Factorization and Criticality in Spin Systems

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Exactly Solvable Quantum Chains June 22, 2018 @ Natal



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Outline	Motivation	Factorization	The XYZ case	The XXZ case	Separable State Engineering	Conclusions and perspective
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Outline

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 Motivation: Quantum spin systems in the context of Quantum Information and Quantum Computation

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- Quantum Computer capable of simulating quantum systems (Feynmann 1982).
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- New forms of information transmission : quantum teleportation, quantum cryptography.



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Fundamental resource in Quantum Information and Quantum Computation.



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What about spin systems ?

$$H=-\sum_{i,\mu}h^i\cdot S_i-rac{1}{2}\sum_{i,j}S_i\cdot \mathcal{J}^{ij}S_j$$
 $H=-\sum_{i,\mu}h^i_\mu S^\mu_i-rac{1}{2}\sum_{i,j,\mu,
u}J^{ij}_{\mu
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Some (basic) considerations



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In the absence of magnetic fields:

$$H = -\sum_{i,\mu} h^{i}_{\mu} S^{\mu}_{i} - \frac{1}{2} \sum_{i,j,\mu,\nu} J^{ij}_{\mu\nu} S^{\mu}_{i} S^{\nu}_{j},$$

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the ground state (GS) is typically entangled.

With pairwise entanglement's range similar to that of the interactions .

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the ground state (GS) is typically entangled.

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When spin systems are immersed in **finite magnetic fields**, the **GS** still remains an **entangled state**.

If we want a separable GS (initialization): "turn off" the interaction or apply strong magnetic fields ($h^i_\mu >> J^{ij}_{\mu\nu}$):

$$H=-\sum_{i,\mu}h^i_\mu S^\mu_i-rac{1}{2}\sum_{i,j,\mu,
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the spins align with the magnetic fields.



Factorizing Field

Is it possible to have a separable GS in the presence of spin interactions and finite magnetic fields?



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Factorized states can be used as **Sector** initial states for quantum information protocols.



A brief history of factorization

Physica 112A (1982) 235-255 North-Holland Publishing Co. Received 12 October 1981

ANTIFERROMAGNETIC LONG-RANGE ORDER IN THE ANISOTROPIC QUANTUM SPIN CHAIN

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VOLUME 93, NUMBER 16	PHYSICAL	REVIEW	LETTERS	week ending 15 OCTOBER 2004	IAS asel, Switzerland
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Salvatore M, Giampaolo, 1.2 Gerardo Adesso, 1.2 and Fabrizio Illuminati 1.2.3,* ¹Dipartimento di Matematica e Informatica, Università deeli Studi di Salerno, Via Ponte don Melillo, I-84084 Fisciano (SA), Italy ²CNR-INFM Coherentia, Napoli, Italy; CNISM, Unità di Salerno, Italy; and INFN, Sezione di Napoli-Gruppo Collegato di Salerno, Italy ³ISI Foundation for Scientific Interchange, Viale Settimio Severo 65, 1-10133 Turin, Italy (Received 31 March 2008; published 13 May 2008)

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¹ Dipartimento di	Salvatore N Matematica e Info	I. Giampaolo, ^{1,2} Gerar rmatica. Università deel	do Adesso, ^{1,2} and PRL 104	, 207202 (2010)	PHYSIC	AL REVIEW	LETTER	S	week ending 21 MAY 2010
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¹ Dipartimento di	Salvatore M Matematica e Infor ² CNR-I and ³ ISI Foundation	 Giampaolo,^{1,2} Gerau matica, Università degl NFM Coherentia, Napo INFN, Sezione di Napoi for Scientific Interchan (Received 31 March) 	rdo Adesso, ^{1,2} and PRL 104 i Studi di Salerno, 1 li, Italy: CNISM, Ur li-Gruppo Collega ge, Vale Settimio S 2008: rublished 13	, 207202 (2010) Probing Quantum	PHYSIC Frustrated	AL REVIEW	LETTERS	week ending 21 MAY 2010 f the Ground State
	Entanglem	PHYSICAL REV	VIEW A 77, 052322 (2003 yclic chains at fac	³⁾ torizing fields		», ¹ Gerardo Adesso Iniversità degli Studi gato di Salerno, Via of Nottingham, Unive Ianuscrint received 20	, ² and Fabrizio II di Salerno, CNR-S Ponte don Melillo, rsity Park, Notting, 0 April 2010; publ.	luminati ^{1,59} SPIN, CNISM, Unità di Salerno, I-84084 Fisciano (SA), Italy ham NG7 2RD, United Kingdom lished 19 May 2010)
Departamer	nto de Física-IF (Re	R. Rossignoli, N. LP, Universidad Na ceived 29 Novembe	Canosa, and J acional de La Pl er 2007; publishe	ization and entangle	PHYSIC/	AL REVIEW A 80, 0 eneral XYZ spin	062325 (2009) n arrays in no	onuniform transverse fields
	Transv	verse fie	eld!	Departamento de Física (1	R. Rossign <i>IFLP, Univer</i> Received 22 N	oli, N. Canosa, and rsidad Nacional de L fay 2009; published	d J. M. Matera a Plata, CC 67, La 10 December 2009	a Plata 1900, Argentina D)

Factorization general equations

System of *N* spins s_i interacting through XYZ Heisenberg couplings of arbitrary range in the presence of a general magnetic fields h^i

$$H=-\sum_i h^i\cdot S_i-rac{1}{2}\sum_{i,j}S_i\cdot \mathcal{J}^{ij}S_j$$

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The completely separable state

$$|\Theta\rangle = \otimes_{i=1}^n e^{-\imath\phi_i S_i^z} e^{-\imath\theta_i S_i^y} |\uparrow_i\rangle = |\nearrow\swarrow\searrow\wedge,$$

is an exact eigenstate iff it satisfies¹ :

¹MC, R. Rossignoli, N. Canosa, Phys. Rev. B **92**, 224422 (2015).

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Field independent equations: Which state?

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2 The field-dependent conditions: What fields?

$$\boldsymbol{n}_i imes (\boldsymbol{h}_i + \sum_j \mathcal{J}^{ij} \langle \boldsymbol{S}_j \rangle) = \boldsymbol{0},$$

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which implies $\mathbf{h}_i = \mathbf{h}_i^{\perp} + \mathbf{h}_i^{\parallel}$. $(\mathbf{h}_i^{\parallel} = h_i \mathbf{n}_i, \quad \mathbf{n}_i \cdot \mathbf{h}_i^{\perp} = 0)$

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What is it good for?

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The determination of factorized ground states is a useful **tool** towards further improving our understanding of spin systems.

• Detect entanglement phase transitions and critical points.

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- Determine ordered phases .

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The XYZ case

Spin array with anisotropic XYZ couplings in a general fields.

$$H = -\sum_{i} \boldsymbol{h}^{i} \cdot \boldsymbol{S}_{i} - \sum_{i \neq j} J_{x}^{ij} S_{i}^{x} S_{j}^{x} + J_{y}^{ij} S_{i}^{y} S_{j}^{y} + J_{z}^{ij} S_{i}^{z} S_{j}^{z}$$

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Recap results in FM and AFM systems

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Recap results in FM and AFM systems \Rightarrow Revise them with our general equations.

Antiferromagnetic spin chain, Recap

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Anisotropic XYZ spin chain with first neighbour couplings¹

The Hamiltonian

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Ferromagnetic spin systems, Recap

Anisotropic XYZ systems with first neighbor couplings immersed in transverse fields ¹

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- $|\Theta\rangle$ breaks parity symmetry ($[H, P_z] = 0$, $P_z = e^{i\pi S_z}$). The GS is two-fold degenerate : linear combinations of the symmetry preserving entangled crossing states.

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Anisotropic FM and AFM XYZ systems, Revised

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is an exact eigenstate iff $\langle {\bf S} \rangle$ is parallel to a principal plane . The XZ plane solution requires again $\cos^2\theta = \frac{J^y_{ij} - J^z_{ij}}{J^x_{ij} - J^z_{ij}}$, while the factorizing fields of lowest magnitude are $h^i_\perp = \sin\theta\cos\theta\sum_j s_j (J^x_{ij} - J^z_{ij})\sqrt{\chi} \ (\text{also in the } XZ \text{ principal plane }).$

 $h_i = h_i^{\perp} + h_i^{\parallel}$, $|\Theta\rangle$ is always a non-degenerate GS in FM and <u>AFM</u> chains if h_i^{\parallel} is sufficiently large: factorization lines in field space.



Feasible with a uniform field !

¹MC, R. Rossignoli, and N. Canosa, Phys. Rev. B **92**, 224422 (2015).

Anisotropic FM and AFM XYZ systems, Revised

Néel-type solution¹ The Néel-type state $| \swarrow \nearrow \checkmark \ldots \rangle$ is an exact eigenstate iff the factorizing field $h \in$ ellipsoid $\frac{h_x^2}{(J_x+J_z)(J_x+J_y)} + \frac{h_y^2}{(J_y+J_z)(J_y+J_x)} + \frac{h_z^2}{(J_z+J_x)(J_z+J_y)} = 1$.

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FM: Just UGS AFM: NGS+UGS !!

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Anisotropic FM and AFM XYZ systems, Revised



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Factorization and entanglement

Spin-1/2 chain of N = 12 spins.



¹MC, R. Rossignoli, and N. Canosa, Phys. Rev. B **92**, 224422 (2015).

The XYZ case

General arrays of spins s_i with XXZ couplings immersed in **nonuniform transverse fields**

$$H = -\sum\limits_{i} h^{i}S_{i}^{z} - \sum\limits_{i < j} J^{ij}(S_{i}^{x}S_{j}^{x} + S_{i}^{y}S_{j}^{y}) + J_{z}^{ij}S_{i}^{z}S_{j}^{z}$$

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Since $[H, S^z] = 0$, its eigenstates can be characterized by their total magnetization M along z.

Factorization General Equations

The completely separable state

$$|\Theta\rangle = \otimes_{i=1}^{n} e^{-\imath\phi_i S_i^z} e^{-\imath\theta_i S_i^y} |\uparrow_i\rangle = |\nearrow \swarrow \land \dots \rangle,$$

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1 Field independent equations: Which state ?

$$\eta_{ij} \equiv \frac{\tan(\theta_j/2)}{\tan(\theta_i/2)} = \Delta_{ij} \pm \sqrt{\Delta_{ij}^2 - 1}, \qquad \Delta_{ij} = J_z^{ij}/J^{ij}$$

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2 The field-dependent conditions: What fields?

$$h^i_{
m s} = \sum_j s_j
u_{ij} J^{ij} \sqrt{\Delta^2_{ij} - 1}$$

with $\nu_{ij} = \pm 1$ the sign in (1). This Eq. is independent of the angles θ_i and must fulfill the zero sum condition $\sum_i s_i h_s^i = 0$.

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Fundamental Properties

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- ${\bullet}~$ The components of $|\Theta\rangle~$ with definite M~ are also eigenstates of H~ with the same energy E_{Θ} .

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$$P_M|\Theta\rangle \propto \sum_{\substack{m_1,\ldots,m_N\\\sum_i m_i=M}} \left[\prod_{i=1}^N \sqrt{\binom{2s_i}{s_i-m_i}} \eta_{i,i+1}^{\sum_{j=1}^i m_j}\right] |m_1\ldots m_N\rangle, \quad M = -S\ldots S,$$

where $P_M = \frac{1}{2\pi} \int_0^{2\pi} e^{i\varphi(S^z - M)} d\varphi$ is the projector onto total magnetization M. These states are entangled $\forall |M| \leq S - 1$.

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In the vicinity of factorization, entanglement reaches full range.

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Building separable solutions, or how playing with LEGOS finally paid off

Chain of N spins s with first neighbor interactions.

$$\frac{\tan(\theta_j/2)}{\tan(\theta_i/2)} = \frac{J_z}{J} \pm \sqrt{\left(\frac{J_z}{J}\right)^2 - 1}, \quad J_z > J, \qquad h^{ij} = \pm h_s = \pm sJ\sqrt{\Delta^2 - 1}, \quad h^i = \sum_j h^{ij}$$

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Fundamental Properties (II)



 $^{^1}$ MC, R. Rossignoli, N. Canosa, and E. Rios, Phys. Rev. Lett. **119**, 220605 (2017).



Fundamental Properties (II)



By projecting onto magnetization M we can determine **analytical** expressions for the reduce state of any spin pair. For a d-dimensional spin-s system with uniform anisotropy Δ and alternating fields, there are just 3 distinct reduced pair states ρ_{oe}^{M} (odd-even), ρ_{oe}^{M} y ρ_{ee}^{M}

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$$(\rho_{ij}^{M})_{m_{j},m_{j}'}^{m} = \eta^{f_{ij}} \frac{\sqrt{C_{m_{j}}^{s,m} C_{m_{j}'}^{s,m} Q_{Ns-2s-M+m}^{M-m,(\delta+2l_{ij})s}(\eta)}}{Q_{Ns-M}^{M,\delta s}(\eta)}$$

with $m = m_i + m_j = m'_i + m'_j$ the pair magnetization $([\rho^M_{i,j}, S^z_i + S^z_j] = 0)$, $Q_n^{m,k}(\eta) = (\eta^2 - 1)^n P_n^{m-k,m+k}(\frac{\eta^2+1}{\eta^2-1})$ with $P_n^{\alpha,\beta}(x)$ the Jacobi polynomials . $C_k^{s,m} = {2s \choose s-k} {2s \choose s-m+k}$ and $f_{ij} = 2s - m_j - m'_j, 0, 4s - 2m$, $l_{ij} = 0, -1, 1$ for the *oe*, *oo*, *ee* pairs, and $\delta = 0(1)$ if N is even (odd).

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Magnetization and Entanglement



• Factorizing fields correspond to critical points in the the multidimensional field space $\{h^1, \ldots, h^N\}$.

F. C. Alcaraz, and A. L. Malvezzi, J. Phys. A 28, 1521 (1995).

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Other field configuration



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How many state / field configurations are there?

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How many state / field configurations are there?





How many state / field configurations are there?



Outline	Motivation	Factorization	The XYZ case	The XXZ case	Separable State Engineering	Conclusions and perspective
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For $1 \times N$	1,	2,	4,	8,	16
For $2 imes N$	2,	6,	18,	54,	162
For $3 \times N$	4,	18,	82,	374,	1706

MC, R. Rossignoli, N. Canosa, and E. Rios, Phys. Rev. Lett. **119**, 220605 (2017). J. Ginepro, and T.C. Hull, J. Integer Seq. **17**, Art. 14.10.8 (2014).

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Bonus

For $1 \times N$ 2, 8. 16 1. 6, For $2 \times N$ 2, 18, 54, 162 For $3 \times N$ 4. 18, 82, 374, 1706



Counting Miura-ori Foldings

Jessica Ginepro¹ Department of Mathematics University of Connecticut 196 Auditorium Road, Unit 3009 Storrs, CT 06269-3009 USA Figure 1: A 4×4 Miura-ori with the standard MV assignment. Bold creases are mountains and non-bold creases are valleys.

1 Introduction

In the mathematics of origami (paper folding), enumerating the number of ways in which a crease pattern can fold up is often difficult. Even the seemingly simple postage-stamp

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Bonus

For $1 \times N$ 2, 8. 16 1. 6, For $2 \times N$ 2, 18, 54, 162 For $3 \times N$ 4. 18. 82, 374. 1706



Counting Miura-ori Foldings



Jessica Ginepro¹ Department of Mathematics University of Connecticut 196 Auditorium Road, Unit 3009 Storrs, CT 06269-3009 USA Figure 1: A 4×4 Miura-ori with the standard MV assignment. Bold creases are mountains and non-bold creases are valleys.

1 Introduction

In the mathematics of origami (paper folding), enumerating the number of ways in which a crease pattern can fold up is often difficult. Even the seemingly simple postage-stamp

By defining A(1) = (1) and

$$A(M+1) = \begin{pmatrix} A(M) & A(M)^T \\ 0 & A(M) \end{pmatrix}$$

with $B(M) = A(M) + A(M)^T$, the total number of configurations is

$$L(M,N) = \sum_{i,j} (B^{N-1}(M))_{ij}$$

MC, R. Rossignoli, N. Canosa, and E. Rios, Phys. Rev. Lett. 119, 220605 (2017).

J. Ginepro, and T.C. Hull, J. Integer Seq. 17, Art. 14.10.8 (2014).

Separable state engineering

Given a spin system (i.e., given the \mathcal{J}^{ij}) What separable eigenstates can the system posses?

 $\boldsymbol{n}_{i}^{x'} \cdot \mathcal{J}^{ij} \boldsymbol{n}_{j}^{x'} = \boldsymbol{n}_{i}^{y'} \cdot \mathcal{J}^{ij} \boldsymbol{n}_{j}^{y'}, \quad \boldsymbol{n}_{i}^{x'} \cdot \mathcal{J}^{ij} \boldsymbol{n}_{j}^{y'} = -\boldsymbol{n}_{i}^{y'} \cdot \mathcal{J}^{ij} \boldsymbol{n}_{j}^{x'} \quad (1)$



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If some control over the couplings and the fields is feasible, then YES!

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oFactorization
oThe XYZ case
oThe XXZ case
oSeparable State Engineering
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Properties (I)

• Lemma 1 : Given n_i and n_j arbitraries, there always exists a nonzero XYZ -type coupling: $J^{ij}_{\mu\nu} = J^{ij}_{\mu}\delta_{\mu\nu}$ satisfying (1).



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• Lemma 2 : Given \mathcal{J}^{ij} y n_j , there always exists at least one n_i satisfying (1).

$$\boldsymbol{n}_i = \alpha [\boldsymbol{a} \times \boldsymbol{b} \pm (\eta \lambda_+ \boldsymbol{a} + \lambda_- \boldsymbol{b})],$$

 $a = \mathcal{J}^{ij} n_j^{x'}, b = \mathcal{J}^{ij} n_j^{y'}, \lambda_{\pm}^2 = \frac{\sqrt{(|a|^2 - |b|^2)^2 + 4|a \cdot b|^2} \pm (|a|^2 - |b|^2)}{2}$

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[•] Lemma 3 : $|\Psi_s\rangle$ can always become a nondegenerate GS of H with a controllable gap . h_i

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• Lemma 4 : If $s_i = s_j$ and the coupling is of the XYZ -type. Given n_i and n_j , there always exists a uniform factorizing field:

$$oldsymbol{h}^{ij}_{\parallel}+oldsymbol{h}^{ij}_{\perp}=oldsymbol{h}^{ji}_{\parallel}+oldsymbol{h}^{ji}_{\perp}$$

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We can now think of **bulk** separable state engineering.

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■ Lemma 5 : Pairwise entanglement reaches full range in the vicinity of factorization. → Quantum critical point.

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Fixed and tunable couplings

Tunable couplings

Fixed couplings



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• General conditions for the existence of separable eigenstates in spin arrays.

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- General conditions for the existence of separable eigenstates in spin arrays.
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- Factorization feasible with simple architectures .

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- Factorization feasible with simple architectures .
- Engineer separable ground states .

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Conclusions and perspective

Thanks for your attention!

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Our works:

M. Cerezo, R. Rossignoli, N. Canosa, and E. Rios, Phys. Rev. Lett. 119, 220605 (2017). M. Cerezo, R. Rossignoli, and N. Canosa, Phys. Rev. A 94, 042335 (2016). M. Cerezo, R. Rossignoli, and N. Canosa, Phys. Rev. B 92, 224422 (2015).