Two dimensional black hole solutions in Horava-Lifshitz gravity

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Introduction

- Why alternative theory of gravity ?
- The Horava-Lifshitz theory
- Black hole solutions: there several solutions in 3+1 and 2+1 HL gravity in the literature
- Why two-dimensional black holes ?

The HL gravity

$$S_{EH} = \frac{M_{Pl}^2}{2} \int d^{D+1}x \sqrt{g} R \qquad \Lambda = 0$$

$$G_N \sim M_{Pl}^{-(D-1)}, \qquad [M_{Pl}] = 1$$

Einstein Gravity:
Einstein-Hilbert action

The HL gravity

$$S_{EH} = \frac{M_{Pl}^2}{2} \int d^{D+1}x \sqrt{g} R \qquad \Lambda = 0$$

$$S_{EH} = \frac{M_{Pl}^2}{2} \int d^D x \, dt N \sqrt{g} (R + K_{ij} K^{ij} - K^2)$$

ADM decomposition

$$R = g^{ij} R_{ij}$$

The HL gravity

$$K_{ij} = \frac{1}{2N}(\dot{g}_{ij} - \nabla_i N_j - \nabla_j N_i)$$

$$K = g^{ij} K_{ij}$$

extrinsic curvature tensor

shift vector field

$$ds^2 = -N^2 dt^2 + g_{ij}\left(dx^i + N^i dt\right)\left(dx^j + N^j dt\right)$$

lapse function

The HL gravity

$$S_{EH} = \frac{M_{Pl}^2}{2} \int d^D x \, dt N \sqrt{g} (R + K_{ij} K^{ij} - K^2)$$

$$[M_{Pl}] = \frac{D-z}{2}$$

Horava, PRD(2009).

$$S_{HL} = \frac{M_{Pl}^2}{2} \int d^D x \, dt N \sqrt{g} (K_{ij} K^{ij} - \lambda K^2 - \mathcal{V})$$

UV *anisotropic* scaling between space and time:

$$t \to b^{-z} t, \qquad x^i \to b^{-1} x^i, \qquad i = 1, 2, ..., D$$

The HL gravity

$$S_{EH} = \frac{M_{Pl}^2}{2} \int d^D x \, dt N \sqrt{g} (R + K_{ij} K^{ij} - K^2)$$

$$[M_{Pl}] = \frac{D - z}{2}$$

Horava, PRD(2009).

$$S_{HL} = \frac{M_{Pl}^2}{2} \int d^D x \, dt N \sqrt{g} (K_{ij} K^{ij} - \lambda K^2 - \mathcal{V})$$

Power-counting renormalizable for $z \geq D$
and $z = 1$ **corresponds to the GR regime (IR)**

The HL gravity

$$S_{EH} = \frac{M_{Pl}^2}{2} \int d^D x \, dt N \sqrt{g} (R + K_{ij} K^{ij} - K^2)$$

Horava, PRD(2009).

$$S_{HL} = \frac{M_{Pl}^2}{2} \int d^D x \, dt N \sqrt{g} (K_{ij} K^{ij} - \lambda K^2 - \mathcal{V})$$

HL gravity: kinetic part

The HL gravity

$$S_{EH} = \frac{M_{Pl}^2}{2} \int d^D x \, dt N \sqrt{g} (R + K_{ij} K^{ij} - K^2)$$

↓

Horava, PRD(2009).

$$S_{HL} = \frac{M_{Pl}^2}{2} \int d^D x \, dt N \sqrt{g} (K_{ij} K^{ij} - \lambda K^2 - \mathcal{V})$$

HL gravity:
potential part

The HL gravity

$$S_{EH} = \frac{M_{Pl}^2}{2}\int d^D x\, dt N\sqrt{g}(R + K_{ij}K^{ij} - K^2)$$

Horava, PRD(2009).

$$S_{HL} = \frac{M_{Pl}^2}{2}\int d^D x\, dt N\sqrt{g}(K_{ij}K^{ij} - \lambda K^2 - \mathcal{V})$$

Horava-Lifshitz gravity:

λ ≤ 1

The HL gravity

$$S_{EH} = \frac{M_{Pl}^2}{2} \int d^D x \, dt N \sqrt{g} (R + K_{ij} K^{ij} - K^2)$$

↓

$$S_{HL} = \frac{M_{Pl}^2}{2} \int d^D x \, dt N \sqrt{g} (K_{ij} K^{ij} - \lambda K^2 - \mathcal{V})$$

Horava-Lifshitz gravity: **$\lambda > 1$**
("healthy extension")

Blas, Pujolas and Sibiryakov, PRL(2010); Jacobson, PRD(2010).

The HL gravity

$$\mathcal{V} = -\xi R - \eta a^i a_i + \frac{1}{M_{Pl}^2} L_4 + \frac{1}{M_{Pl}^4} L_6$$

$$a_i = \partial_i \ln N$$

"proper acceleration"
(healthy non-projectable
HL gravity: N(t,x))

Blas, Pujolas and Sibiryakov, PRL(2010); Jacobson, PRD(2010).

The HL gravity

$$\mathcal{V} = -\xi R - \eta a^i a_i + \frac{1}{M_{Pl}^2} L_4 + \frac{1}{M_{Pl}^4} L_6$$

(dim 2) $\quad R,\ a_i a^i$

Blas, Pujolas and Sibiryakov, PRL(2010);
Jacobson, PRD(2010).

The HL gravity

$$\mathcal{V} = -\xi R - \eta a^i a_i + \frac{1}{M_{Pl}^2} L_4 + \frac{1}{M_{Pl}^4} L_6$$

$$(\text{dim } 4) \quad R_{ij}R^{ij},\ R^2,\ R\nabla_i a^i,\ a_i \Delta a^i$$

Blas, Pujolas and Sibiryakov, PRL(2010);
Jacobson, PRD(2010).

The HL gravity

$$\mathcal{V} = -\xi R - \eta a^i a_i + \frac{1}{M_{Pl}^2} L_4 + \frac{1}{M_{Pl}^4} L_6$$

$$\text{(dim 6)} \quad (\nabla_i R_{jk})^2,\ (\nabla_i R)^2,\ \Delta R \nabla_i a^i,\ a_i \Delta^2 a^i$$

Blas, Pujolas and Sibiryakov, PRL(2010);
Jacobson, PRD(2010).

The HL gravity

$$S = S_{HL} + S_\phi$$

Matter (scalar) contribution

$$S_\phi = \int d^D x\, dt N \sqrt{g} \Big[\frac{1}{2N} (\partial_t \phi - N^i \nabla_i \phi)^2 - \alpha (\nabla_i \phi)^2 - V(\phi) - \beta \phi \nabla^i a_i - \gamma \phi a^i \nabla_i \phi) \Big]$$

with the GR regime at α=1, β=γ=0

Two dimensional HL gravity

$$S_{HL} = \frac{M_{Pl}^2}{2}\int dx dt N\sqrt{g}[(1-\lambda)K^2 + \eta g^{11} a_1 a_1]$$

$$[M_{Pl}] = \frac{D-z}{2} = 0, \qquad D = 1, z = 1$$

where at one spatial dimension : $R = 0,\ K_{ij}K^{ij} \equiv K^2$

and

$$S_\phi = \int dx\, dt N\sqrt{g}\Big[\frac{1}{2N}(\partial_t\phi - N^1\nabla_1\phi)^2 - \alpha(\nabla_1\phi)^2 - V(\phi) - \beta\phi\nabla^1 a_1 - \gamma\phi a^1\nabla_1\phi)\Big]$$

Li, Wang, Wu and Wu, PRD(2014)

α=1, β=γ=0 (dimensionless parameters)

Two dimensional HL gravity

There are two "free" parameters left in HL sector: λ, η

$$S_{HL} = \frac{M_{Pl}^2}{2} \int dx dt N \sqrt{g} [(1-\lambda)K^2 + \eta g^{11} a_1 a_1]$$

This term vanishes for static solutions → no dependence on **λ**

Li, Wang, Wu and Wu, PRD(2014)

Two dimensional HL gravity

There are two "free" parameters left in HL sector: λ, η

$$S_{HL} = \frac{M_{Pl}^2}{2}\int dx dt N\sqrt{g}[(1-\lambda)K^2 + \eta g^{11} a_1 a_1]$$

This term vanishes for *homogeneous (time-dependent solutions)* → no dependence on **η**

Li, Wang, Wu and Wu, PRD(2014)

Two dimensional black holes in HL gravity

$$ds^2 = -N^2dt^2 + g_{ij}\left(dx^i + N^i dt\right)\left(dx^j + N^j dt\right)$$

where $g_{ij} \equiv g_{11} = \frac{1}{N^2}$ $\quad N \equiv N(x)$

$$N\sqrt{g} = 1$$

$N_1(x) = 0$ **(gauge)**

static solutions

Bazeia, Brito and Costa, PRD(2015)

$$K = \frac{N}{2}(\dot{g}_{11} - 2\nabla_1 N_1) \longrightarrow K = 0$$

Two dimensional black holes in HL gravity

$$S = \frac{M_{Pl}^2}{2} \int dx dt \left(\eta N^2 a_1^2 - \frac{2}{M_{Pl}^2} \alpha N^2 \phi'^2 - \frac{2}{M_{Pl}^2} V(\phi) \right)$$

Equations of motion

$$\frac{\delta S}{\delta N} = 0 \rightarrow \eta a_1^2 - \frac{2}{M_{Pl}^2} \alpha \phi'^2 = 0$$

$$\frac{\delta S}{\delta \phi} = 0 \rightarrow \frac{d}{dx}(N^2 \phi') = \frac{1}{2\alpha} \frac{\partial V}{\partial \phi}$$

Bazeia, Brito and Costa, PRD(2015)

Two dimensional black holes in HL gravity

$$S = \frac{M_{Pl}^2}{2} \int dx dt \left(\eta N^2 a_1^2 - \frac{2}{M_{Pl}^2} \alpha N^2 \phi'^2 - \frac{2}{M_{Pl}^2} V(\phi) \right)$$

Equations of motion

$$\eta a_1^2 - \frac{2}{M_{Pl}^2} \alpha \phi'^2 = 0 \rightarrow a_1 = \pm \phi', \qquad \alpha = \eta M_{Pl}^2 / 2$$

$$\frac{d}{dx} (\eta N^2 \phi') = \frac{1}{M_{Pl}^2} \frac{\partial V}{\partial \phi}$$

Bazeia, Brito and Costa, PRD(2015)

Two dimensional black holes in HL gravity

$$S = \frac{M_{Pl}^2}{2} \int dx dt \left(\eta N^2 a_1^2 - \frac{2}{M_{Pl}^2} \alpha N^2 \phi'^2 - \frac{2}{M_{Pl}^2} V(\phi) \right)$$

Equations of motion

$$a_1 \equiv \frac{d \ln N}{dx} = \frac{1}{N} \frac{dN}{dx} = \pm \phi' \rightarrow N = e^{\pm \phi}$$

$$\frac{d}{dx} (\eta N^2 \phi') = \frac{1}{M_{Pl}^2} \frac{\partial V}{\partial \phi}$$

Bazeia, Brito and Costa, PRD(2015)

Two dimensional black holes in HL gravity

Solutions

i) $V(\phi) = const. \rightarrow N^2\phi' = C_1/\eta \rightarrow N\frac{dN}{dx} = \pm M$

$$N(x)^2 = 2M|x| - 1 \qquad N(x) = \sqrt{2\,(M|x| + C)}.$$

$$\phi(x) = \ln\sqrt{2M|x| - 1}.$$ **(positive branch)**

$$ds^2 = -\left(2M|x| - 1\right)dt^2 + \frac{1}{\left(2M|x| - 1\right)}dx^2$$

Here we patched together the two branches along a delta source

Bazeia, Brito and Costa, PRD(2015)

Mann, Shiekh and Tarasov, NPB(1990)

Two dimensional black holes in HL gravity

Solutions

ii) $$V_\phi(\phi(x)) = M_{Pl}^2 \left(A + \frac{B}{x^3} + \frac{C}{x^4}\right)$$

$$\frac{d}{dx}\left(\eta N \frac{dN}{dx}\right) = \pm \frac{1}{M_{Pl}^2} V_\phi$$

Picking just the positive branch:

$$N(x)^2 = 2C_2 + \frac{A}{\eta}x^2 - 2C_1 x + \frac{B}{\eta x} + \frac{C}{3\eta x^2}$$

$$\phi(x) = \ln N^2$$

$$ds^2 = -N^2 dt^2 + \frac{1}{N^2}dx^2$$

Bazeia, Brito and Costa, PRD(2015)

Mann, Shiekh and Tarasov, NPB(1990)

Two dimensional black holes in HL gravity

Solutions

ii.a) Schwarzschild-like solution

$$C_2 = 1/2,\ B = -2M,\ \eta = 1 \text{ and } A = C = C_1 = 0.$$

$$V_\phi(\phi(x)) = -\frac{2M}{x^3} M_{Pl}^2$$

$$\boxed{N(x)^2 = 1 - \frac{2M}{x}}$$

$$ds^2 = -N^2 dt^2 + \frac{1}{N^2} dx^2$$

Bazeia, Brito and Costa, PRD(2015)

Mann, Shiekh and Tarasov, NPB(1990)

Two dimensional black holes in HL gravity

Solutions

ii.a) **Schwarzschild-like solution**

$$C_2 = 1/2,\ B = -2M,\ \eta = 1 \text{ and } A = C = C_1 = 0.$$

$$V_\phi(\phi(x)) = \left(-\frac{2M}{x^3}\right) M_{Pl}^2$$

In this case one can invert the funtion $V_\phi(x)$ and integrate in ϕ to find V(ϕ):

$$V(\phi) = \frac{B}{8M^3}\phi - \frac{3B}{16M^3}e^{2\phi} + \frac{3B}{32M^3}e^{4\phi} - \frac{B}{48M^3}e^{6\phi}$$

Bazeia, Brito and Costa, PRD(2015)

Christensen and Fulling, PRD(1977)

Two dimensional black holes in HL gravity

Solutions

ii.b) Reissner-Nordström-like solution

$C_2 = 1/2, B = -2M,\ C = 3Q^2,\ \eta = 1$ and $A = C_1 = 0$

$$V_\phi(\phi(x)) = \left(-\frac{2M}{x^3} + \frac{3Q^2}{x^4}\right) M_{Pl}^2$$

$$N(x)^2 = 1 - \frac{2M}{x} + \frac{Q^2}{x^2}$$

$$ds^2 = -N^2 dt^2 + \frac{1}{N^2} dx^2$$

Bazeia, Brito and Costa, PRD(2015)

Trivedi, PRD(1993)

Two dimensional black holes in HL gravity

Solutions

ii.c) **A new black hole solution**

$$f(x) = 2C_2 + \frac{A}{\eta}x^2 - 2C_1 x + \frac{B}{\eta x} + \frac{C}{3\eta x^2} \qquad f(x) \equiv N(x)^2,$$

$C_1 \neq 0,\ C_2 \neq 0,\ B \neq 0$ and $A = C = 0$

$$\boxed{f(x) = 2C_2 - 2C_1 x + \frac{B}{\eta x}}$$

$$ds^2 = -f(x)dt^2 + \frac{1}{f(x)}dx^2$$

Bazeia, Brito and Costa, PRD(2015)

Two dimensional black holes in HL gravity

Solutions

ii.c) **A new black hole solution**

This solution develops the following horizons

$$x_h^{\pm} = \frac{C_2}{C_1} \pm \sqrt{\Delta}, \qquad \Delta = \frac{C_2^2}{C_1^2} + \frac{2B}{\eta C_1}$$

The Hawking temperature

$$T_H = \left.\frac{f'(x)}{4\pi}\right|_{x=x_h^+}$$

Bazeia, Brito and Costa, PRD(2015)

Two dimensional black holes in HL gravity

Solutions

ii.c) **A new black hole solution**

For the special case

$$C_2 = 0,\ C_1 = -M \text{ and } B = -2M$$

The horizons are independent of the mass

$$x_h^{\pm} = \pm\frac{2}{\sqrt{\eta}} \ \ (\eta > 0)$$

Bazeia, Brito and Costa, PRD(2015)

Two dimensional black holes in HL gravity

Solutions

ii.c) **A new black hole solution**

and the Hawking temperature is

$$T_H = \frac{1}{4\pi}\left(-2C_1 - \frac{B}{(\frac{2}{\sqrt{\eta}})^2}\right)$$

or simply

$$T_H = \frac{1}{8\pi}\left(4+\eta\right) M$$

A typical relation between Hawking temperature and BH mass in 1+1 dimensions

Bazeia, Brito and Costa, PRD(2015)

Gravitational collapse

Interior solution

Gravitational collapse of a certain mass of dust confined into a region of the unidimensional space [−r, r] which metric is given in co-moving coordinates by

$$ds^2 = -N(\tau)^2 d\tau^2 + a(\tau)^2 d\rho^2$$

The action

$$S = \frac{M_{Pl}^2}{2} \int d^2x N \sqrt{g_{11}} \left[(1-\lambda) K^2 + \eta g^{11} \phi'^2 \right] + S_m$$

Bazeia, Brito and Costa, PRD(2015)

$$a_1 \equiv \frac{d \ln N}{dx} = 0 \qquad \alpha = \eta M_{Pl}^2 / 2$$

Gravitational collapse

Interior solution

Gravitational collapse of a certain mass of dust confined into a region of the unidimensional space [−r, r] which metric is given in co-moving coordinates by

$$ds^2 = -N(\tau)^2 d\tau^2 + a(\tau)^2 d\rho^2$$

The action

$$S = \frac{M_{Pl}^2}{2} \int d^2x \left[\frac{(1-\lambda) a^3 \dot{a}^2}{N} \right] + \frac{M_{Pl}^2}{2} \int d^2x \left[\frac{N \eta \phi'}{a} \right] + \int d^2x N \sqrt{g_{11}} L_m$$

Bazeia, Brito and Costa, PRD(2015)

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Gravitational collapse of a certain mass of dust confined into a region of the unidimensional space [−r, r] which metric is given in co-moving coordinates by

$$ds^2 = -N(\tau)^2 d\tau^2 + a(\tau)^2 d\rho^2$$

The equation of motion

$$\frac{\delta S}{\delta N} = 0 \rightarrow \frac{(\lambda-1)M_{Pl}^2 a^3 \dot{a}^2}{2N^2} + \frac{M_{Pl}^2 \eta \phi'}{2a} = -\frac{1}{\sqrt{g_{11}}}\frac{\delta S_m}{\delta N} = \sigma.$$

ϕ=0

energy density

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}L_m)}{\delta g_{\mu\nu}}$$

Bazeia, Brito and Costa, PRD(2015)

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Gravitational collapse of a certain mass of dust confined into a region of the unidimensional space [−r, r] which metric is given in co-moving coordinates by

$$ds^2 = -N(\tau)^2 d\tau^2 + a(\tau)^2 d\rho^2$$

The equation of motion

$$(\lambda - 1) M_{Pl}^2 a^3 \dot{a}^2 = 2\sigma$$

The energy conservation

$$\nabla_\mu T^{\mu\nu} = 0 \longrightarrow \frac{\partial}{\partial t}(\sigma \sqrt{a^2}) = 0$$

$$N = 1$$

$$\sigma = \frac{\rho_0 a_0}{a}$$

Bazeia, Brito and Costa, PRD(2015)

Gravitational collapse

Interior solution

Gravitational collapse of a certain mass of dust confined into a region of the unidimensional space [−r, r] which metric is given in co-moving coordinates by

$$ds^2 = -N(\tau)^2 d\tau^2 + a(\tau)^2 d\rho^2$$

The interior solution

$$a^2 \dot{a} = \pm \sqrt{\frac{2\sigma_0 a_0}{M_{Pl}^2(\lambda - 1)}} = \pm\beta$$

$$a = (\pm 3\beta\tau + C)^{1/3}$$

$$ds^2 = -d\tau^2 + (1 - 3\beta\tau)^{2/3} d\rho^2$$

Bazeia, Brito and Costa, PRD(2015)

Gravitational collapse

Interior solution

Gravitational collapse of a certain mass of dust confined into a region of the unidimensional space [−r, r] which metric is given in co-moving coordinates by

$$ds^2 = -d\tau^2 + (1 - 3\beta\tau)^{2/3} d\rho^2$$

***The density of the dust given by σ goes to infinity (singularity)* as the scale factor approaches zero. This occurs in the finite time $\tau_c = 1/(3\beta)$, that is**

$$\tau_c = \sqrt{\frac{(\lambda - 1) M_{Pl}^2}{18 \sigma_0 a_0}}$$

Recall that $\lambda > 1$

Bazeia, Brito and Costa, PRD(2015)

Gravitational collapse

Exterior solution

Gravitational collapse of a certain mass of dust confined into a region of the unidimensional space [−r, r] which metric is given in co-moving coordinates by

$$ds^2 = -d\tau^2 + a^2(\tau,\rho)d\rho^2, \qquad \text{(interior solution)}$$

$$x(\tau,\rho) = \rho a(\tau) = \rho(1 - 3\beta\tau)^{2/3}$$

$$ds^2 = -A(x)^2 dt^2 + A(x)^{-2} dx^2 \qquad \text{(exterior solution)}$$

Bazeia, Brito and Costa, PRD(2015)

Gravitational collapse

Exterior solution

The constant of the motion along the geodesic

$$\epsilon = -g_{\mu\nu}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau};$$

$$-\epsilon = -A^2\left(\frac{dt}{d\tau}\right)^2 + A^{-2}\left(\frac{dx}{d\tau}\right)^2$$

Bazeia, Brito and Costa, PRD(2015)

Gravitational collapse

Exterior solution

There is a Killing vector that corresponds to energy conservation satisfying

$$K_\mu \frac{dx^\mu}{d\tau} = K_t \frac{dt}{d\tau} + K_x \frac{dx}{d\tau} = const \qquad K_\mu = (-A^2, 0)$$

$$\downarrow$$

$$\boxed{\frac{dt}{d\tau} = -\frac{E}{A^2}}$$

Bazeia, Brito and Costa, PRD(2015)

Gravitational collapse

Exterior solution

Using the conditions $x = x(\rho, \tau),\, t = t(\rho, \tau)$ **and** $x = \rho a$

$$dx = \frac{\partial x}{\partial \tau} d\tau + \frac{\partial x}{\partial \rho} d\rho = \rho \frac{\partial a}{\partial \tau} d\tau + a d\rho$$

$$dt = \frac{\partial t}{\partial \tau} d\tau + \frac{\partial t}{\partial \rho} d\rho = -\frac{C}{A^2} d\tau + \frac{\partial t}{\partial \rho} d\rho$$

In the limit $x \rightarrow r$ $\quad \epsilon E^2 = C^2$

Bazeia, Brito and Costa, PRD(2015)

Gravitational collapse

Exterior solution

After some algebra … we find

$$A^2 = 1 - \frac{\beta^2 r^6}{x^4}$$

That is

$$ds^2 = -\left(1 - \frac{\beta^2 r^6}{x^4}\right) dt^2 + \left(1 - \frac{\beta^2 r^6}{x^4}\right)^{-1} dx^2$$

where $\rho = r$ and $x = ra$

Bazeia, Brito and Costa, PRD(2015)

Gravitational collapse

Exterior solution

The curvature escalar is

$$R = \frac{20\beta^2 r^6}{x^6}$$

and the two dimensional Schwarzschild radius is given by

$$x_H = r^{3/2}\beta^{1/2} = r^{3/2}\left[\frac{2\sigma_0 a_0}{M_{Pl}^2(\lambda - 1)}\right]^{1/4}$$

Bazeia, Brito and Costa, PRD(2015)

Sikkema and Mann, Class. Quantum Grav.(1991)

Conclusions

- We have considered two-dimensional black holes in HL gravity
- Tough the simplicity of our approach due to some assumptions, the scenario is rich enough to reveal several black hole physics
- The parameters of the HL theory affects the fate of collapsing dust
- We did not consider higher derivatives. One should consider this elsewhere

Thanks a lot !