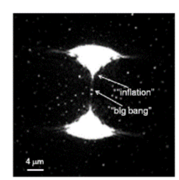
- **Hyperbolic metamaterials**

$$\frac{\omega^2}{c^2} = \frac{k_z^2}{\varepsilon_1} + \frac{k_x^2 + k_y^2}{\varepsilon_2}$$

- Rindler and event horizons, black/white/worm-holes,
- inflation, Big Bang, Rip, and Crunch,
- metric signature transitions, end-of-time, multiverse,…







I.Smolyaninov et al,
1201.5348, 1510.07137

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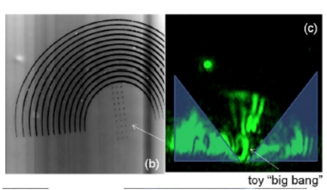
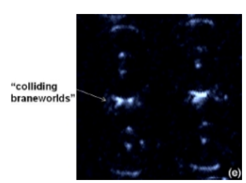
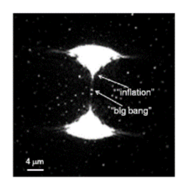






I.Smolyaninov et al,
1201.5348, 1510.07137

- **Analogue holography** DVK 1411.1693

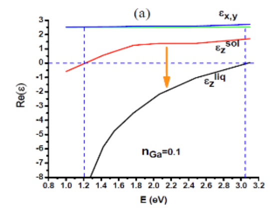
Attainable geometries

- **Dispersion** of extraordinary waves

$$\frac{\omega^2}{c^2}\vec{D}_\omega = \vec{\nabla}\times\vec{\nabla}\times\vec{E}_\omega \quad \text{and} \quad \vec{D}_\omega = \overset{\leftrightarrow}{\varepsilon}_\omega \vec{E}_\omega$$

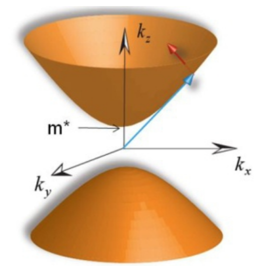
$$\omega^2 = k_z^2/\epsilon_{xy} + k_{xy}^2/\epsilon_{zz}$$

$$ds^2 = -\epsilon_{xy} dz^2 - \epsilon_{zz}(dx^2 + dy^2)$$



I.Smolyaninov, E.Narimanov,'09…

$\epsilon_{xy} > 0$
$\epsilon_{zz} < 0$



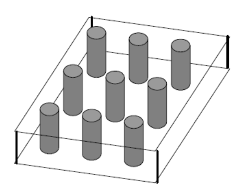
- **Attainable 2+1 geometries**

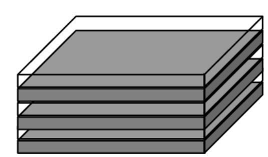
$$ds^2 = \frac{d\tau^2}{u^{2\alpha}} + R^2\frac{du^2}{u^{2\beta}} + \frac{dx^2}{u^{2\gamma}}$$

$$ds^2 = u^{2\theta/d}\left(\frac{d\tau^2}{u^{2\zeta}} + \frac{L^2 du^2 + d\mathrm{x}^2}{u^2}\right)$$

$$\zeta = \frac{1-\beta+\alpha}{1-\beta+\gamma}, \quad \theta = \frac{1-\beta}{1-\beta+\gamma}$$

Hyperscaling-violation metrics





n(x,y) n(z)

Prospective boundary dual

- Fluctuating elastic membrane: $$F = \int d^d\mathbf{x}[\frac{\kappa}{2}(\nabla^2 h)^2 + \mu v_{\alpha\beta}^2 + \frac{\lambda}{2}v_{\alpha\alpha}^2]$$

  (coupled in- and out-of-plane modes) $$v_{\alpha\beta} = \partial_\alpha \xi_\beta + \partial_\beta \xi_\alpha + \partial_\alpha h \partial_\beta h$$

- Effective out-of-plane action: $$\Delta F \sim \int d^d\mathbf{k}\mathbf{k}^{4-\eta}|h_\mathbf{k}|^2$$

- Cf.: boundary theory: $$S_{boundary} = \frac{1}{2\nu}\int d^2\mathbf{k}\mathbf{k}^{2+\theta/\zeta}|\phi_\mathbf{k}|^2$$

- Boundary 'vertex' operators: $$\psi(\mathbf{x}) \sim \exp[i\phi(\mathbf{x})]$$

- Optical field correlations: (speckle interferometry) $$G_\omega(\mathbf{x}) \sim \exp[-\sqrt{c}L\omega|\mathbf{x}/cL|^{\theta/\zeta}]$$

  cf.: $< E_\omega(\mathbf{x})E^*_{-\omega}(0) > \propto \exp(-\omega|\mathbf{x}|)$

- Noise power spectrum and other moments of the boundary field distribution function can be related to the bulk ‘metric’

- Practical realizations: Co nanoparticles in kerosene, PMMA on gold, InGaAs (m)/GaAs(d) ,…

The rise of SYK model

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- spin glasses (Georges/Parcollet/Sachdev ´89; Sachdev/Ye ´92),
- randomized Majoranas (Kitaev ´15),
- toy holography (Sachdev ´15, Maldacena, Stanford, Shenker, Gross, Polchinski, Rosenhaus ´16...)

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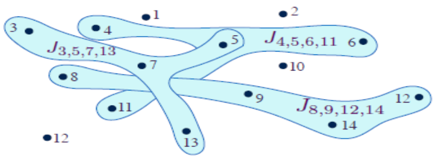
Original, Dirac:

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell$$



S.Sachdev, 1506.05111

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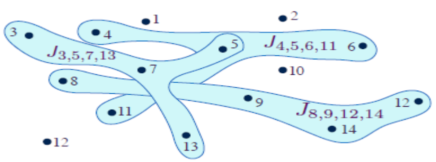

q-Generalized, Majorana: $$H = i^{q/2} \sum_{\alpha_1 \dots \alpha_q}^{N} J^{\alpha_1 \dots \alpha_q} \chi^{\alpha_1} \dots \chi^{\alpha_q}$$

S.Sachdev, 1506.05111

$$S = \sum_i^L \sum_\alpha^N \chi_i^\alpha \partial_\tau \chi_i^\alpha - i^{q/2} \sum_{i_a, \alpha_a} J_{i_1 \dots i_q}^{\alpha_1 \dots \alpha_q} \chi_{i_1}^{\alpha_1} \dots \chi_{i_q}^{\alpha_q}$$

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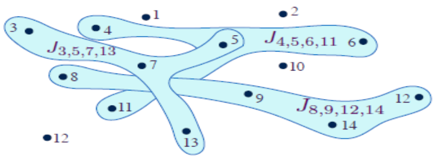

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Disorder averaging: $< J^{\alpha_1 \ldots \alpha_q} J^{\beta_1 \ldots \beta_q} >= \frac{J^2 (q-1)!}{N^{q-1}} \prod_a^q \delta^{\alpha_i \beta_i}$

Spreading SYK-ness: non-random models

-

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- QM of tensors with $(D+1)n^D$ components, vector rep. of $O(n)^D$) :

$$S = \int d\tau \left[\frac{1}{2} \sum_c \left(\sum_{\mathbf{a}^c} \psi^c_{\mathbf{a}^c} \frac{d}{d\tau} \psi^c_{\mathbf{a}^c} \right) - \imath^{(D+1)/2} \frac{J}{n^{\frac{D(D-1)}{4}}} \sum_{\mathbf{a}^0,\dots,\mathbf{a}^D} \psi^0_{\mathbf{a}^0} \dots \psi^D_{\mathbf{a}^D} \prod_{c_1<c_2} \delta_{a^{c_1c_2}a^{c_2c_1}} \right] .$$

Witten '16, Gurau '16...

- Tetrahedron model (D=3):

$$H_1^t = \frac{g}{(N_a N_b N_c)^{1/2}} c^\dagger_{a_1 b_1 c_1} c^\dagger_{a_2 b_2 c_1} c_{a_1 b_2 c_2} c_{a_2 b_1 c_2} .$$

a =1,..N_a; b=1,..N_b, c=1,...N_c , symmetry $U(N_a)$ $U(N_b)$ $O(N_c)$

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$a = 1,..N_a$; $b=1,..N_b$, $c=1,...N_c$, symmetry $U(N_a)\ U(N_b)\ O(N_c)$

- $N_a=N=3$, $N_{b=2=}=M=2$, $N_c=L>>1$ designer unit cell

$$H = \sum_j \sum_{r,r'=-(N-1)/2}^{(N-1)/2} \sum_{\alpha,\beta,\gamma,\sigma=1}^{M} -\frac{g\eta_{r,r'}}{N\sqrt{M}} \times \mathcal{J}_{\alpha\beta}\mathcal{J}_{\gamma\sigma} c^\dagger_{j_x,j_y,\alpha} c^\dagger_{j_x+r,j_y+r',\beta} c_{j_x,j_y+r',\gamma} c_{j_x+r,j_y,\sigma}.($$

$$\begin{aligned} H &= \sum_j H_j, \\ H_j &= U\hat{n}_j^2 + \sum_{\hat{e}=\hat{x},\hat{y}} J\left(\vec{S}_j \cdot \vec{S}_{j+\hat{e}} - \frac{1}{4}\hat{n}_j\hat{n}_{j+\hat{e}}\right) \\ &\quad - K\left(\epsilon_{\alpha\beta}\epsilon_{\gamma\sigma} c^\dagger_{j,\alpha} c^\dagger_{j+\hat{x}+\hat{y},\beta} c_{j+\hat{y},\gamma} c_{j+\hat{x},\sigma} + H.c.\right) \end{aligned}$$

Wu et al, 1802.04293

SYK model: key properties

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- N>>1: **simple diagrammatics** (´melonic´ graphs)

- **Replica-symmetric** (not a spin-glass) mean-field states

- **Reparametrization invariance** $t \rightarrow f(t)$, Liouville quantum mechanics and Schwarzian action for fluctuations about mean-field

- **Maximally chaotic** behavior and fast scrambling (akin to black holes)

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Prospective holographic dual:

- Pure AdS_2 (naïve)

- Dilaton (Jackiw-Teitelboim) gravity in AdS_2 (+ infinite number of massive scalars)?

- AdS_3? (Jevicki et al)

Saddle-point analysis of (generalized) SYK model

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DVK 1905.04381

- **G-Σ functional**:

$$Z = \int DGD\Sigma(Det[F[\partial_\tau] + \Sigma])^N$$

$$\exp(N \int_{\tau_1,\tau_2} G\Sigma - A[G]))$$

$$A = N \sum_k^{\infty} \int_{\tau_1,\dots\tau_k} J_k^2(\tau_1,\dots\tau_k) G^q(\tau_1,\tau_2)\dots G^q(\tau_{k-1},\tau_k)$$

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$$\exp(N\int_{\tau_1,\tau_2} G\Sigma - A[G]))$$

- **Equations of motion**:

$$\int_\tau (F(\partial_\tau)\delta(\tau_1,\tau)+\Sigma(\tau_1,\tau))G(\tau,\tau_2) = \delta(\tau_1-\tau_2)$$

$$\Sigma(\tau_1,\tau_2) = \frac{1}{N}\frac{\delta A}{\delta G(\tau_1,\tau_2)}$$

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Saddle-point analysis of (generalized) SYK model

DVK 1905.04381

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- **Residual invariance**:
  (SL(2,R) for T=0)

$$\tan\frac{\pi f(\tau)}{\beta} \to \frac{a\tan\frac{\pi f(\tau)}{\beta}+b}{c\tan\frac{\pi f(\tau)}{\beta}+d} \qquad \text{ad-bc=1}$$

Holographic matching

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- **Equivalent geometry** (charged BH in $AdS_2 \times R^d$.):

$$ds^2 = \frac{R_2^2}{\zeta^2}\left[-\left(1-\zeta^2/\zeta_0^2\right)dt^2 + \frac{d\zeta^2}{\left(1-\zeta^2/\zeta_0^2\right)}\right]$$

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$$S(\omega,q) = 2\int_{z_0}^{z_t} dz\sqrt{g(z)V(z)} \qquad V(z) = m^2 + \frac{q^2}{h(z)} + \frac{\omega^2}{f(z)}$$

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- **Thermodynamics**: F(T), S(N -> ∞ , T -> 0) > 0

- **Four-point functions** (OTOC): <[O(t,x), O(o,o)]²>, etc.

Beyond mean-field: Schwarzian dynamics

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- **Correction to mean-field propagator**:

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- **Next order** O(T/J) correction: (non-local) $$\delta A \sim \frac{N}{J^2}\int_{\tau_1\tau_2} \frac{(f_1' f_2')^2}{(\delta\tau_{12})^4}\ln\left(\frac{J^2(\delta\tau_{12})^2}{f_1' f_2'}\right)$$

Generalized SYK models

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- **Generalized SYK**: other symmetry breaking terms are possible:

- **Time-dependent** SYK coupling J_2: $$J_k^2(\delta\tau) = \delta_{k,2}\frac{J^{2-2\gamma}}{(\delta\tau)^{2\gamma}}$$

- **Scale-invariant** IR solution with dimension: $$\Delta = \frac{1-\gamma}{2q}$$

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-Reparametrization symmetry is broken s**pontaneously** AND **explicitly**:

$$\delta A = \frac{\Gamma}{J}\int_{\tau_1\tau_2}(\delta\tau_{12})^2\ln(J\delta\tau_{12})G_f^{2q}(\tau_1,\tau_2)Sch\{\tan\frac{\pi f}{\beta},\tau\}$$
$$\approx \frac{\Gamma}{J}\int_{\tau_1\tau_2}\frac{(f_1'-1)(f_2'-1)}{(\delta\tau_{12})^2} \quad ($$

- Cf. Caldeira-Leggett with Ohmic dissipation

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- **Further reduction** (Liouville-like): $H = \frac{1}{2}\pi_\phi^2 + \frac{b}{2}\pi_f^2 e^{2\phi} - ae^\phi\pi_f + \frac{1}{2}c$

$A(\phi) = 2ae^\phi$, $B(\phi) = be^{2\phi}$, and $C(\phi) =$ C

$$ds^2 = d\phi^2 + e^{-2\phi}df^2 \qquad H^2$$

Liouvillian quantum mechanics

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D.Bagrets, A.Altland, A.Kamenev, 1607.00694, 1702.08902

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  (next O(T/J) order)

- **Higher order** functions (e.g., OTOC)

$$< G_f(\tau_1, \tau_2) \ldots G_f(\tau_{2p-1}, \tau_{2p}) >=$$

$$= \int D\phi \prod_{i=1}^{p} \frac{e^{\Delta(\phi(\tau_{2i-1}) + \phi(\tau_{2i}))}}{(\int_{\tau_{2i-1}}^{\tau_{2i}} e^\phi)^{2\Delta}} e^{-\int_\tau L(\phi)}$$

Sick SYK cousin: Morse potential

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- **Quantization**: $$(-\frac{\partial^2}{\partial\phi^2} + \lambda^2(e^{2\phi} - 2e^\phi sgn\mu))\psi = (\epsilon - \lambda^2)\psi$$

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- Quantization: $(-\frac{\partial^2}{\partial\phi^2} + \lambda^2(e^{2\phi} - 2e^\phi sgn\mu))\psi = (\epsilon - \lambda^2)\psi$

- **Eigenstates**: (incl. bound)

$\psi_k \sim e^{-\phi/2} W_{\lambda,ik}(z)$ $\quad z = 2\lambda e^\phi$ $\quad \epsilon_n = -(n - \lambda + 1/2)^2$

$\psi_n(z) \sim z^{\lambda - n - 1/2 - z/2} L_n^{2\lambda - 2n - 1}(z)$ $\quad \lambda = \mu\beta = O(\beta J) \gg 1$

$\Omega \sim \mu\beta/M$

Sick SYK cousin: Morse potential

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- **Density of states**: $$\rho(\epsilon) \approx \frac{\lambda - \frac{1}{2}}{2\pi}\sum_n \delta(E_n - \Omega(n + \frac{1}{2})) \sim M$$

Schwarzian correlations: Liouville vs Morse

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DVK 1905.04381

- **Energy-stress**: $T(\tau) = M(f''' - (2\pi/\beta)^2 f')$

- **Correlation**: $$< T(\tau)T(0) >= M \sum_n \frac{e^{2\pi\tau/\beta}(\omega_n^2 - (2\pi/\beta)^2)\omega_n^2}{\omega_n^2 + \Omega^2 + \Gamma|\omega_n|}$$
$$\sim M max[1/\beta^3, \Omega^3] \sin \Omega\tau e^{-\Gamma\tau/2}$$

- **Energy fluctuations**: $$< (\delta E)^2 >= \frac{\partial^2}{\partial\beta^2} \ln Z(\beta) \sim M max[1/\beta^3, \Omega^3]$$

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$$< G_f(\tau_1, \tau_2)G_f(\tau_3, \tau_4) >\sim 1/t^4$$ cf. $1/t^{3/2}$ and $1/t^6$

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- **Lyapunov exponents**: $$\frac{< G_f(\tau_1, \tau_3)G_f(\tau_2, \tau_4) >}{< G_f(\beta/2, 0) >^2} = 1 - O(\frac{\beta}{M})e^{\lambda_L t}$$

Liouville $\lambda_L = 2\pi/\beta(1 - O(1/\beta J))$

Morse $\lambda_L = \frac{2\pi}{\beta}(1 - O(\alpha))$

Bulk theory: JT gravity and beyond

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- **JT gravity**: $I_{\mathrm{JT}}[g,\Phi] = -\frac{1}{4\pi}\int_D \Phi(R+2)\sqrt{g}\,d^2x - \frac{1}{2\pi}\int_{\partial D}\Phi K\,d\ell$ R=-2

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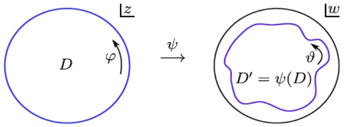
- **Boundary dynamics** only:  $f' = e^{\phi}$

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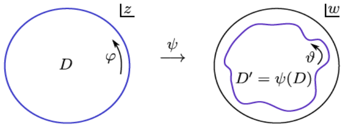

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- '**Particle in magnetic field**' problem (effectively 1D, too):

$$I[X] = \int_0^\beta d\tau\,\left(\frac{1}{2}\,g_{\alpha\beta}\dot{X}^\alpha\dot{X}^\beta - \gamma\omega_\alpha\dot{X}^\alpha\right)$$

$$G(x_1,x_0;\beta) = \int_{\substack{X(0)=x_0\\X(\beta)=x_1}} DX\,e^{-I[X]}.$$

A.Kitaev and J.Suh,
1711.08467, 1808.07032

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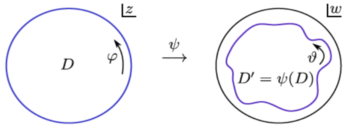

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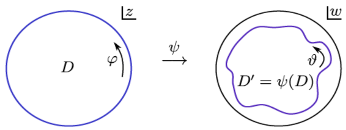

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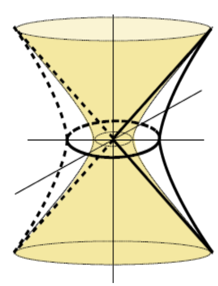
- Other equivalent **AdS_3 sections**?



$$\left(-\frac{\partial^2}{\partial\theta^2} - \frac{k^2+\frac{1}{4}}{\cos^2\theta} + 2\lambda m\tan\theta\right)\chi = (m^2-\lambda^2)\chi$$

Seeking to develop global SYK-ness

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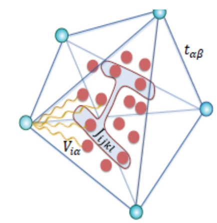
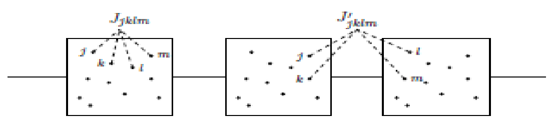
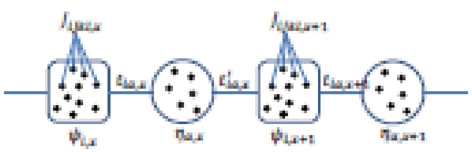


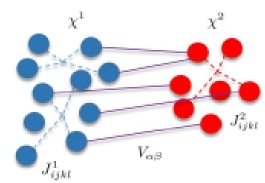
S.Banerjee and E.Altman, 1610.04619



Y.Gu, X.-L.Qi, and D.Stanford, 1609.07832



S.-K.Jian and H.Yao, 1703.02051



X.Chen et al, 1705.03406

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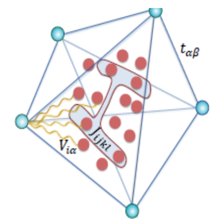

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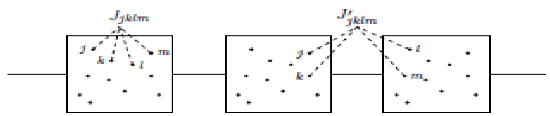

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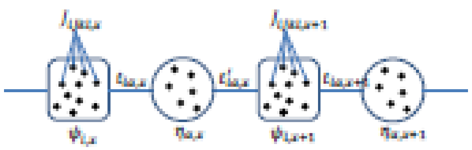

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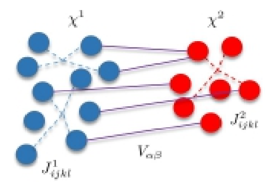

X.Chen et al, 1705.03406

- **Diffusive** energy transport, hydrodynamic **universal** bound

$D \sim v_b^2 t_L$ (Sachdev, Hartnoll, Lucas,...)

- Despite (**postulated**) single-particle ultra-locality: $G_{ij}(\tau) \sim \frac{sgn\tau}{|\tau|^{2/q}}\delta_{ij}$

Thickening and sickening the SYK model

Thickening and sickening the SYK model

DVK, 1705.03956,1805.00870

$$S = \sum_{i}^{L} \sum_{\alpha}^{N} \chi_i^\alpha \partial_\tau \chi_i^\alpha - i^{q/2} \sum_{i_a, \alpha_a} J_{i_1 \dots i_q}^{\alpha_1 \dots \alpha_q} \chi_{i_1}^{\alpha_1} \dots \chi_{i_q}^{\alpha_q}$$

Time-dependent and/or non-local disorder correlations:

$$< J_{i_1 \dots i_q}^{\alpha_1 \dots \alpha_q}(\tau_1) J_{j_1, \dots j_q}^{\beta_1 \dots \beta_q}(\tau_2) >= \frac{F_{i_1 \dots i_q j_1, \dots j_q}(\tau_{12})(q-1)!}{N^{q-1}} \prod_a^q \delta^{\alpha_a \beta_a}$$

- standard SYK on NL sites: $F_{i_1 \dots i_q j_1, \dots j_q}(\tau_{12}) = J^2 \prod_a^q \delta_{i_a j_a}$
- L copies of N-site SYK: $F_{i_1 \dots i_q j_1, \dots j_q}(\tau_{12}) = J^2 \prod_a^q \delta_{i_a j_a} \prod_a^{q-1} \delta_{i_a i_a}$

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- **Algebraic** space and/or time correlations:

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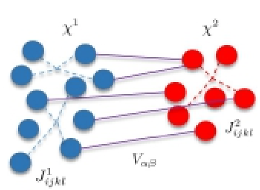
$$J^2_{ij}(\tau) \sim \tau^{-2\alpha}$$

$$J^2_{ij}(\tau) \sim |i-j|^{-2\beta}$$

$$J^2_{ij}(\tau) \sim (\tau^2 + a^2|i-j|^2)^{-\gamma}$$



Mean-field analysis

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DVK, 1705.03956,1805.00870

- **Partition function**:

$$Z = \int DG_{ij}(\tau) D\Sigma_{ij}(\tau) Pf(\partial_\tau - \Sigma)$$

$$\exp(N \sum_{ij} \int_{\tau_1,\tau_2} (G_{ij}(\tau_{12})\Sigma_{ij}(\tau_{12}) - \frac{1}{q} J^2{}_{ij}(\tau_{12}) G^q_{ij}(\tau_{12})))$$

- **Saddle-point equation**: $\sum_j \int_{\tau_3} (\delta_{ij}\partial_{\tau_1}\delta(\tau_{13}) - \Sigma_{ij}(\tau_{13})) G_{jk}(\tau_{32}) = \delta_{ik}\delta(\tau_{12})$

$$\Sigma_{ij}(\tau_{12}) = J^2_{ij}(\tau_{12}) G^{q-1}_{ij}(\tau_{12})$$

- **Asymptotic** IR regime: $\int_{\tau_3,\mathbf{x}_3} G_{\mathbf{x}_{13}}(\tau_{13}) J^2_{\mathbf{x}_{32}}(\tau_{32}) G^{q-1}_{\mathbf{x}_{32}}(\tau_{32}) = \delta(\tau_{12})\delta(\mathbf{x}_{12})$

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- **Scaling-invariant** IR behavior for: $\frac{d}{z}(q-2) + 2[J] - 2 < 0$

z= ∞ or **q**=2: holds for **any** [J]<1
z=1 and **q**>2: **only** for **d**=1 and [J]=0
Generic **z**>1, **q**>2, **d**>0 : [J]<0 **(non-unitary?)**

Mean-field solutions

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DVK, 1705.03956,1805.00870

- **Ultra-local** (original SYK): $G_{ij}(\tau) \sim \frac{sgn\tau}{|\tau|^{2/q}}\delta_{ij}$ (Hartree-type contractions)

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$$J_{ij}^2(\tau) \sim \tau^{-2\alpha} \quad |i-j|^{-2\beta}$$

$$G(\tau, \mathrm{x}) \sim \frac{sgn\tau}{\tau^{2\Delta_\tau}} \frac{1}{|\mathrm{x}|^{2\Delta_x}} \qquad \Delta_\tau = (1-\alpha)/q \qquad \Delta_x = (d-\beta)/q$$

$$G(\omega, \mathrm{k}) \sim |\omega|^{2\Delta_\tau - 1} \mathrm{k}^{2\Delta_x - d}$$

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- **Lorentz-invariant**: $J^2_{ij}(\tau) \sim (\tau^2 + a^2|i-j|^2)^{-\gamma}$

$$G(\tau, \mathrm{x}) \sim \frac{sgn\tau}{(\tau^2 + \mathrm{x}^2)^{\Delta}} \qquad \Delta = (D-\gamma)/q$$

D=d+1

$$G(\omega, \mathrm{k}) \sim (\omega^2 + \mathrm{k}^2)^{\Delta - D/2}$$

(bosonic case: Patashinsky, Pokrovsky ´64)

- Other?

Fluctuations about mean-field

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DVK, 1705.03956

- **Reparametrization invariance** (for α=β=γ=0 only):

$$G(x_1, x_2) \to |g(x_1)g(x_2)|^{D/2q} G(f(x_1), f(x_2)) \qquad g = |det \partial f^\mu / \partial x^\nu|^2$$

μ,ν = 1,.., D=d+1

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DVK, 1705.03956

- Reparametrization invariance (for α=β=γ=0 only):

$$G(x_1, x_2) \to |g(x_1)g(x_2)|^{D/2q} G(f(x_1), f(x_2))$$

$$g = |det\partial f^\mu / \partial x^\nu|^2$$

μ,ν = 1,.., D=d+1

- **Original** SYK (d=0): Schwarzian

$$S(f) = \frac{N}{J}\int_\tau \{f, x\} = \frac{N}{J}\int_\tau (\frac{f'''}{f'} - \frac{3}{2}(\frac{f''}{f'})^2)$$

-Generalized SYK (d>0):
non-local action

$$\delta S(f) = \frac{N}{2}\int_k (k_\mu f^\mu)^2 (C\omega + \omega^2/J)|\mathbf{k}|^d$$

$$C = O(\alpha) + O(\beta)$$

-Stress-energy correlations:
no diffusive pole

$$< T_{\mu\nu}(\omega, \mathbf{k}) T_{\mu\nu}(-\omega, -\mathbf{k}) >= \frac{i\omega|\mathbf{k}|^d (C + i\omega/J)^2}{C + (i\omega + D_\epsilon \mathbf{k}^2)/J}$$

2-body problem

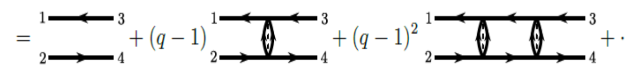
DVK, 1705.03956

- **Fluctuations**: $G = G_0 + g|G_0|^{(2-q)/2} \qquad \Sigma = \Sigma_0 + \sigma|G_0|^{(q-2)/2}$

$$\delta S(g,\sigma) = N \int \left(g_{12}\sigma_{12} - \frac{q-1}{2} F_{12} g_{12}^2 - \frac{\sigma_{12}\hat{K}_{12,34}\sigma_{34}}{2(q-1)}\right)$$

$$\delta S(g) = \frac{N(q-1)}{2} \int g_{12}(\hat{K}^{-1}_{12,34} - \hat{1}_{13}\hat{1}_{24}F_{12}) g_{34}$$

- **Quadratic kernel**: $\hat{K}_{12,34} = (q-1) G_{13} G_{24} |G_{34}|^{q-2}$



- **Diagonalization**: $\displaystyle\int_{x_3,x_4} \hat{K}_{12,34} F_{34} \Psi_{34}(h|\omega,\mathbf{k}) = \lambda_h(\omega,\mathbf{k}) \Psi_{12}(h|\omega,\mathbf{k})$

- **Eigenstates** (spin-zero): $\Psi_{12}(h|\omega,\mathbf{k}) \sim |x_{12}|^{h-2\Delta} e^{ik_\mu(x_1^\mu + x_2^\mu)/2}$

- **Eigenvalue** equation:
(Lorentz-invariant)

$$\frac{\lambda_h}{x_{12}^{2\Delta-h}} = \int_{x_3,x_4} \frac{1}{x_{13}^{2\Delta} x_{24}^{2\Delta} x_{34}^{2D-2\Delta-h}} \qquad \lambda_h = \lambda_h(0,0)$$

$$(1-q)\frac{\Gamma(D-\Delta)\Gamma(\frac{D}{2}-\Delta)\Gamma(-\frac{D}{2}+\Delta+\frac{h}{2})\Gamma(\Delta-\frac{h}{2})}{\Gamma(-\frac{D}{2}+\Delta)\Gamma(\Delta)\Gamma(D-\Delta-\frac{h}{2})\Gamma(\frac{D}{2}-\Delta+\frac{h}{2})} = 1$$

Bosonic SYK: $$(1-q)\frac{\Gamma(D-\Delta)\Gamma(\frac{D}{2}-\Delta)\Gamma(-\frac{D}{2}+\Delta+\frac{h}{2})\Gamma(\Delta-\frac{h}{2})}{\Gamma(-\frac{D}{2}+\Delta)\Gamma(\Delta)\Gamma(D-\Delta-\frac{h}{2})\Gamma(\frac{D}{2}-\Delta+\frac{h}{2})} = 1$$

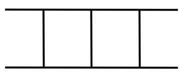
No solutions for **h**=2, D or D+1 (stress-energy operator)
Prospective dual is **not dominated** by gravity?

OTOC functions and chaos

DVK, 1705.03956

- **Generic 2-body** amplitude: $\mathcal{F}_{12,34} = < \chi_i^\alpha(\tau_1)\chi_j^\beta(\tau_2)\chi_k^\gamma(\tau_3)\chi_l^\delta(\tau_4) >$

-**Expansion** over eigenstates: $\mathcal{F}_{12,34} = \frac{1}{1-\hat{K}}\mathcal{F}^0_{12,34} = \sum_\lambda \Psi_{12}\frac{1}{1-\lambda} < \Psi_{34}|\mathcal{F}^{(0)} >$



$$\mathcal{F}^{(0)}_{12,34} = G_{13}G_{24} - G_{14}G_{23}$$

- **Finite temperature** basis: $\Psi_{12}(h|\mathrm{k}) \sim \frac{e^{i\mathrm{k}(\mathrm{x}_1+\mathrm{x}_2)/2-\pi T h(\tau_1+\tau_2)}}{\cosh(\pi T\tau_{12})^{2\Delta_\tau - h}|\mathrm{x}_1-\mathrm{x}_2|^{2\Delta_x - h}}$

- **OTOC** functions: $\mathcal{F}(\tau,\mathrm{x}) = < u\chi_\mathrm{x}^\alpha(\tau)u\chi_0^\beta(0)u\chi_\mathrm{x}^\alpha(\tau)u\chi_0^\beta(0) >$ $\quad u = e^{-H/4T}$

(Larkin and Ovchinnikov ´69),

-**Chaos** spreading: $\mathcal{F}(\tau,\mathrm{x}) \sim 1 - \frac{1}{N}e^{\lambda_L(\tau-|\mathrm{x}|/v_B)}$

- **Lyapunov** index: $\lambda_L = -2\pi h T$ where **h** solves $\frac{\Gamma(3-2\Delta_\tau)\Gamma(2\Delta_\tau - h)}{\Gamma(1+2\Delta_\tau)\Gamma(2-2\Delta_\tau-h)} = 1$
(ladder equation)

- Original SYK (**d**=0): h = -1 (maximal chaos)
- For **d** > 0 and/or **α, β, γ** $\neq$0: h > -1 (**no chaotic** bound saturation)

New horizons

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- **Resonant** SYK model in momentum space

$$H_k = \int_k \sum_\alpha \epsilon_k c^\dagger_{k\alpha} c_{k\alpha},$$

$$H_U = \frac{1}{(2N)^{3/2}} \sum_{\alpha_a} \int_{k_a} U_{\alpha_a}(k_a) c^\dagger_{k_1\alpha_1} c^\dagger_{k_2\alpha_2} c_{k_3\alpha_3} c_{k_4\alpha_4}$$

$$\mathcal{K}(k_a, k'_{a'}) = \mathcal{K}_0(k_a, k'_{a'}) \, \frac{1}{2} \Big[\mathcal{K}_1(k_a)\delta(\epsilon_{k_1} + \epsilon_{k_2} - \epsilon_{k_3} - \epsilon_{k_4})$$

A. Patel and S.Sachdev, 1906.03265

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$$\rho = \frac{m^*}{ne^2} \frac{1}{\tau} \qquad \frac{1}{\tau} = \alpha \frac{k_B T}{\hbar}$$

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Material		n (10^{27} m^{-3})	m^* (m_0)	A_1 / d (Ω / K)	$h / (2e^2 T_F)$ (Ω / K)	α
Bi2212	$p = 0.23$	6.8	8.4 ± 1.6	8.0 ± 0.9	7.4 ± 1.4	1.1 ± 0.3
Bi2201	$p \sim 0.4$	3.5	7 ± 1.5	8 ± 2	8 ± 2	1.0 ± 0.4
LSCO	$p = 0.26$	7.8	9.8 ± 1.7	8.2 ± 1.0	8.9 ± 1.8	0.9 ± 0.3
Nd-LSCO	$p = 0.24$	7.9	12 ± 4	7.4 ± 0.8	10.6 ± 3.7	0.7 ± 0.4
PCCO	$x = 0.17$	8.8	2.4 ± 0.1	1.7 ± 0.3	2.1 ± 0.1	0.8 ± 0.2
LCCO	$x = 0.15$	9.0	3.0 ± 0.3	3.0 ± 0.45	2.6 ± 0.3	1.2 ± 0.3
TMTSF	$P = 11$ kbar	1.4	1.15 ± 0.2	2.8 ± 0.3	2.8 ± 0.4	1.0 ± 0.3

SYK = 1

S.Sachdev, Montreal, July '19

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- While not providing water-proof examples of **genuine holographic correspondence**, the d=0 SYK-like models offer an important insight into the properties of a whole sequence of the SL(2,R)-symmetric QM systems and their JT-like (effectively 1D) 'bulk' duals.

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- While not providing water-proof examples of genuine holographic correspondence, the d=0 SYK-like models offer an important insight into the properties of a whole sequence of the SL(2,R)-symmetric QM systems and their JT-like (effectively 1D) 'bulk' duals.
- **Higher-dimensional** ´thickening' tends to ´sicken´ the salient SYK behavior. Still, the d>0 - dimensional SYK-like models can be viewed as interesting examples of soluble (super)strongly-interacting many-body systems with markedly NFL properties.