Behind the geon horizon

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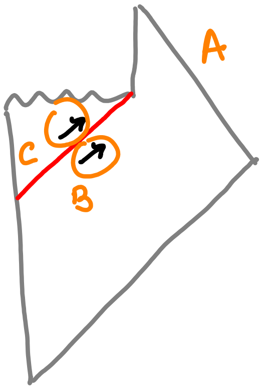
· based on 1412.1084, w/ Simon Ross

Motivation

- Black hole information paradox Mathur '09, AMPS '12



- recovery of information $\Rightarrow$ $S_{AB} < S_A$
- smoothness of horizon $\Rightarrow$ $S_{BC} \simeq 0$
- strong subadditivity $S_A + S_C \leq S_{AB} + S_{BC}$

$\downarrow$

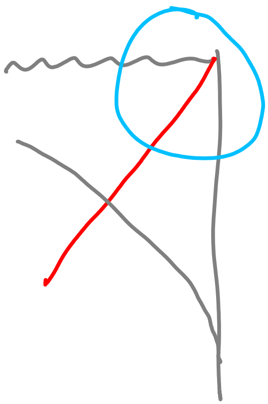
contradiction !

- possible way out : $C \subset A$ $\rightarrow$ black hole complementarity

$\rightarrow$ How is the black hole interior encoded outside ?

The black hole interior in AdS/CFT

- large black hole in AdS w/ a single exterior



- dual to pure state $|\psi\rangle \subset$ CFT thermalises
- i.e. when probed by a small algebra of observables $\mathcal{O}_i$

$$\langle \psi | \mathcal{O}_i \ldots | \psi \rangle = \mathrm{Tr}(\rho_{th} \mathcal{O}_i \ldots) + \mathcal{O}(e^{-S})$$

- Papadodimas - Raju (PR) → quantitative proposal for reconstructing the b.h. interior
- this talk → concrete example and check of the PR construction ($\mathbb{R}P^2$ geon black hole)

Plan

- review : reconstruction of bulk from the boundary
    - the PR proposal
    - the $\mathbb{RP}^2$ geon & properties
- construction of mirror operators
- modifications of the geon state
- future directions

Reconstructing the black hole interior

Reconstructing the bulk from the boundary in AdS/CFT

- CFT → large N, few operators of low dimension
- correlation functions of single-trace operators factorize

$$\langle \mathcal{O}\,\mathcal{O}\,\mathcal{O}\,\mathcal{O} \rangle = \langle \mathcal{O}\mathcal{O} \rangle \langle \mathcal{O}\mathcal{O} \rangle + \text{perm.} + \mathcal{O}(1/N)$$

- generalized free field operators → free scalar in AdS

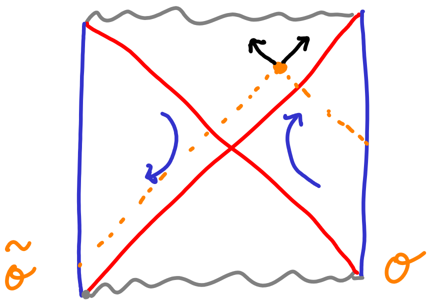
$$(\Box_{AdS} - m^2)\Phi = 0$$

$$\Phi(z, x^\mu) = \int d^d x' \, K(z, x; x') \, \mathcal{O}(x')$$

- reproduces local EFT in the bulk, pert. in $1/N$, around vacuum + few excitations

- breaks down if we compute very "long" correlators

Reconstructing the black hole interior : eternal b.h



- entangled state in 2 copies of the CFT

$$\psi_{tfd} = \sum_i e^{-\frac{\beta E_i}{2}} |E_i\rangle |\tilde{E}_i\rangle$$

- $[O, \tilde{O}] = 0$ $\langle O O \dots \rangle$ - thermal

- $\langle \psi_{tfd} | O(t_1, x_1) \dots \tilde{O}(t_n, x_n) \dots | \psi_{tfd} \rangle = Z_\beta^{-1} \mathrm{Tr}\left[e^{-\beta H} O(t_1, x_1) \dots O(t_n + \frac{i\beta}{2}, x_n) \dots \right]$

$$\tilde{O} | \psi_{tfd} \rangle = e^{-\frac{\beta H}{2}} O^{+} e^{\frac{\beta H}{2}} | \psi_{tfd} \rangle$$

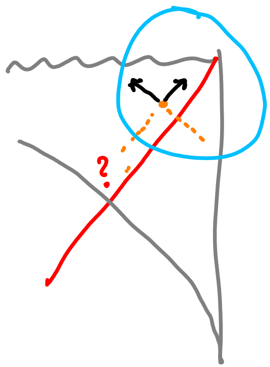
- bulk field in interior

$$\Phi(t, \vec{x}, r) = \int_{\omega > 0} d\omega\, d^{d-1}k \left[O_{\omega k} K^{(1)}_{\omega k}(t, \vec{x}, r) + \tilde{O}_{\omega k} K^{(2)}_{\omega k}(t, \vec{x}, r) + h.c. \right]$$

BTZ

- reproduces local EFT in the eternal black hole

Reconstruction of the black hole interior – single-sided b.h



- $|\psi\rangle$ – pure state that thermalizes
- need right-moving modes!
- PR proposal:

same CFT!

$$\Phi(t,\vec{x},r) = \int_{\omega>0} d\omega\, d^{d-1}k \left[\mathcal{O}_{\omega k}\, K^{(1)}_{\text{eternal}}(t,\vec{x},r) + \tilde{\mathcal{O}}_{\omega k}\, K^{(2)}_{\text{eternal}}(t,\vec{x},r) + \text{h.c.} \right]$$

Conditions on $\tilde{\mathcal{O}}$:

- "correctly entangled" $\tilde{\mathcal{O}}|\psi\rangle = e^{-\frac{\beta H}{2}} \mathcal{O}^{\dagger} e^{\frac{\beta H}{2}} |\psi\rangle$ state dependent
- commute when acting on $|\psi\rangle$ + excitations $[\tilde{\mathcal{O}}, \mathcal{O}]\, \mathcal{O} \dots |\psi\rangle = 0$
- solution always exists if $P(\mathcal{O}_i)|\psi\rangle \neq 0$

The $\mathbb{RP}^2$ geon example

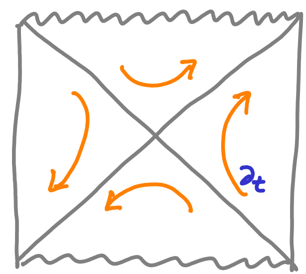
BTZ warm-up

AdS$_3$: embedding in $\mathbb{R}^{2,2}$ $\quad ds^2 = -dT_1^2 - dT_2^2 + dX_1^2 + dX_2^2$

- hyperboloid $-T_1^2 - T_2^2 + X_1^2 + X_2^2 = -\ell^2$

- $\xi \equiv X_1 \partial_{T_1} + T_1 \partial_{X_1}$; $\eta \equiv X_2 \partial_{T_2} + T_2 \partial_{X_2}$

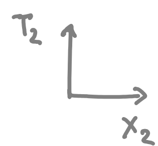
BTZ : quotient by $e^{2\pi r_+ \xi/\ell}$



$$ds^2 = -\frac{(r^2 - r_+^2)}{\ell^2} dt^2 + \frac{\ell^2}{r^2 - r_+^2} dr^2 + r^2 d\varphi^2$$

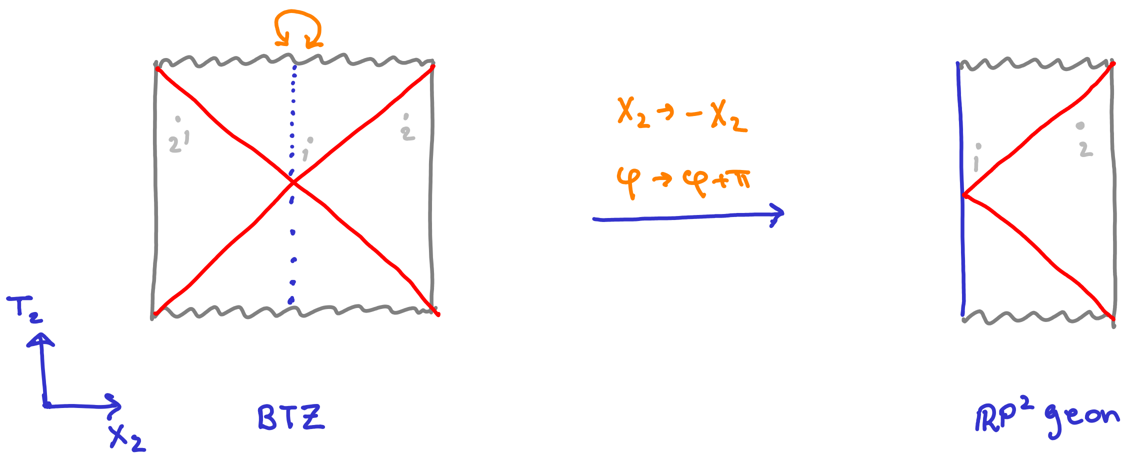
$$\xi = \ell/r_+ \, \partial_\varphi \; ; \; \eta = \ell^2/r_+ \, \partial_t$$

- r-t plane $\leftrightarrow$ X_2, T_2 plane



Definition of the $\mathbb{RP}^2$ geon

Louko & Marolf '98



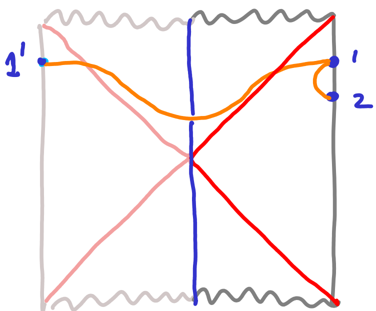
- correlator obtained via method of images

$$\langle \Phi(P_1)\,\Phi(P_2)\rangle_{\text{geon}} = \langle \Phi(P_1)\,\Phi(P_2)\rangle_{BTZ} + \langle \Phi(P_1)\,\Phi(P_2')\rangle_{BTZ}$$

- analyticity $\to$ geodesic approximation

Thermality

- $|\Psi_g\rangle \rightarrow$ pure state that thermalizes at late times



- late-time correlators

$$\langle O(t_1)\, O(t_2)\rangle_{geon} = \underbrace{\langle O(t_1)\, O(t_2)\rangle_{BTZ}}_{\text{thermal}}$$

$$+ \langle O(t_1')\, O(t_2)\rangle_{BTZ}$$

$$\propto e^{-(t_1+t_2)\Delta/\beta}$$

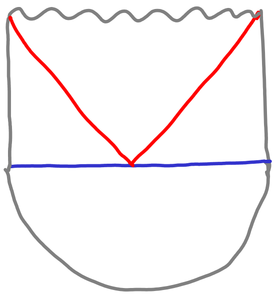
- for $t > t_* = \frac{\beta}{2\pi} \ln S_{BH}$ scrambling time supressed

- geon correlators are not thermal for $t \simeq 0$

Path integral construction

Maldacena '01

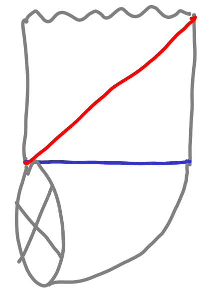
- eternal BTZ
- $\mathbb{R}P^2$ geon

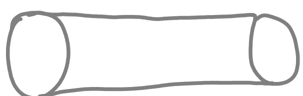


$t=0$

Lorentzian

Euclidean



$Z_{CFT}\Big[$  $\Big]$

$\beta/2$

$\rightarrow \begin{cases} \tau \sim \beta/2 - \tau \\ \varphi \sim \varphi + \pi \end{cases} \rightarrow$ $Z_{CFT}\Big[$  $\Big]$

$\beta/4$

$$\Psi_{tfd} = \sum_E e^{-\frac{\beta E}{2}} |E\rangle_1 |E\rangle_2$$

$$|\Psi_g\rangle = e^{-\frac{\beta H}{4}} |C\rangle$$

crosscap

Properties of the geon state

- entanglement structure

$$|\psi_g\rangle = e^{-\beta H/4}|C\rangle \quad ; \quad (L_n - (-1)^n \bar{L}_{-n})|C\rangle = 0$$

→ entangled state between left & right-movers (in single CFT)

- e.g. free boson CFT $\qquad |C\rangle = \exp\left[-\sum_{n=1}^{\infty} \frac{(-1)^n}{n} j_{-n}\bar{j}_{-n}\right]|0\rangle$

- generally, crosscap expected to satisfy

$$\mathcal{A}^{+}(t,\varphi)|C\rangle = \mathcal{A}(-t,\varphi+\pi)|C\rangle \qquad \mathcal{A} = e^{\frac{\beta H}{4}} \mathcal{O}\, e^{-\frac{\beta H}{4}}$$

$$\Rightarrow \quad \boxed{e^{-\frac{\beta H}{2}} \mathcal{O}^{+}(t,\varphi)\, e^{\frac{\beta H}{2}} |\psi_g\rangle = \mathcal{O}(-t,\varphi+\pi)|\psi_g\rangle}$$

- exact
- matches gravity

Properties of the geon state

- for GFF operators, can solve

$$|\psi_g\rangle \sim \prod_{\omega,k} \exp\left[\alpha_{\omega,k}(-1)^k O^\dagger_{\omega,k}\, O^\dagger_{\omega,-k}\right] \sim \prod_{\substack{\omega_0,t_0\\ k_0,\varphi_0}} \exp\left[\alpha_{\omega_0,k_0}\, O^\dagger_{\omega_0, t_0;\, k_0\, \varphi_0}\, O^\dagger_{\omega_0,-t_0;\, -k_0,\varphi_0+\pi}\right]$$

$\rightarrow$ thermal density matrix at late times

- support at high energies

$$|C\rangle = \sum_i c_{i,m_i}\, |i,m_i\rangle_L\, |i,m_i\rangle_R \quad \Rightarrow \quad |\psi_g\rangle = \sum_i e^{-\frac{\beta E_i}{4}} c_{i,m_i}\, |i\rangle_L |i\rangle_R$$

- partition function $\rightarrow$ CFT on Klein bottle 

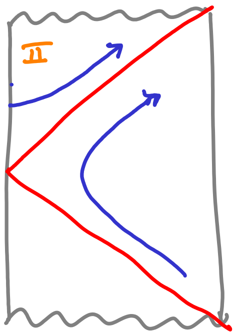
$$Z_K = \langle \Theta C|\, e^{-\frac{\beta H}{2}}\, |C\rangle = \sum_i e^{-\frac{\beta E_i}{2}}\, d_c(E_i) \qquad d_c = \sum_{m_i} |c_{i,m_i}|^2$$

- "modular invariance" $\Rightarrow$ $d_c(E) \sim e^{\pi\sqrt{cE/3}}$ Cardy growth

- matches geon black hole entropy @ $t=0$, not $t>0$

Constructing the mirror operators

Mirror operators – method I (direct construction)



$t>0$

- region II wavefunctions → symmetric $\begin{cases} t \to -t \\ \varphi \to \varphi + \pi \end{cases}$

- $$\Phi^{\mathrm{II}}_{geon}(t,r,\varphi) = \sum_m \int d\omega \left[\mathcal{O}_{\omega,m} \left(e^{-i\omega t + im\varphi} + (-1)^m e^{i\omega t + im\varphi} \right) K^{(1)}_{BTZ} + h.c \right]$$

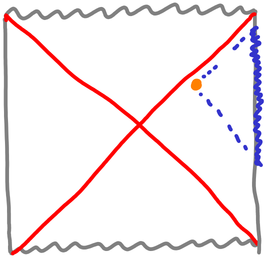
- mirror operators $\tilde{\mathcal{O}}^{g}_{\omega,m} = (-1)^m \mathcal{O}_{\omega,-m}$

- Fourier transform $\tilde{\mathcal{O}}_g(t,\varphi) = \mathcal{O}(-t, \varphi+\pi)$

  ↳ agrees w/ field theory expectation

- $[\mathcal{O}(-t), \mathcal{O}(t')] \approx 0$ only for t, t' large

Distinguishing $\mathcal{O}$ & $\tilde{\mathcal{O}}$

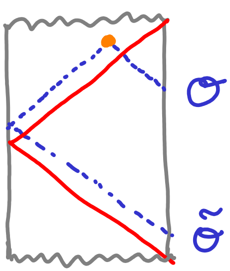


• for $r \to r_+$ $\quad \Phi_{bulk}(t,r,\varphi) = \int dt'\, d\varphi'\, \underbrace{K(x,x')}\, \mathcal{O}(t',\varphi')$

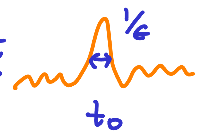
boundary support diverges ↵

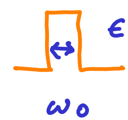
• $K_{\omega m}(t,r,\varphi) \underset{r\to r_+}{\sim} \frac{1}{\omega}\, e^{\frac{\beta |m|}{4}} \cos(\omega r_* + \delta_{\omega m})$

• smear $\quad \Phi_{\omega_0, m_0}(t_0, r) = \int dt\, d\varphi \left[\xi^*_{\omega_0 t_0}(t)\, \Phi^+(t,r,\varphi) + h.c. \right]$



• $\xi_{\omega_0 t_0}$: $\quad e^{-i\omega_0 t}$





$\cdot\, e^{i\omega t_0}$

• for $\omega_0 \gg \epsilon^{-1}, M \to$ ray tracing

Mirror operators – method II (PR)

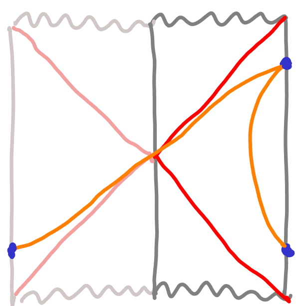
- PR conditions:
$$\begin{cases} \tilde{O}\,|\psi_g\rangle = e^{-\beta H/2}\, O\, e^{\beta H/2}\,|\psi_g\rangle \\ [\tilde{O}, O]\,|\psi_g\rangle = 0 \end{cases}$$

stays

bks dn

- bulk geon state $d^g_{\omega,m}|\psi_g\rangle \propto \left(O_{\omega,m} - e^{-\beta\omega/2}(-1)^m O^+_{\omega,-m}\right)|\psi_g\rangle = 0$

↳ consistent w/ $\tilde{O} = O(-t, \varphi+\pi)$



- $\langle O\tilde{O}\rangle_{geon} = \langle O(t)\, O(-t)\rangle_{BTZ} + {}$ $\sim e^{-t/\beta}$

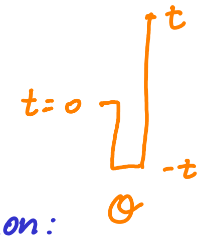
$+ \langle O(t)\,\tilde{O}(t)\rangle_{BTZ}$

time-indep, $[O,\tilde{O}]_{BTZ} = 0$

- $\langle [O,\tilde{O}]\rangle_{geon} \sim e^{-t/\beta}$ ← correct

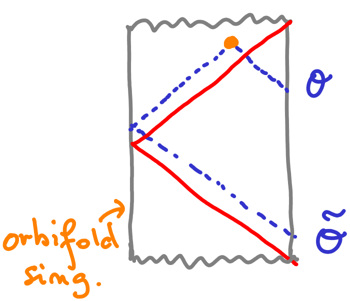
Comments

- $\tilde{\mathcal{O}}$ simple $\to$ precursor sense



very complicated

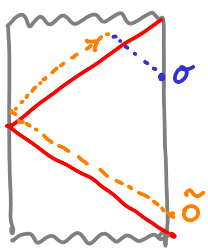
- ray tracing v. special to geon:
    - no transplanckian problem
    - reflects back to boundary
- another simple example : J-quotient of BTZ $(x_2 \to -x_2)$



$$\mathcal{O}(-t,\varphi)\,|\psi_B\rangle = e^{-\frac{\beta H}{2}}\,\mathcal{O}^{+}(t,\varphi)\,e^{\frac{\beta H}{2}}\,|\psi_B\rangle$$

Modifications of the geon state

- PR prescription → assumes smooth horizon → can one predict when horizon not smooth?
- prescription only applies to equilibrium states



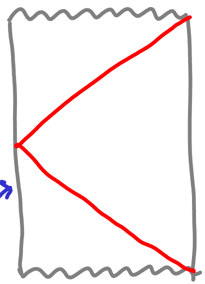
- $\mathcal{O}|\psi_g\rangle$ → out of equilibrium
- $\tilde{\mathcal{O}}|\psi_g\rangle \to \mathcal{H}$. should horizon still be smooth?
- $\mathcal{U}(\tilde{\mathcal{O}})|\psi_g\rangle$, e.g. $e^{i\alpha \tilde{\mathcal{O}}_w^\dagger \tilde{\mathcal{O}}_w}|\psi_g\rangle$

  $d^0_{wm}|\tilde{\mathcal{U}}\psi_g\rangle \neq 0 \Rightarrow$ horizon not smooth?

- $e^{i\alpha \mathcal{O}_w^\dagger \mathcal{O}_w}|\psi_g\rangle \mapsto$ ambiguity of the PR proposal?

A simple example

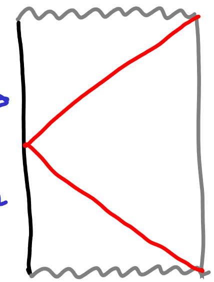
$\mathbb{R}P^2$ geon



$\varphi \to \varphi + \pi$

smooth

J- quotient of BTZ



not smooth

$\begin{cases} x_2 \to -x_2 \\ \varphi \to \varphi \end{cases}$

$$|\psi_g\rangle = e^{-\frac{\beta H}{4}} |C\rangle$$

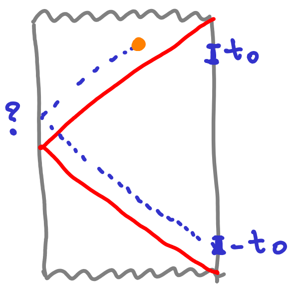
$$|\psi_B\rangle = e^{-\frac{\beta H}{4}} |B\rangle$$

$|\psi_g\rangle = U\,|\psi_B\rangle$, however : horizon is smooth!

- only interior changes

Future directions

- $|\psi_g\rangle \sim \prod_{\omega_0, t_0} \exp\left[\alpha_{\omega_0} \mathcal{O}^{\dagger}_{\omega_0, t_0} \mathcal{O}^{\dagger}_{\omega_0, -t_0}\right] |0\rangle$ + unitary rot $\alpha_{\omega_0} \to e^{i\varepsilon} \alpha_{\omega_0}$



- what happens to the geometry?

- backreaction $\tilde{\mathcal{O}} |\psi_g\rangle$

- more general geons, w/ non-trivial topology behind the horizon?

Thank you !