Holography for Black Hole Microstates

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Outline

- A quick overview of black hole paradoxes:
  - the entropy problem
  - the Strominger-Vafa “solution”
- Holography for BPS black hole microstates:
  - review of the D1-D5 CFT
  - gravity-CFT map for D1-D5 geometries
  - a class of D1-D5-P microstates
- Supergravity construction of D1-D5-P microstates:
  - superstrata
- Geometry from CFT:
  - 1-point functions
  - entanglement entropy

The information paradox

- Hawking: classical horizons coupled to quantum matter emit particle pairs in an entangled state
- When the black hole has completely evaporated the outside radiation is entangled with nothing
  $\Rightarrow$ one cannot associate to it a definite quantum state
- Mathur, AMPS: to restore unitarity one has to either
  - assume remnants
  - modify the classical horizon (fuzzballs, firewalls) $\leftarrow$
  - introduce non-localities (ER=EPR, Papadodimas-Raju)

The entropy problem

- One aspect of the information paradox survives in the susy limit:
  - the final radiation is described by a mixed state $\Rightarrow$ entropy
  - this is the Bekenstein-Hawking entropy

$$S_{BH} = \frac{A_H}{4\,G}$$

  - S_{BH} can be non-vanishing also for BPS black holes
- The usual statistical interpretation of entropy should imply

$$S_{BH} \stackrel{?}{=} \log(\#\text{microstates})$$

- "No-hair" theorem: for fixed mass and charges there is a unique classical geometry with a regular horizon $\Rightarrow$

$$\#\,(\text{classical microstates}) = 1$$

Strominger-Vafa counting

- In string theory the black hole is described by a bound state of N D-branes ($N \gg 1$)
- Example:

$$\text{D1-D5-P on } \mathbb{R}^{4,1} \times S^1 \times T^4$$

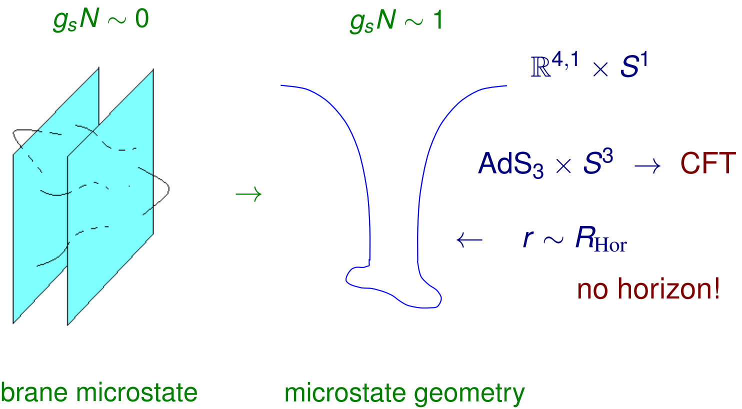
- At small gravitational coupling ($g_s \to 0$) the bound state of D-branes is described by a CFT
- Microstates of the CFT can be counted

$$\log(\#\text{microstates}) = 2\pi\sqrt{n_1 n_5 n_p} = S_{BH}$$

What happens to the microstates at finite gravitational coupling ($g_s N \sim 1$)?

Microstate geometries

- For finite $g_s N$, D-branes backreact on spacetime
- For particular microstates (coherent states), the backreaction is well described by supergravity

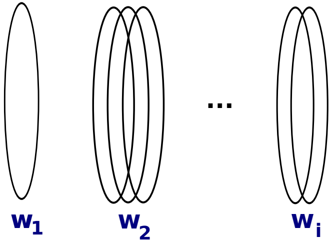


The D1-D5 CFT

- At a special point in moduli space, the low energy limit of the D1-D5 system is described by the

  $$(T^4)^N/S_N \text{ orbifold with (4,4) susy}$$

  where $N = n_1 n_5$
- The orbifold Hilbert space includes twist sectors:
  $\phi_i(y + 2\pi R) = \phi_{i+1}(y)\,, \quad i = 1, \ldots, w - 1$

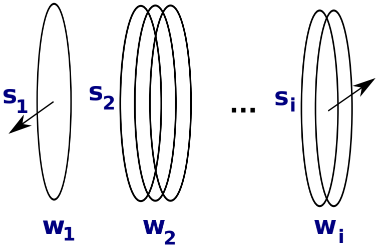


$\sum_i N_i\, w_i = N$

N_i # strands of winding w_i

Ramond ground states $\Leftrightarrow$ D1-D5 states

- The CFT contains fermions $\psi^{\alpha\dot{A}}$, $\tilde{\psi}^{\dot{\alpha}\dot{A}}$ $(\alpha\,,\,\dot{\alpha}\,,\,\dot{A}\,=1,2)$
- Fermions transform under R-symmetry $SU(2)_\alpha \times SU(2)_{\dot{\alpha}} \sim SO(4)$ $\Rightarrow \mathbb{R}^4$ rotations
- Black hole states are in the RR sector $\Rightarrow$ fermion zero-modes
- We restrict to ground states which are bosonic and T^4-invariant $\Rightarrow$ 5 "spin" states: $s = (0,0)\,,\,(\pm 1/2, \pm 1/2)$



$\Leftrightarrow \quad \prod_i \left(|s_i\rangle_{w_i}\right)^{N_i}$

D1-D5 geometries (Lunin, Mathur; Kanitscheider, Skenderis, Taylor)

- D1-D5 geometries are specified by a profile: $g^A(v)\,,\ A = 1, \ldots, 5$
- The components of g^A map to the different spin states:

$$g^1 + i\,g^2 \leftrightarrow (+,+)\,,\,(-,-)$$
$$g^3 + i\,g^4 \leftrightarrow (+,-)\,,\,(-,+)$$
$$g^5 \leftrightarrow (0,0)$$

- The harmonic modes of g^A map to the winding numbers:

$$g^1 + i\,g^2 = \sum_{w\geq 1} \frac{1}{w}\,(a_w^{(+,+)}\,e^{\frac{2\pi\,i\,w}{L}v} + a_w^{(-,-)}\,e^{-\frac{2\pi\,i\,w}{L}v})\,,\ldots$$

- Gravity-CFT map:

$$\boxed{\{a_{w_i}^{(s_i)}\} \quad \Leftrightarrow \quad {\sum}'_{\{N_i\}} \prod_i (a_{w_i}^{(s_i)})^{N_i}\,(|s_i\rangle_{w_i})^{N_i}}$$

where the sum ${\sum}'_{\{N_i\}}$ is restricted to $\sum_i N_i\, w_i = N$

Coherent states and supergravity

- The sum $\sum'_{\{N_i\}} \prod_i (a_{w_i}^{(s_i)})^{N_i} (|s_i\rangle_{w_i})^{N_i}$ is peaked on

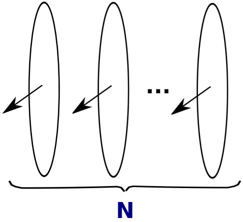
$$w_i \overline{N}_i \propto |a_{w_i}^{(s_i)}|^2 \quad \text{for} \quad \overline{N}_i \gg 1$$

- $\overline{N}_i$ is the average value of strands of winding w_i with “spin” s_i
- When $\overline{N}_i \gg 1\ \forall i$ the state is well described by supergravity
- The sum over N_i has a non-trivial spread (unless all strands are equal, i.e. $N_i = 0$ for $i > 1$)
- Thus generic microstates are not eigenstates of R-charge and the dual geometries are not rotationally invariant

Examples I

- The simplest D1-D5 state is

$$|+,+\rangle_1^N \quad \leftrightarrow$$



- Spectral flow maps this state to the SL(2,$\mathbb{C}$) invariant vacuum
- On the gravity side spectral flow is a change of coordinates mixing AdS_3 (t, y) and S^3 (ϕ, ψ)

$$\phi \to \phi + \frac{t}{R} \quad , \quad \psi \to \psi + \frac{y}{R}$$

- This change of coordinates maps the geometry to

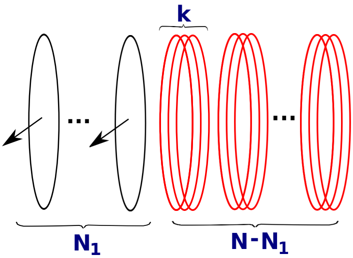
$$AdS_3 \times S^3 \times T^4$$

Examples II

- A non-trivial D1-D5 state is

$$\sum_{N_1} a^{N_1} b^{N-N_1} \left(|+,+\rangle_1\right)^{N_1} \left(|0,0\rangle_k\right)^{N-N_1}$$

$$\updownarrow$$



$$\overline{N}_1 \propto |a|^2$$

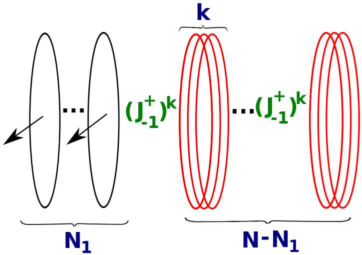
$$k(N-\overline{N}_1) \propto |b|^2$$

- The dual geometry is a deformation of $AdS_3 \times S^3 \times T^4$
- The deformation is controlled by one scalar warp factor

$$Z_4 = b\,R\left(\frac{a}{\sqrt{r^2+a^2}}\right)^k \frac{\sin^k\theta}{r^2+a^2\cos^2\theta}\cos k\phi$$

Adding momentum

- The D1-D5 black hole has vanishing horizon area in classical supergravity
- The simplest BPS black hole with $S_{BH} > 0$ classically is the D1-D5-P hole
- The simplest way to add momentum is to act on a D1-D5 state by the CFT chiral algebra: L_{-n}, J^{α}_{-n}. Example: act with $e^{\chi J^+_{-1}}$ $(\chi = \frac{\pi}{2})$



Note:

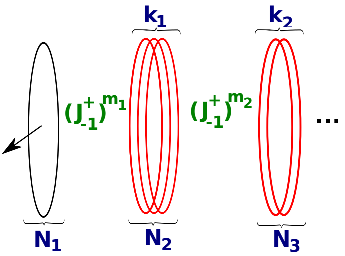
$J^+_{-1}|+,+\rangle_1 = 0$

$(J^+_{-1})^m|0,0\rangle_k = 0 \text{ for } m > k$

- This is a descendant of a D1-D5 state: its geometry is obtained from D1-D5 by a diffeomorphism nontrivial at the AdS boundary

A class of D1-D5-P states

- One can consider a more general class of states



where:

$m_i \leq k_i$

- Generically these states are not descendants of D1-D5 states
- They are not eigenstates of angular momentum nor of momentum $\Rightarrow$ the dual geometries depend on $\phi\,,\psi$ and $v = t + y$

How to construct the dual geometries?

General susy ansatz

- The most general geometry preserving the same supercharges as the D1-D5-P black hole and T^4-invariant is

$$ds_6^2 = -\frac{2}{\sqrt{\mathcal{P}}}(dv+\beta)\Big(du+\omega+\frac{\mathcal{F}}{2}(dv+\beta)\Big)+\sqrt{\mathcal{P}}ds_4^2\,,\ \mathcal{P}=Z_1Z_2-Z_4^2$$

where $v=\frac{t+y}{\sqrt{2}}$, $u=\frac{t-y}{\sqrt{2}}$

- It is encoded by

0) ds_4^2 (4D euclidean metric), β (1-form in 4D)

1) Z_1 , Z_2 , Z_4 (0-forms)

2) ω (1-form in 4D), $\mathcal{F}$ (0-form)

- Susy implies that u is an isometry. Everything depends on v , x^i

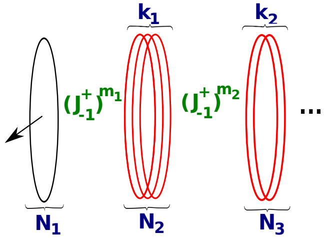
Almost linear structure

0) The sugra equations for ds_4^2 , β are non-linear (they define an "almost hyperkahler" structure)
1) Assuming ds_4^2 , β, the equations for Z_1 , Z_2 , Z_4 are linear and homogeneous
2) The equations for ω , $\mathcal{F}$ are linear and inhomogeneous: the sources are quadratic in Z_i's

Strategy: given ds_4^2 , β, first solve 1) and then solve 2)

A class of D1-D5-P geometries

- Remember we look for the geometry dual to



- Conjecture:
  - strands of type $(J^+_{-1})^m|0,0\rangle_k$ do not affect ds^2_4 , β
  - strands of type $(J^+_{-1})^m|0,0\rangle_k$ contribute linearly to Z_4
    $\Rightarrow Z_4 = \sum_{k,m} b_{k,m} Z_4^{(k,m)}$
    with $b^2_{k,m} \propto \overline{N}_{k,m}$ # strands of type $(J^+_{-1})^m|0,0\rangle_k$

Superstrata

- The contribution $Z_4^{(k,m)}$ of the strands $(J_{-1}^+)^m|0,0\rangle_k$ can be inferred from the change of coordinates corresponding to $e^{\chi J_{-1}^+}$
- Given Z_i's solve the sugra eqs. for ω , $\mathcal{F}$
- Regularity: ω is singular unless one includes in Z_1 terms quadratic in $b_{k,m}$
- Result: for any $\{b_{k,m}\}$ there is a unique regular geometry
- $b_{k,m} \leftrightarrow$ Fourier coefficients of an arbitrary function of two variables

D1-D5-P microstates depend at least on functions of 2 variables
$\Rightarrow$ supestrata

Holographic 1-point functions

- Holography relates terms of order r^{-2-d} in the asymptotic expansion of the geometry towards the AdS boundary with vevs of dimension d operators in the microstate
- The vevs of chiral primary operators (and their descendants) in 1/4 and 1/8 BPS states are protected
- Examples: operators of dimension 1

(Kanitscheider, Skenderis, Taylor)

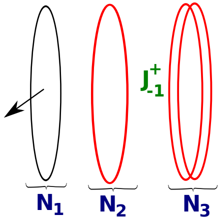
  - $Z_4 \sim \frac{\langle O^{\alpha\dot\alpha}\rangle Y^1_{\alpha\dot\alpha}}{r^3}$ with $O^{\alpha\dot\alpha} = \epsilon_{\dot A\dot B}\,\psi^{\alpha\dot A}\,\tilde\psi^{\dot\alpha\dot B}$
  - $Z_1 \sim \frac{\langle \Sigma_2^{\alpha\dot\alpha}\rangle Y^1_{\alpha\dot\alpha}}{r^3}$ with $\Sigma_2^{\alpha\dot\alpha}$ the twist field with spin $(\pm 1/2, \pm 1/2)$

($Y^1_{\alpha\dot\alpha}$: S^3 scalar spherical harmonics of order 1)

A D1-D5-P example

- $|s\rangle = \sum'_{\{N_i\}} a^{N_1}\, b_1^{N_2}\, b_2^{N_3}\, (|+,+\rangle_1)^{N_1} (|0,0\rangle_1)^{N_2}\, (J^+_{-1}|0,0\rangle_2)^{N_3}$

$\Updownarrow$



- $O^{--}|+,+\rangle_1 = |0,0\rangle_1 \Rightarrow \langle s|O^{--}|s\rangle \propto a\, b_1 \leftrightarrow Z_4$
- $\Sigma_2^{--}(|+,+\rangle_1 \otimes |0,0\rangle_1) = |0,0\rangle_2 \Rightarrow \langle s|\Sigma_2^{--}|s\rangle \propto e^{i\,v}\, a\, b_1\, b_2 \leftrightarrow Z_1$
- Gravity and CFT match!

  Note: the Z_1 coefficient is fixed by regularity on the gravity side

Entanglement entropy

- Consider the EE of one interval of length l in the state $|s\rangle$: $S_l^{(s)}$
- CFT: in the limit of $l \to 0$, $S_l^{(s)}$ is encoded by the vevs $\langle s|\mathcal{O}_K|s\rangle$

$$S_l^{(s)} = -\frac{\partial S_n^{(s)}}{\partial n}|_{n=1}\,,\; S_n^{(s)} = \left(\frac{l}{R}\right)^{-4\Delta_n}\left[1 + \sum_K \left(\frac{l}{R}\right)^{\Delta_K+\bar{\Delta}_K} \mathcal{C}_K \,\langle s|\mathcal{O}_K|s\rangle\right]$$

- Gravity: $S_l^{(s)}$ is given by the area of a minimal co-dimension 2 surface in the 6D geometry (Ryu, Takayanagi; Hubeny, Rangamani)

$$S_l^{(s)} = \frac{\mathrm{area}(\gamma_l)}{4G_N}$$

- $S_l^{(s)}$ is not protected, but if one includes only chiral primary $\mathcal{O}_K$ $\Rightarrow$ gravity and CFT match!

Summary

- We have constructed a family of regular and horizonless D1-D5-P geometries
- We have identified their CFT dual states
- We have checked the gravity-CFT map by computing 1-point functions and entanglement entropy

Note:

- In our states, we act on $|0,0\rangle_k$ strands with powers of J^+_{-1}
- In the NS sector: $J^+_{-1} \to J^+_0$
- One could easily extend the construction to $L_{-1} - J^3_{-1} \to L_{-1}$
- We have the full "graviton gas" (de Boer)

Outlook

- Can one extend the construction to the full chiral algebra $(J^{\alpha}_{-n}, L_{-n})$, $n > 1$?
- Can one include fractional modes $(J^{\alpha}_{-n/w}, L_{-n/w})$?
- If so, one could produce an entropy which scales like $(n_1 n_5 n_p)^{1/2}$
- How well can one resolve typical states in supergravity?
- Need to know the vevs of operators of high enough dimension
- What can one say about non-BPS microstates?