Heavy dark matter and IceCube neutrinos

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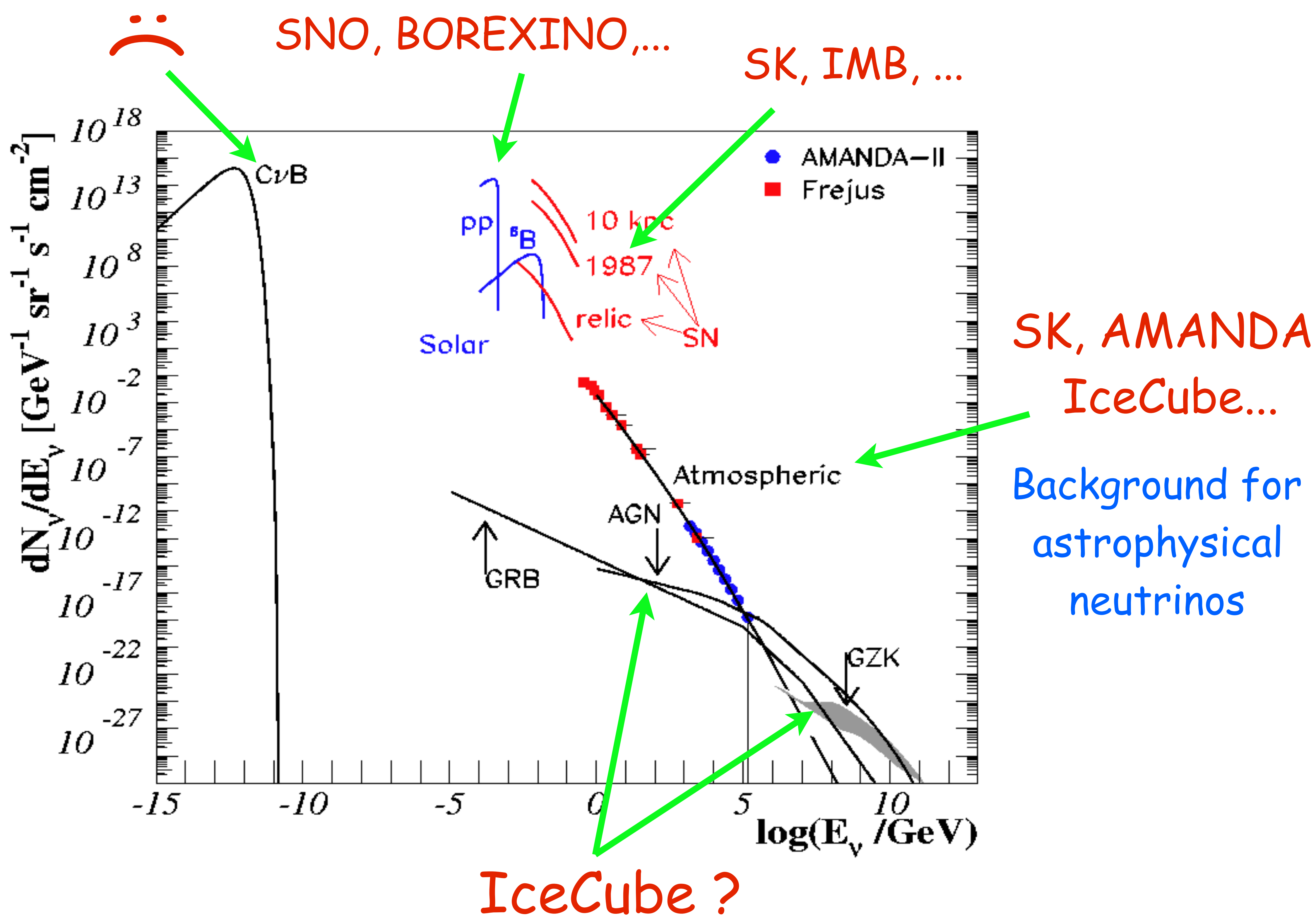
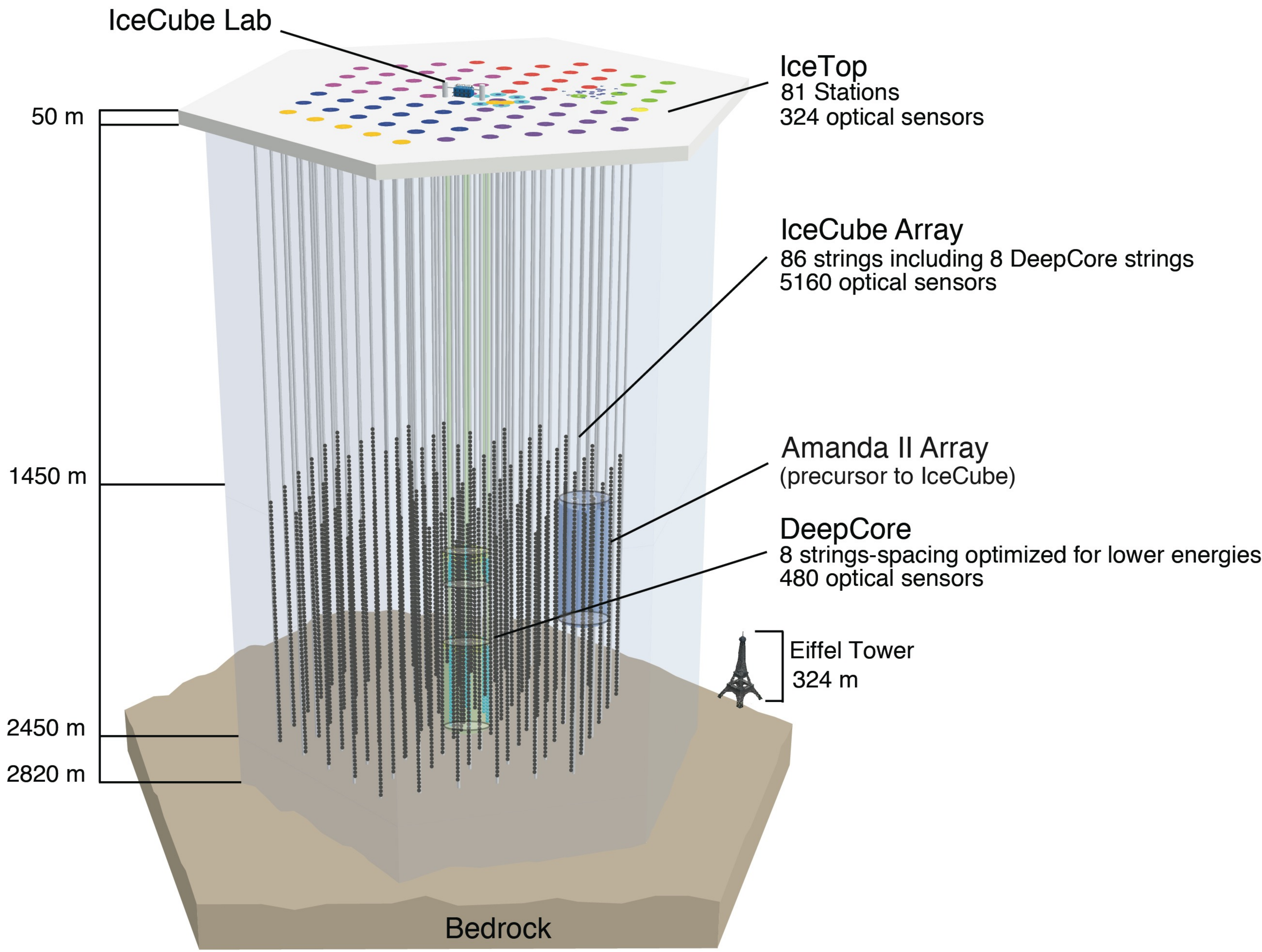
arXiv: 1308.1105 , 1410.5979 , 1505.06486 , 1706.05746 , 1903.12623

Neutrino Sky

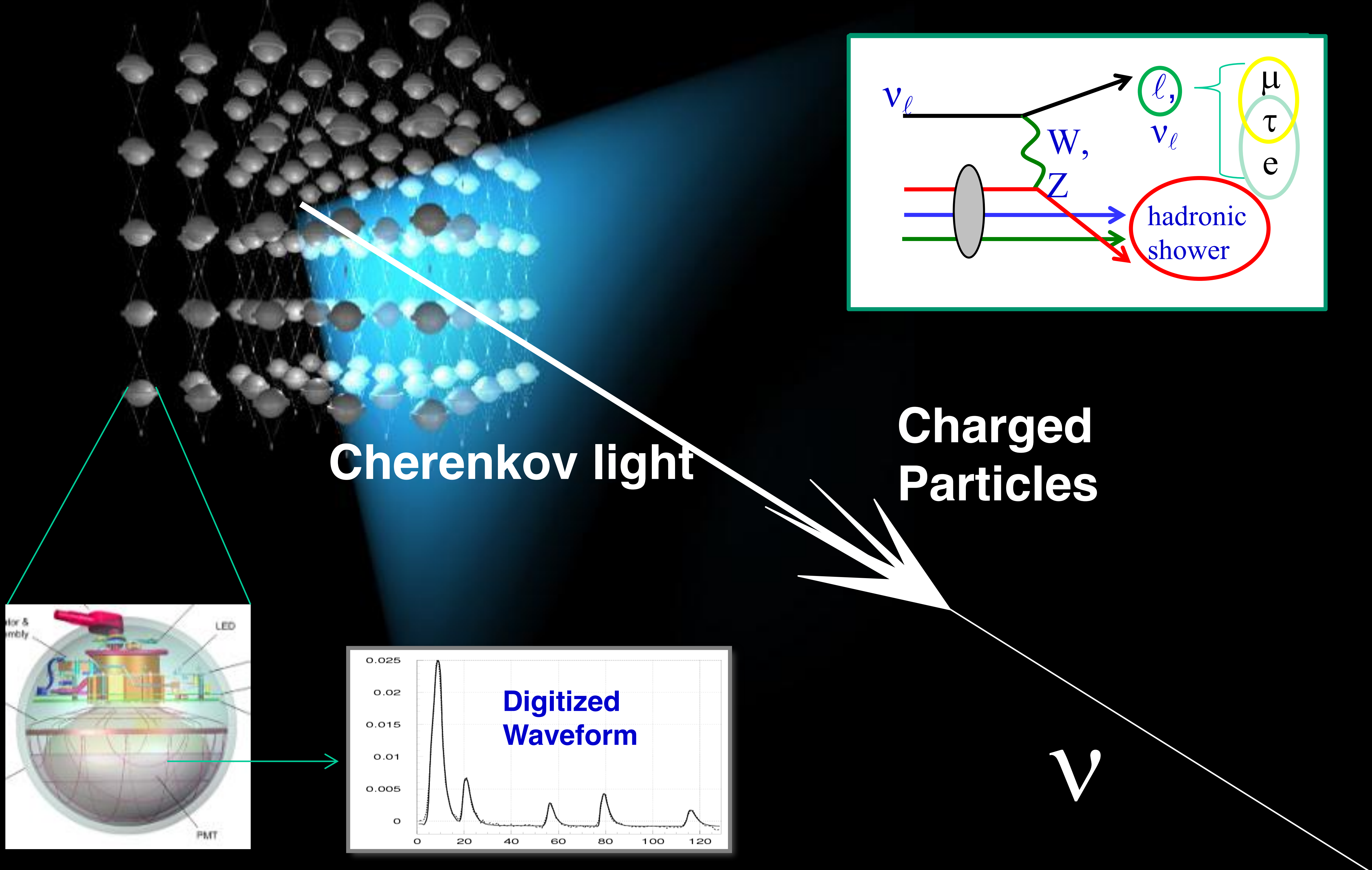



IceCube Lab
IceTop
81 Stations
324 optical sensors
50 m
IceCube Array
86 strings including 8 DeepCore strings
5160 optical sensors
1450 m
Amanda II Array
(precursor to IceCube)
DeepCore
8 strings-spacing optimized for lower energies
480 optical sensors
Eiffel Tower
324 m
2450 m
2820 m
Bedrock

Detection Principle

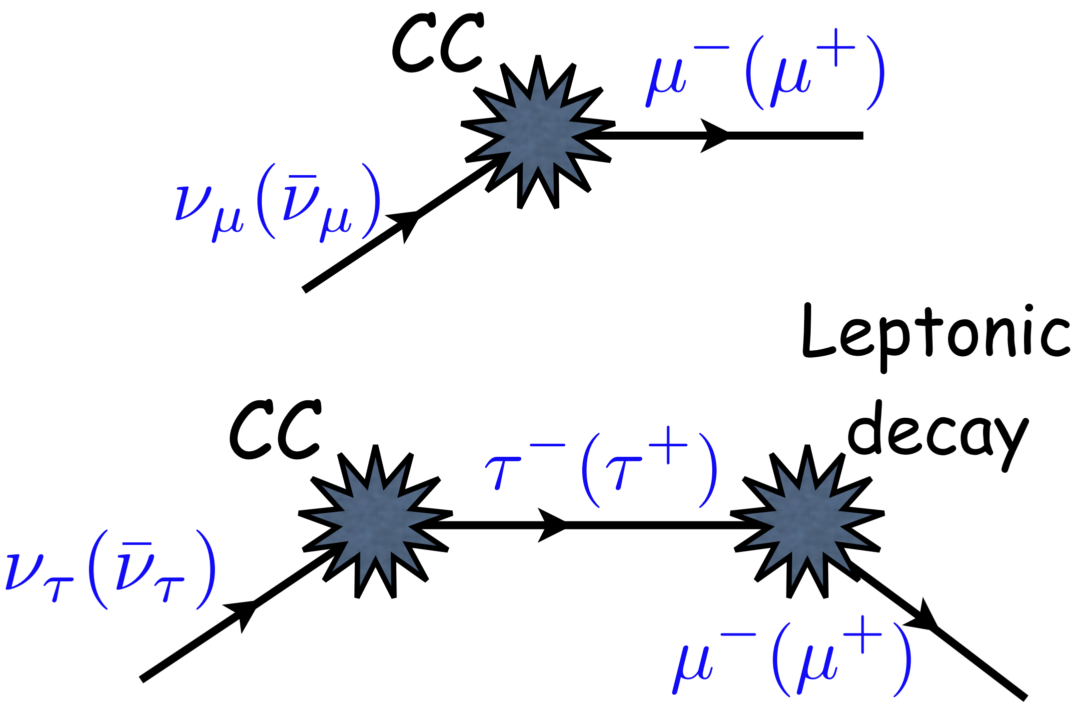
Slide from
A. Ishihara

An array of photomultiplier tubes + Dark and transparent material

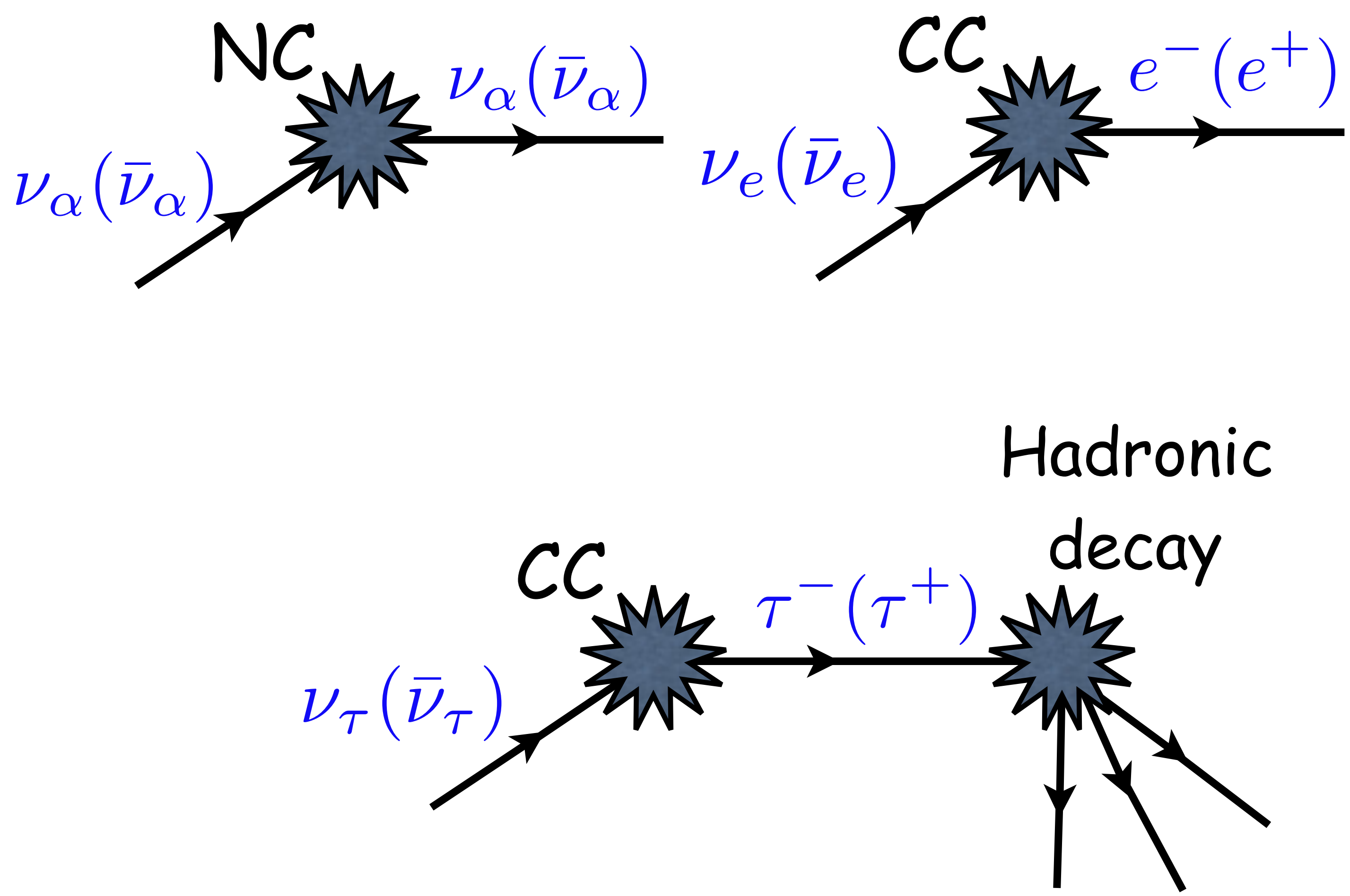


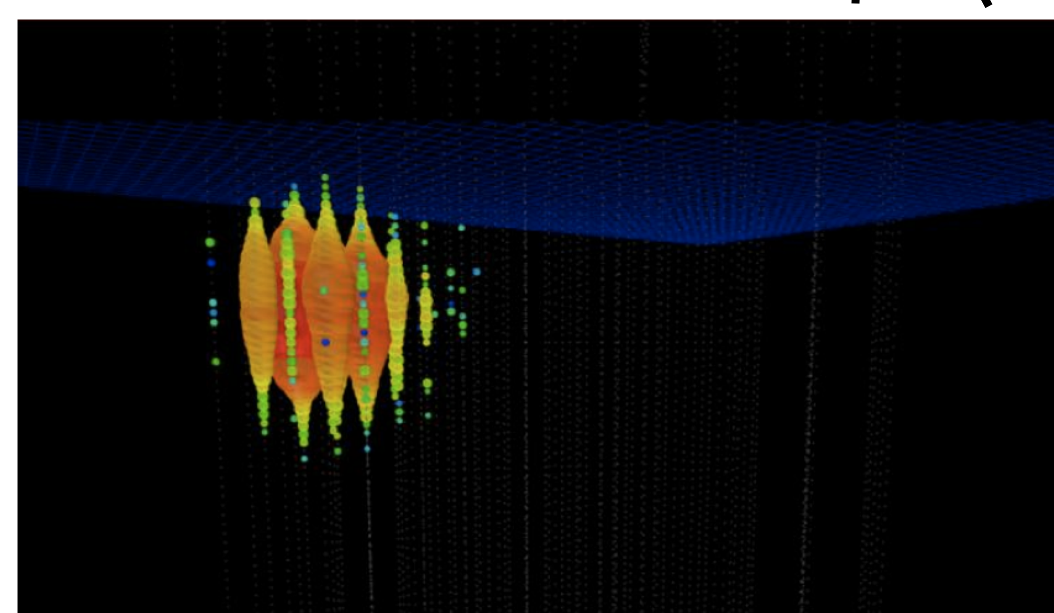
Flavoring at IceCube

muon-track events



cascade events





figures from IceCube website

Flavoring at IceCube

muon-track events

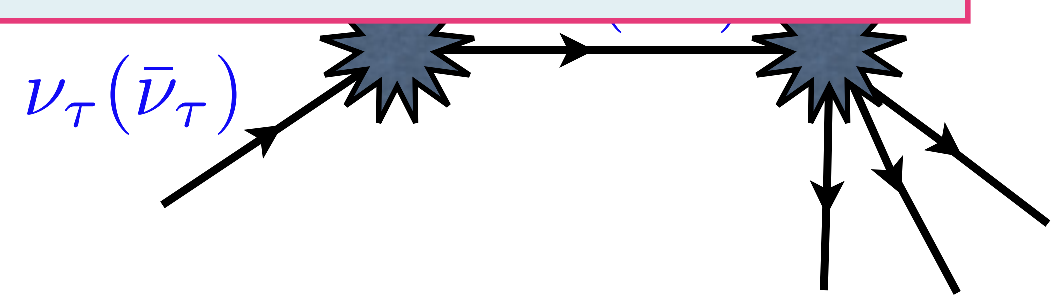
great angular resolution (< 1º)

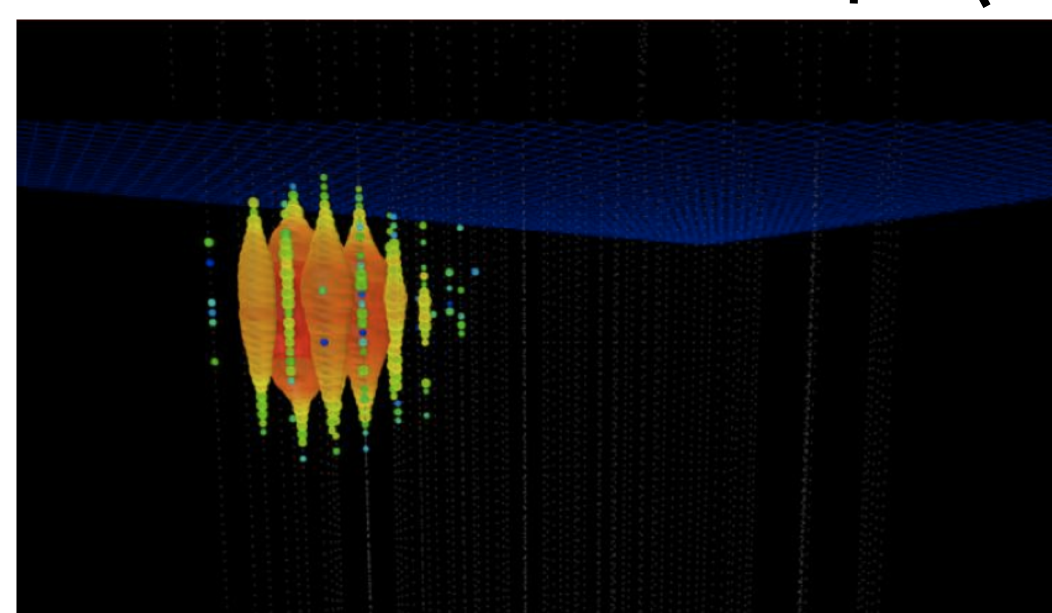
moderate energy resolution ($\sigma_E \sim E$)

cascade events

poor angular resolution (< 10º - 20º)

great energy resolution ($\sigma_E \sim 0.15 \times E$)

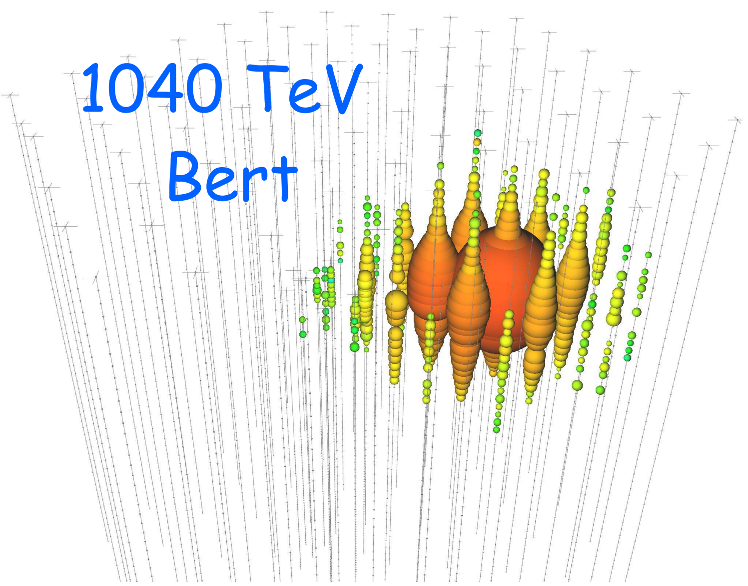




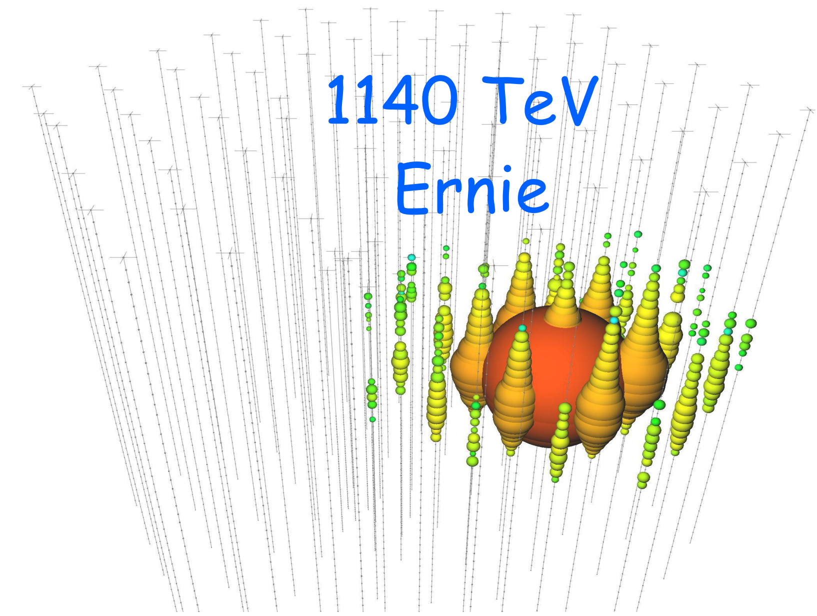
figures from IceCube website

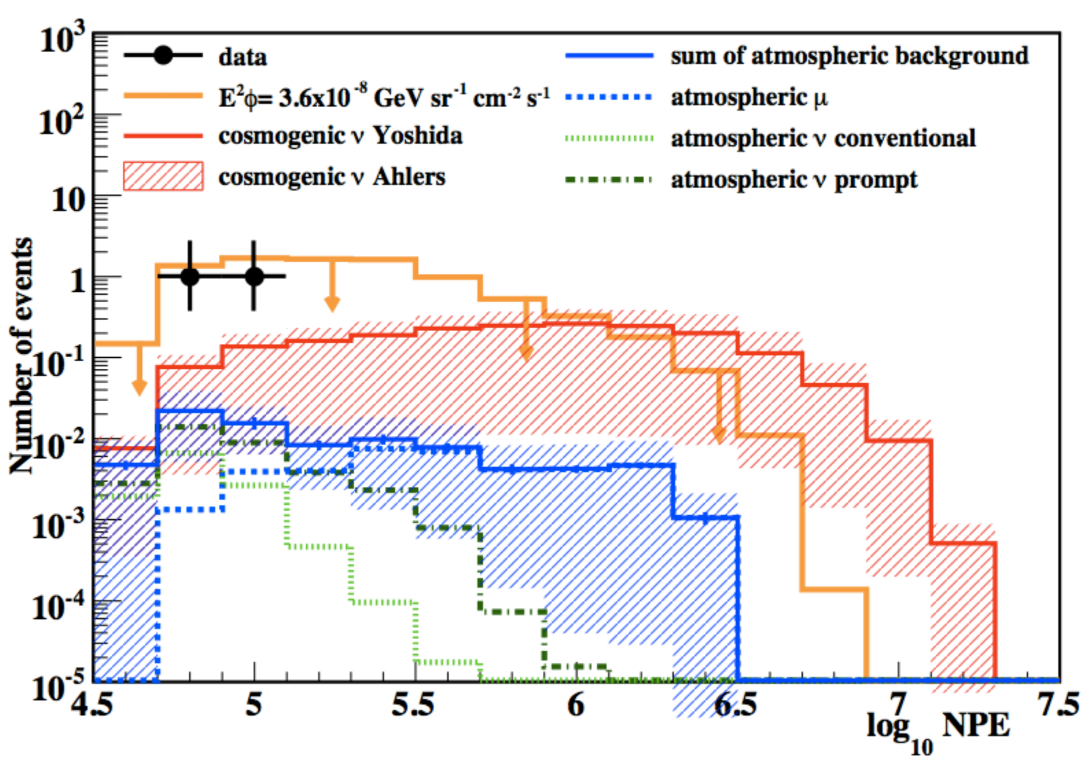
Observation of High Energy Neutrinos in IceCube

✓ The two PeV cascade events, 616 days livetime



M. G. Aartsen et al, PRL (2013)





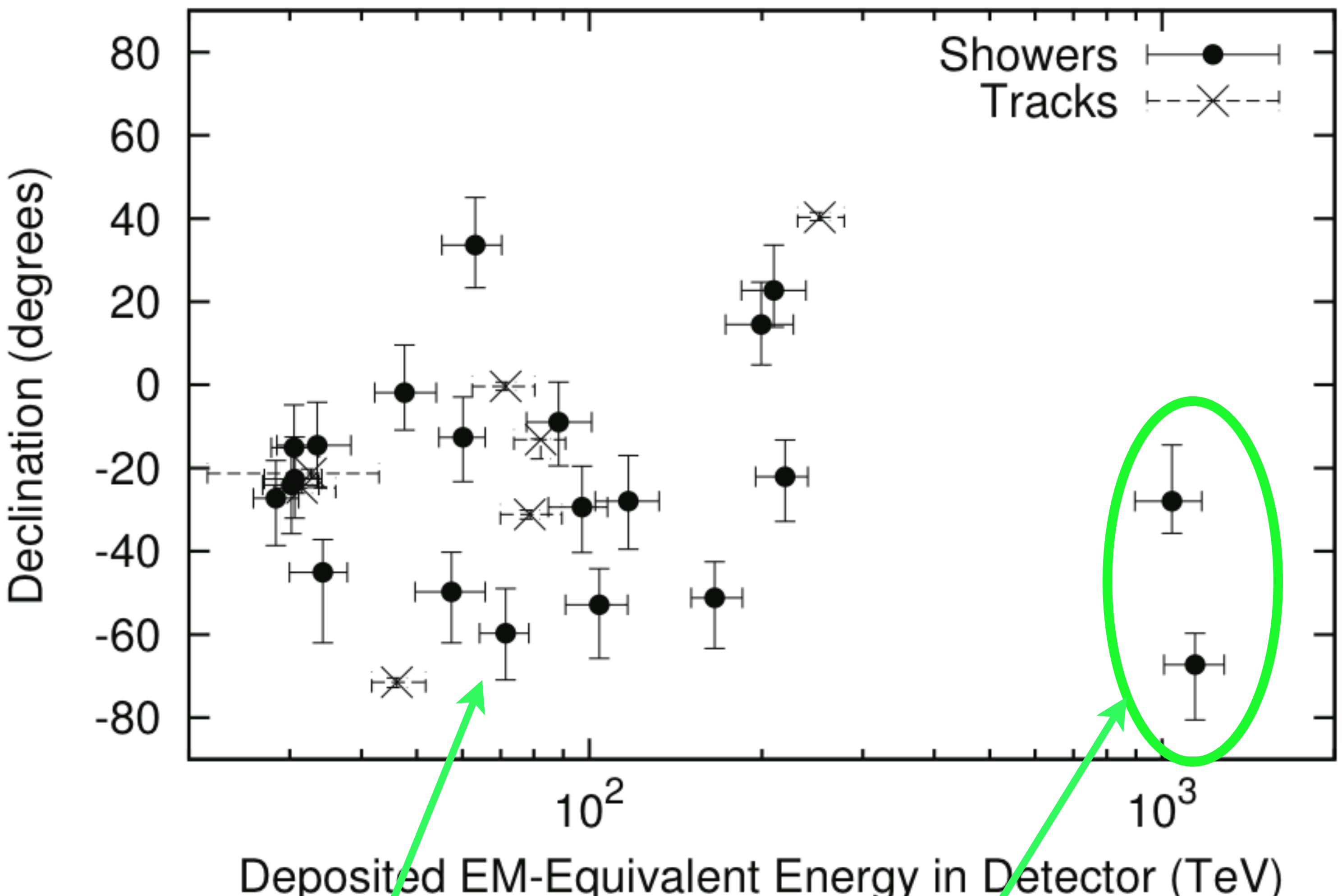
excess of events ~ 2.8σ

cosmogenic? too low energy, more events should be seen in higher energies

Observation of High Energy Neutrinos in IceCube

✓ Looking for lower energy contained events, 662 days livetime

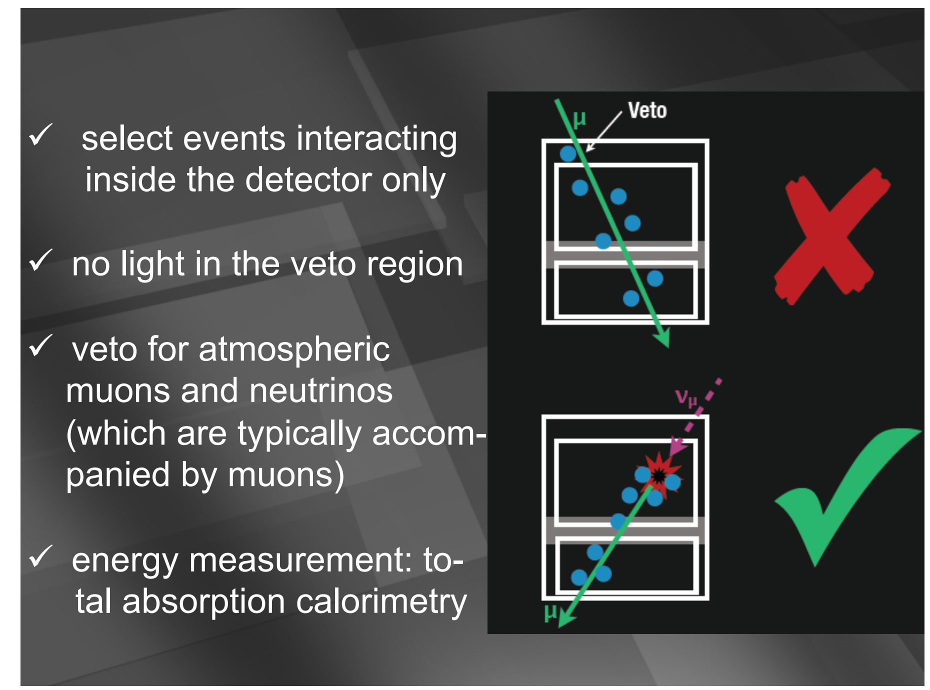
M. G. Aartsen et al. [IceCube Collaboration], Science 342 (2013), [arXiv:1311.5238]



The whole family!

previous PeV cascade events (Bert and Ernie)

HESE analysis



26 more events

excess of events ~ 4.3σ

Source(s) not identified!

Interpreting the IceCube events by decaying dark matter

B. Feldstein, A. Kusenko, S. Matsumoto and T. T. Yanagida,
PRD (2013), [arXiv:1303.7320]

A. E., Pasquale D. Serpico,
JCAP (2013) [arXiv:1308.1105]

Two main diagnostics:

✓ Energy distribution

✓ Angular distribution

A note on Dark Matter

DM exist!

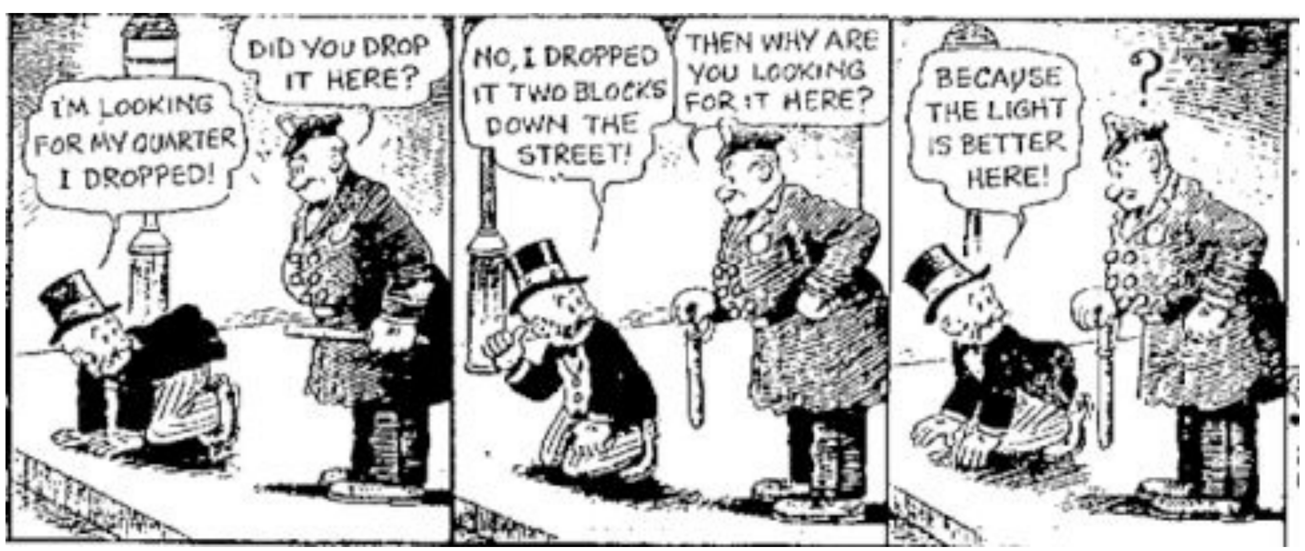
What We Do Not Know?

✓ Mass

axion — Sterile ν — WIMP — Wimpzilla

10^{-6} eV — ~KeV — ~100 GeV — M_{GUT} (10^{16} GeV)

caution: streetlight effect



Mulla Nasreddin

A note on Dark Matter

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✓ Energy distribution

✓ Angular distribution

Energy distribution of neutrinos from decaying DM

✓ Galactic contribution:

NFW $\rho_{\rm halo}(r) \simeq \frac{\rho_h}{r/r_c(1+r/r_c)^2}$

$$\frac{{\rm d}J_{\rm h}}{{\rm d}E_\nu}(l,b) = \frac{1}{4\pi\, m_{\rm DM}\, \tau_{\rm DM}} \frac{{\rm d}N_\nu}{{\rm d}E_\nu} \int_0^\infty {\rm d}s\ \rho_{\rm h}[r(s,l,b)]$$

$r(s,l,b) = \sqrt{s^2 + R_\odot^2 - 2sR_\odot \cos b \cos l}$

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✓ extragalactic contribution:

$$\frac{{\rm d}J_{\rm eg}}{{\rm d}E_\nu} = \frac{\Omega_{\rm DM}\rho_{\rm c}}{4\pi m_{\rm DM}\tau_{\rm DM}} \int_0^\infty {\rm d}z\, \frac{1}{H(z)}\, \frac{{\rm d}N_\nu}{{\rm d}E_\nu}\,[(1+z)E_\nu]$$

Energy distribution of neutrinos from decaying DM

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energy spectrum of neutrinos at production point (including the EW corrections)

quarks

neutrinos, charged leptons

$$\frac{\mathrm{d}N_\nu}{\mathrm{d}E_\nu} = (1-b_{\rm H}) \left.\frac{\mathrm{d}N_\nu}{\mathrm{d}E_\nu}\right|_{\rm S} + b_{\rm H} \left.\frac{\mathrm{d}N_\nu}{\mathrm{d}E_\nu}\right|_{\rm H}$$

Energy distribution of neutrinos from decaying DM

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at the Earth

$$\begin{pmatrix} J_e \\ J_\mu \\ J_\tau \end{pmatrix} = \begin{pmatrix} P_{ee} & P_{e\mu} & P_{e\tau} \\ & P_{\mu\mu} & P_{\mu\tau} \\ & & P_{\tau\tau} \end{pmatrix} \begin{pmatrix} I_e \\ I_\mu \\ I_\tau \end{pmatrix}$$

production point

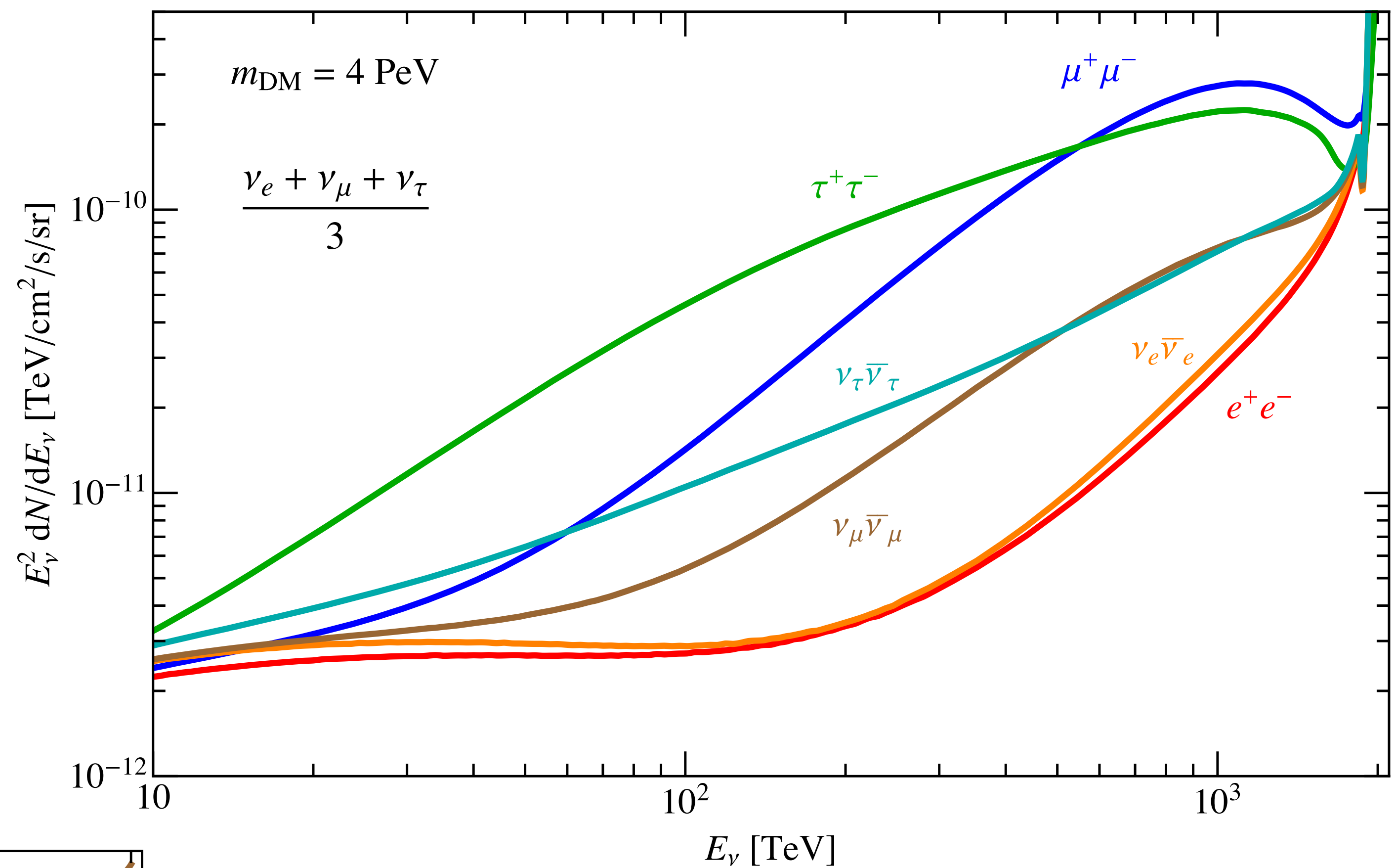
decoherent oscillation

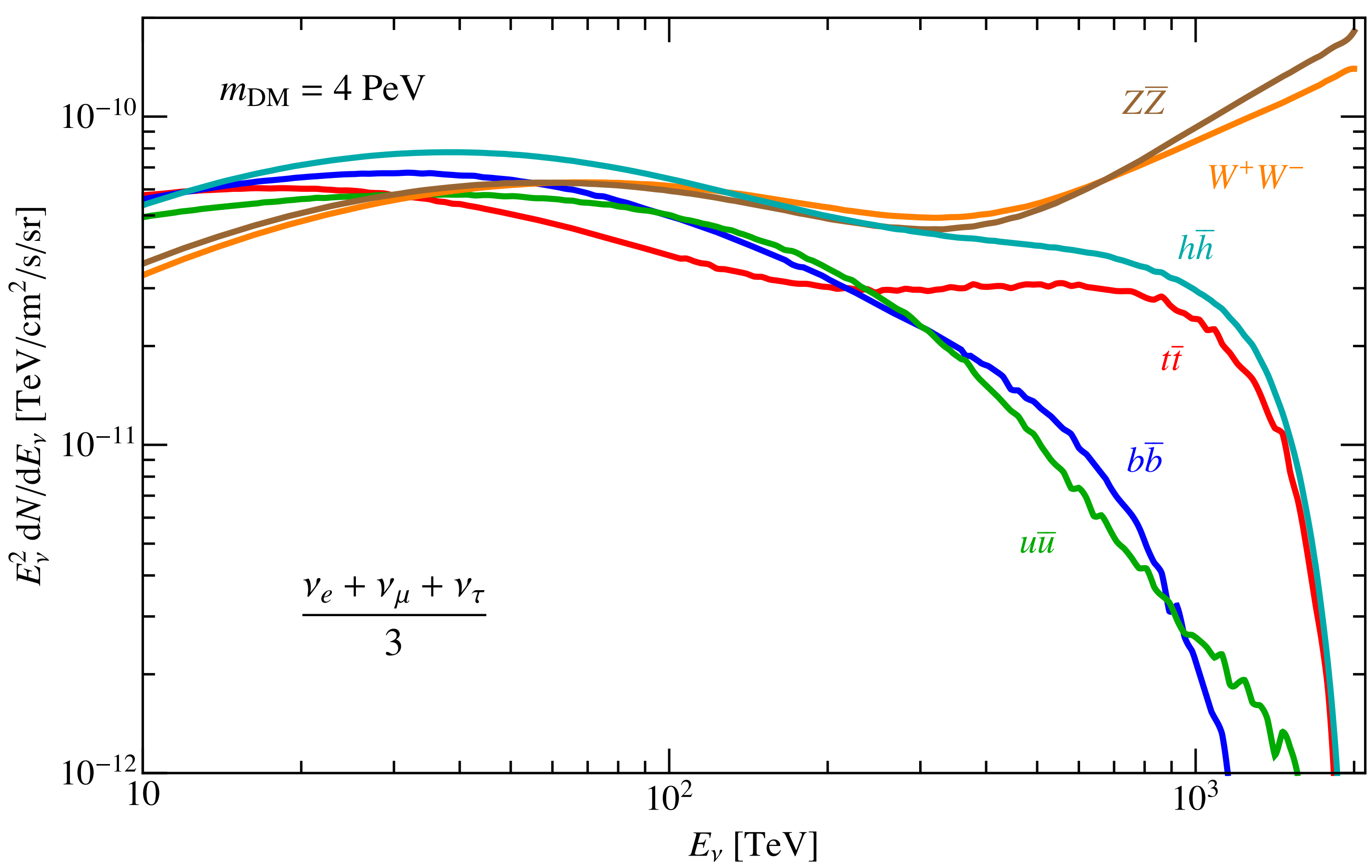
Flux of neutrinos from decaying DM

A. Bhattacharya, A. E., S. Palomares-Ruiz, I. Sarcevic, JCAP (2017) [arXiv:1706.05746]

m_{DM} = 4 PeV

$\tau_{DM} = 10^{27}$ s





EW corrections play an important role

PYTHIA 8.2

Flux of neutrinos from decaying DM

✓ an example:

intriguing features:

a cut-off at $m_{DM}/2$

a peak in ~ PeV

flux is not feature-less

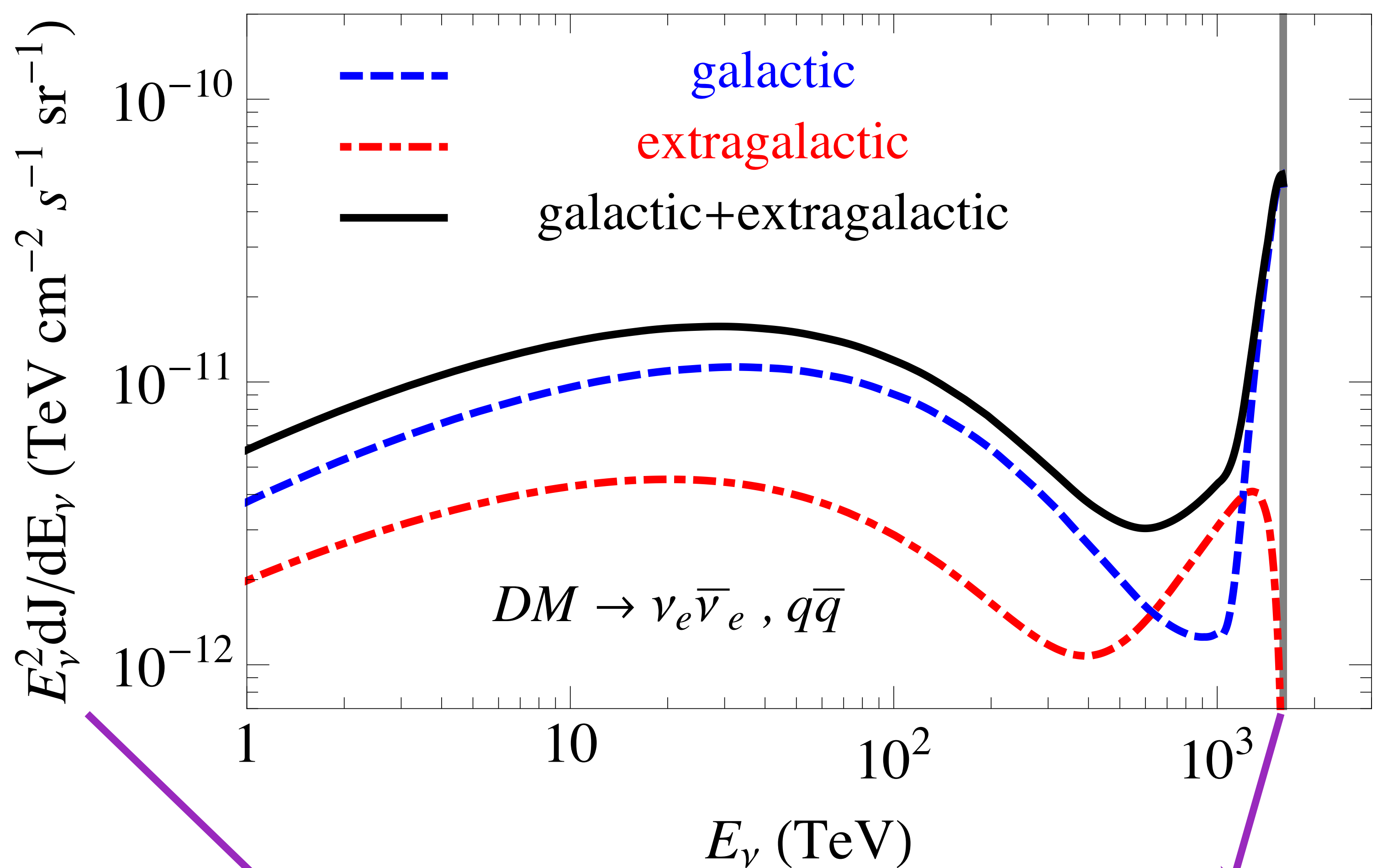
populated spectrum in < 0.4 PeV

due to soft channel and EW cascades

b_H controls the peak height at ~ PeV

τ_{DM} controls the low energy population

A. E., Pasquale D. Serpico, JCAP (2013) [arXiv:1308.1105]



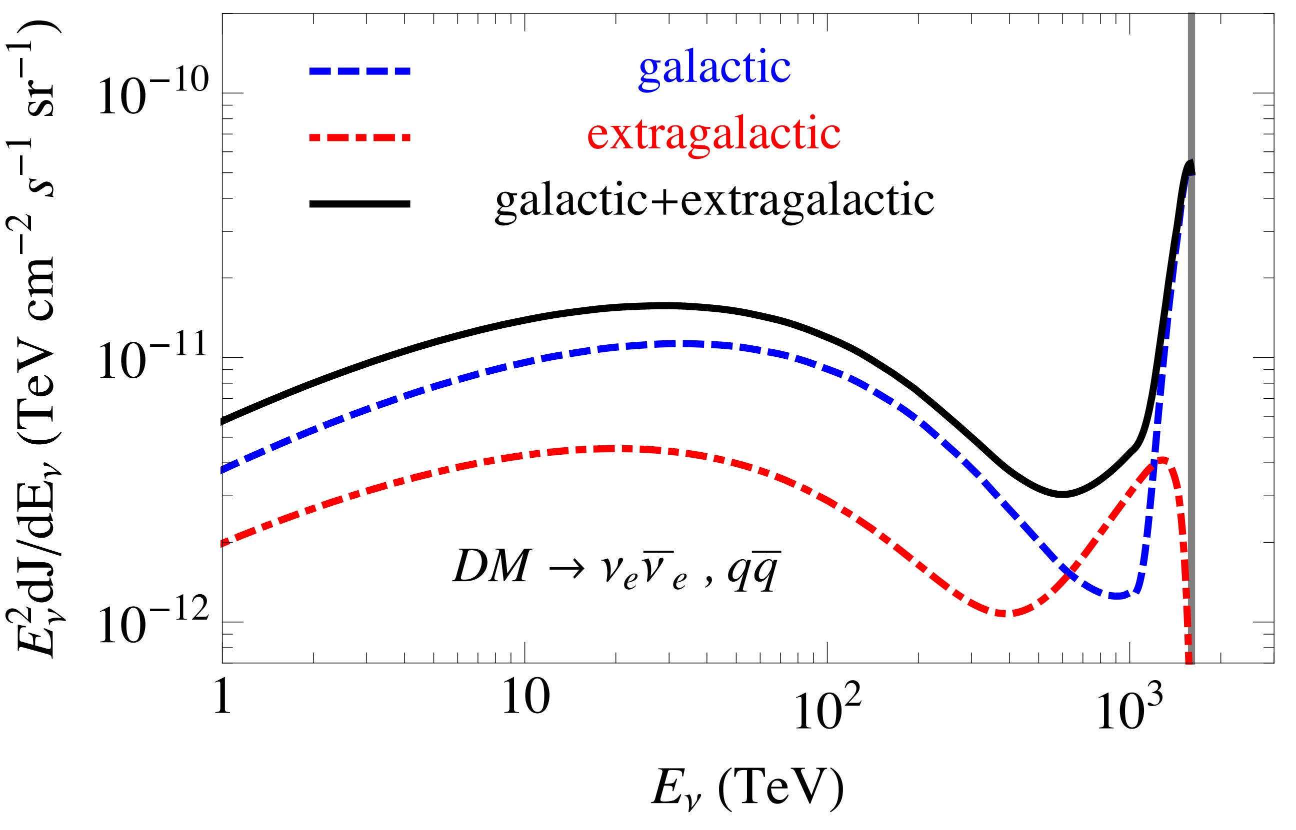
$(\nu_e + \nu_\mu + \nu_\tau)/3$

$m_{DM}/2 = 1.6$ PeV

$b_H = 0.12$ and $\tau_{DM} = 2 \times 10^{27}$ s

Flux of neutrinos from decaying DM

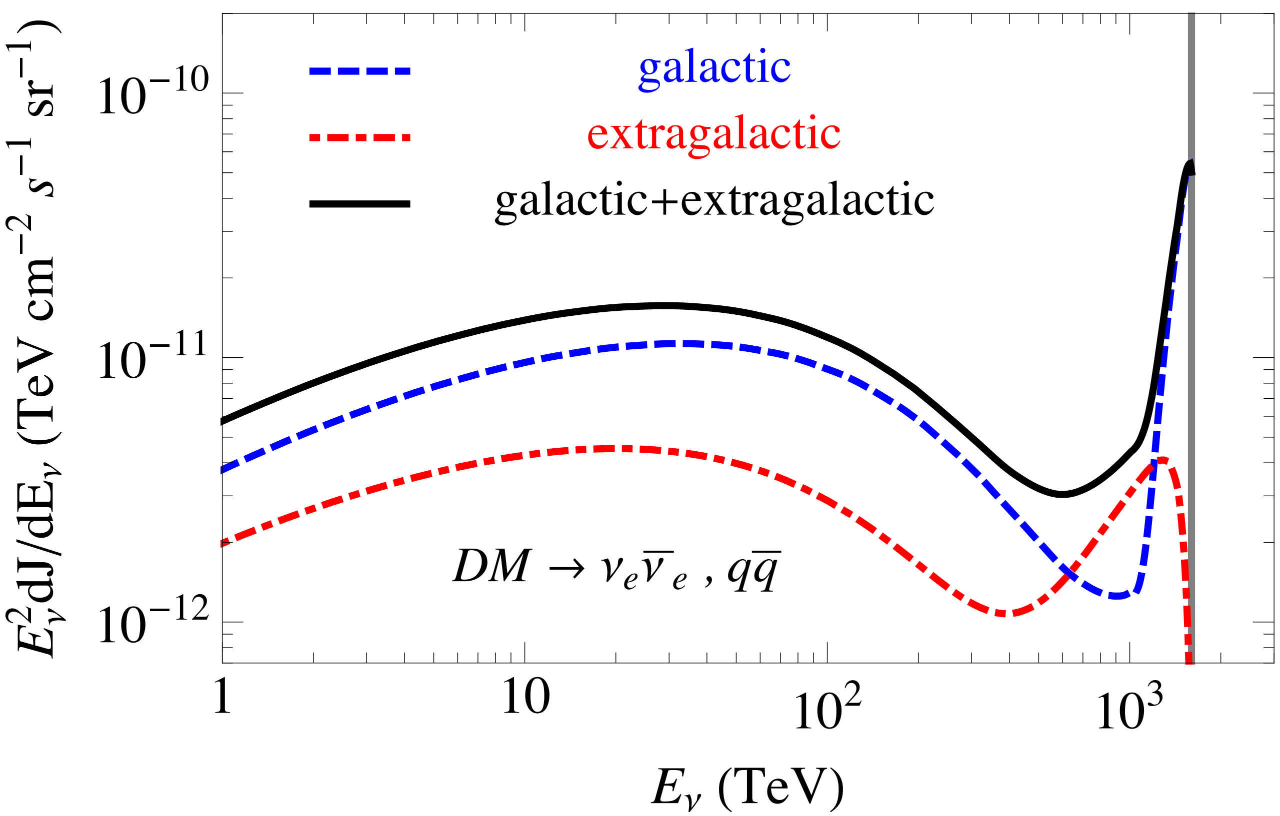
✓ fine-tuned decay channels ?

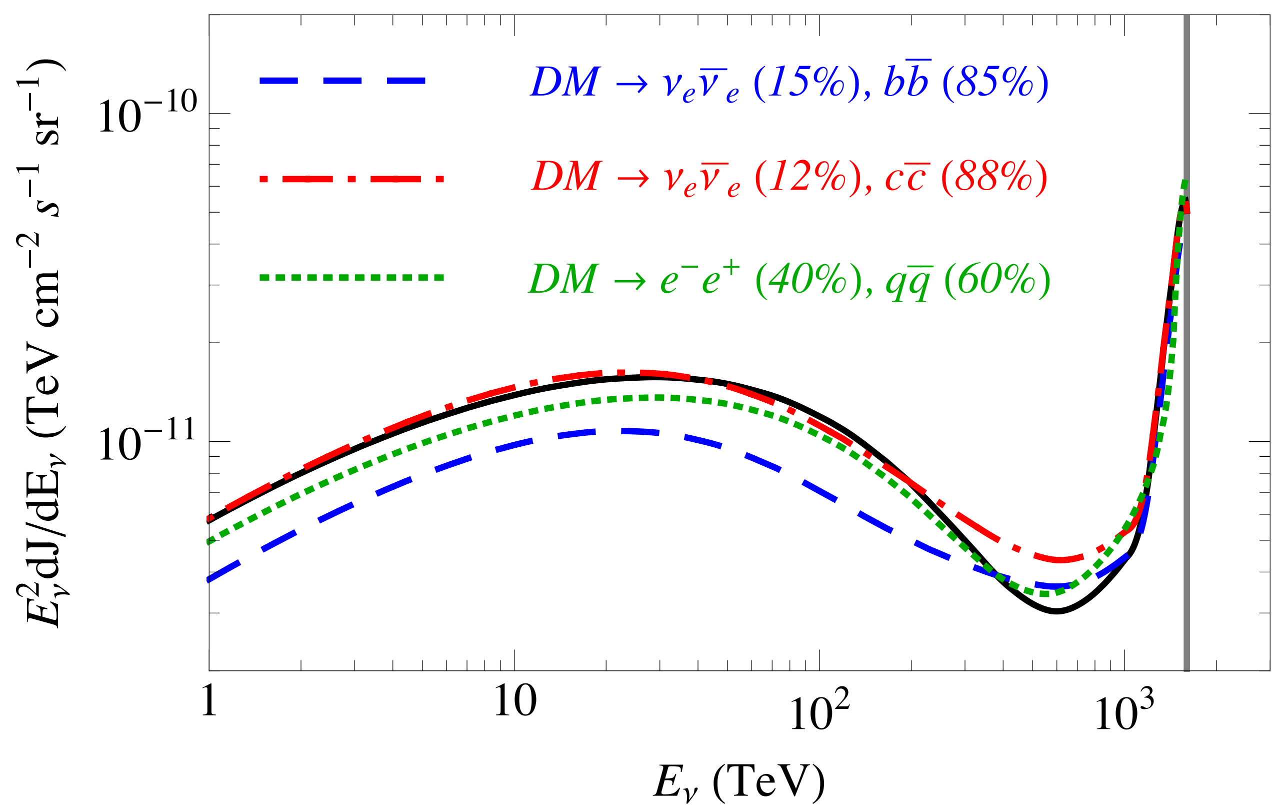


Flux of neutrinos from decaying DM

✓ fine-tuned decay channels ?

τ_{DM} = (1-3) x 10^{27} s



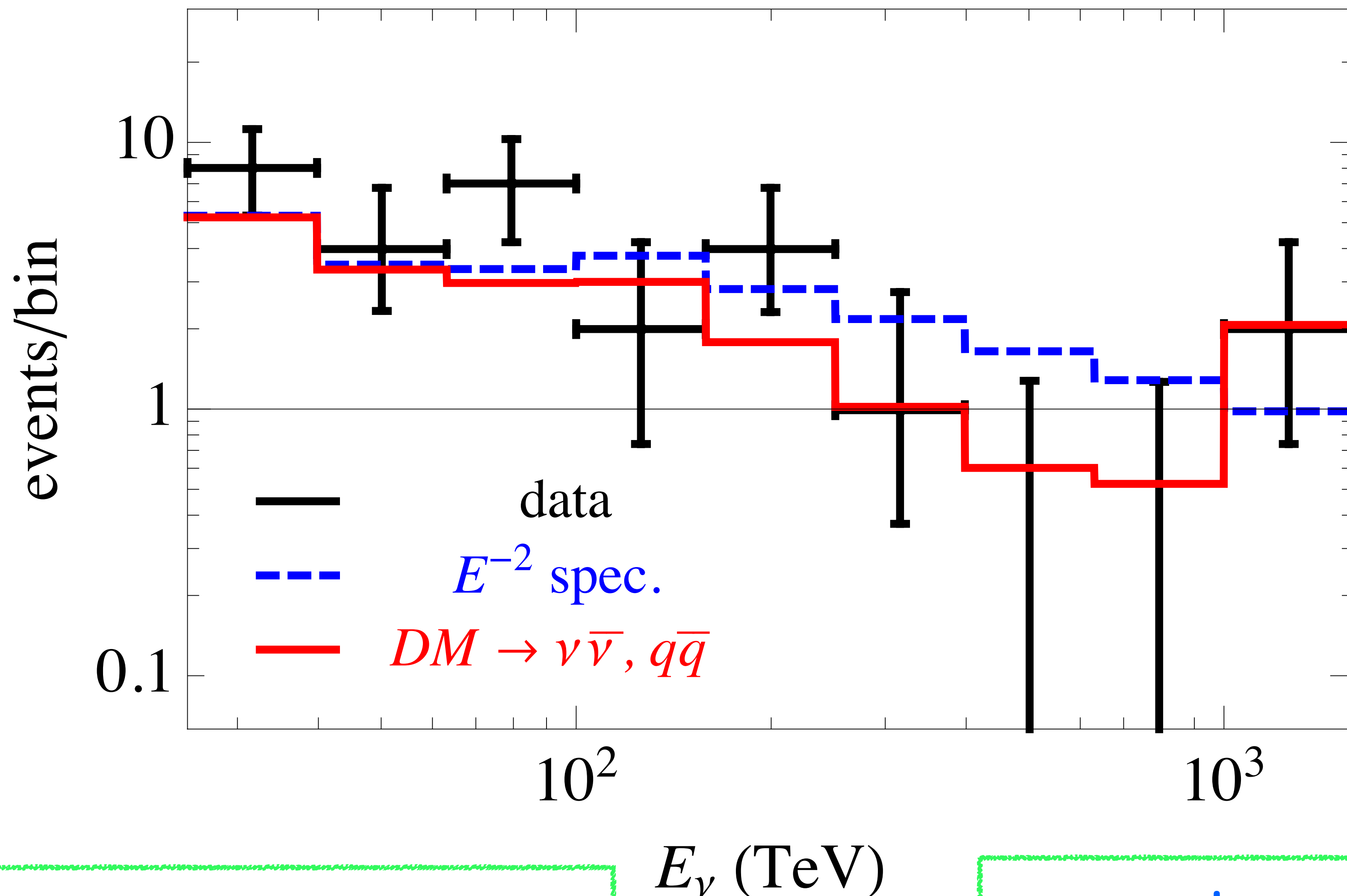


the intriguing features are generic

Confronting with energy distribution of IceCube data

$b_H = 0.12$ and $\tau_{DM} = 2 \times 10^{27}$ s

2 years data set



the low energy bins contain large bkg. contribution

natural explanation for the lack of events > PeV

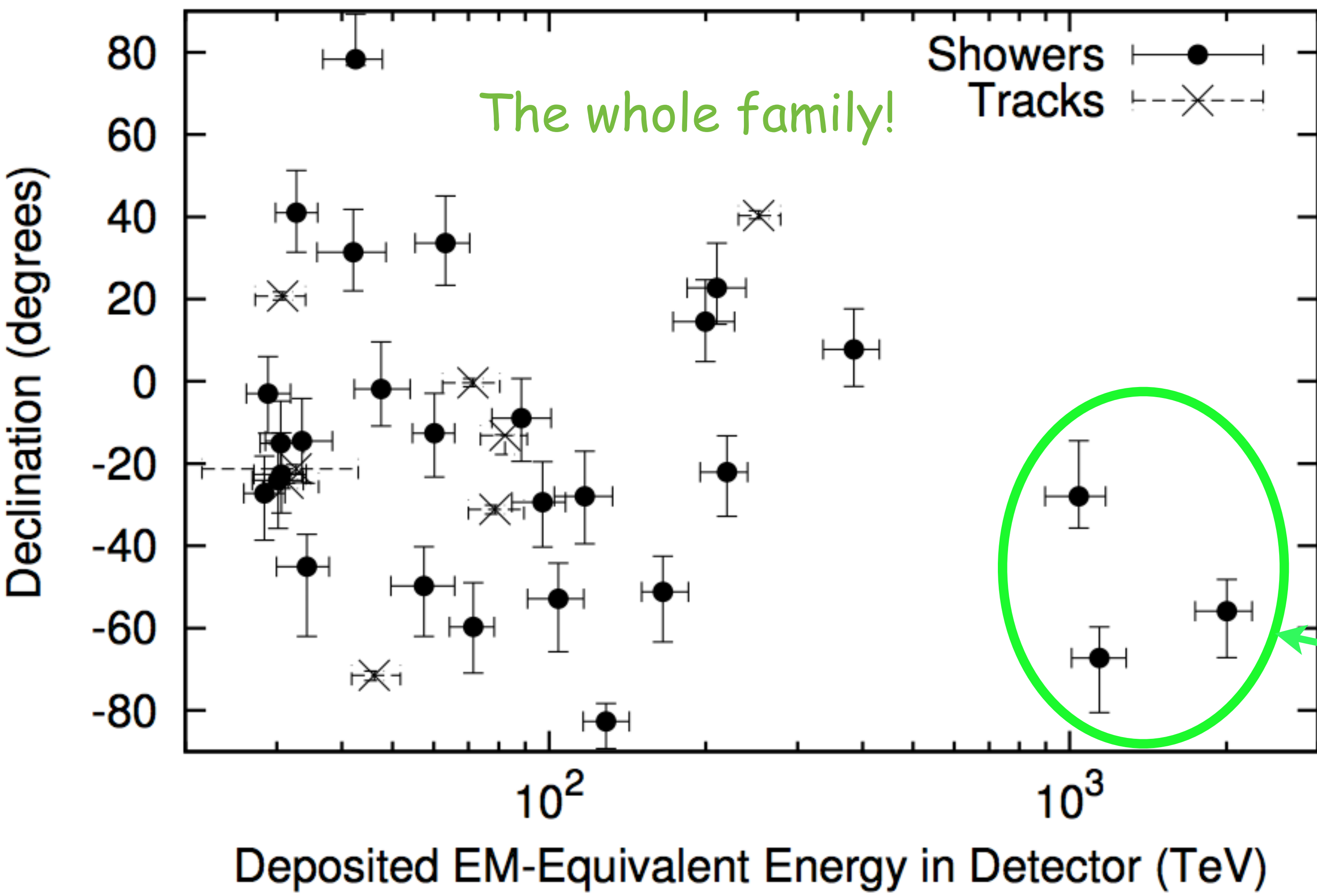
different decay channels lead to qualitatively same result

the value of τ_{DM} is compatible with the bounds derived from neutrinos and gamma rays

Observation of High Energy Neutrinos in IceCube

✓ Looking for lower energy contained events, 988 days livetime

M. G. Aartsen et al. [IceCube Collaboration], PRL 113 (2014), [arXiv:1405.5303]



✓ totally 37 events

✓ three events with energy ~ PeV

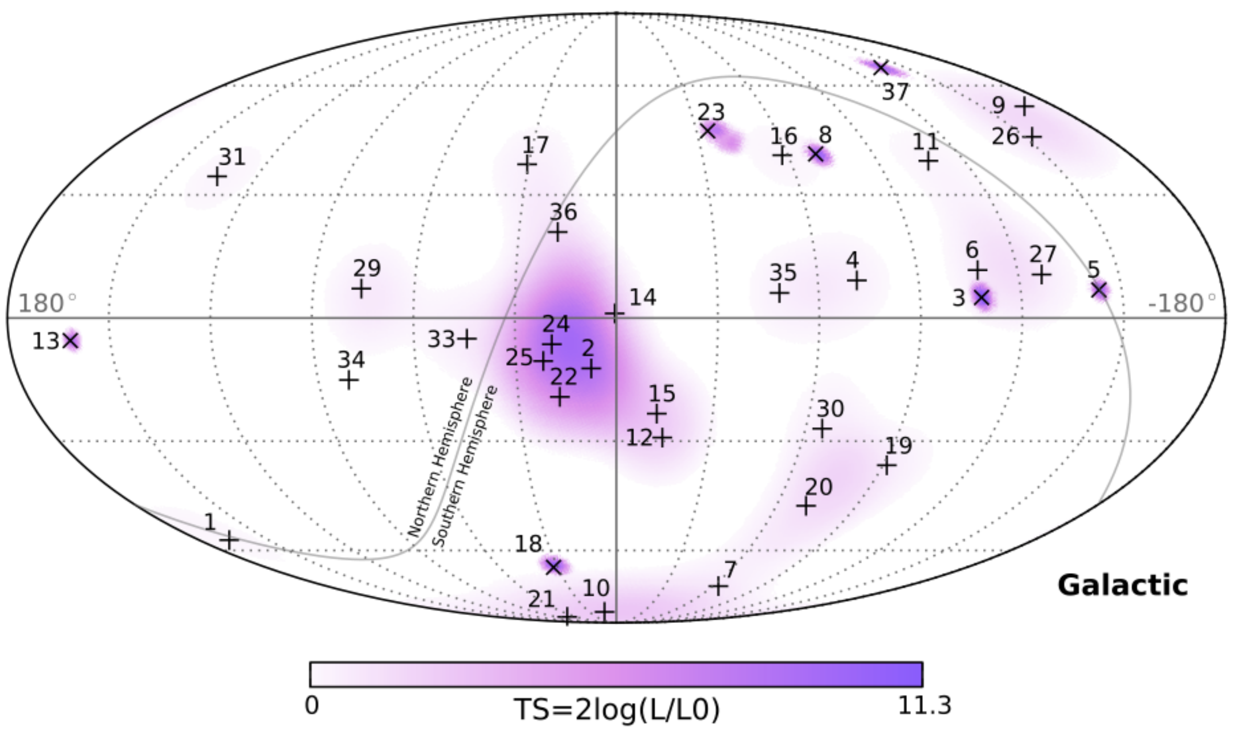
✓ expected bkg. (conventional+prompt) ~ 15.6(+10.1)(-5.8) sys.

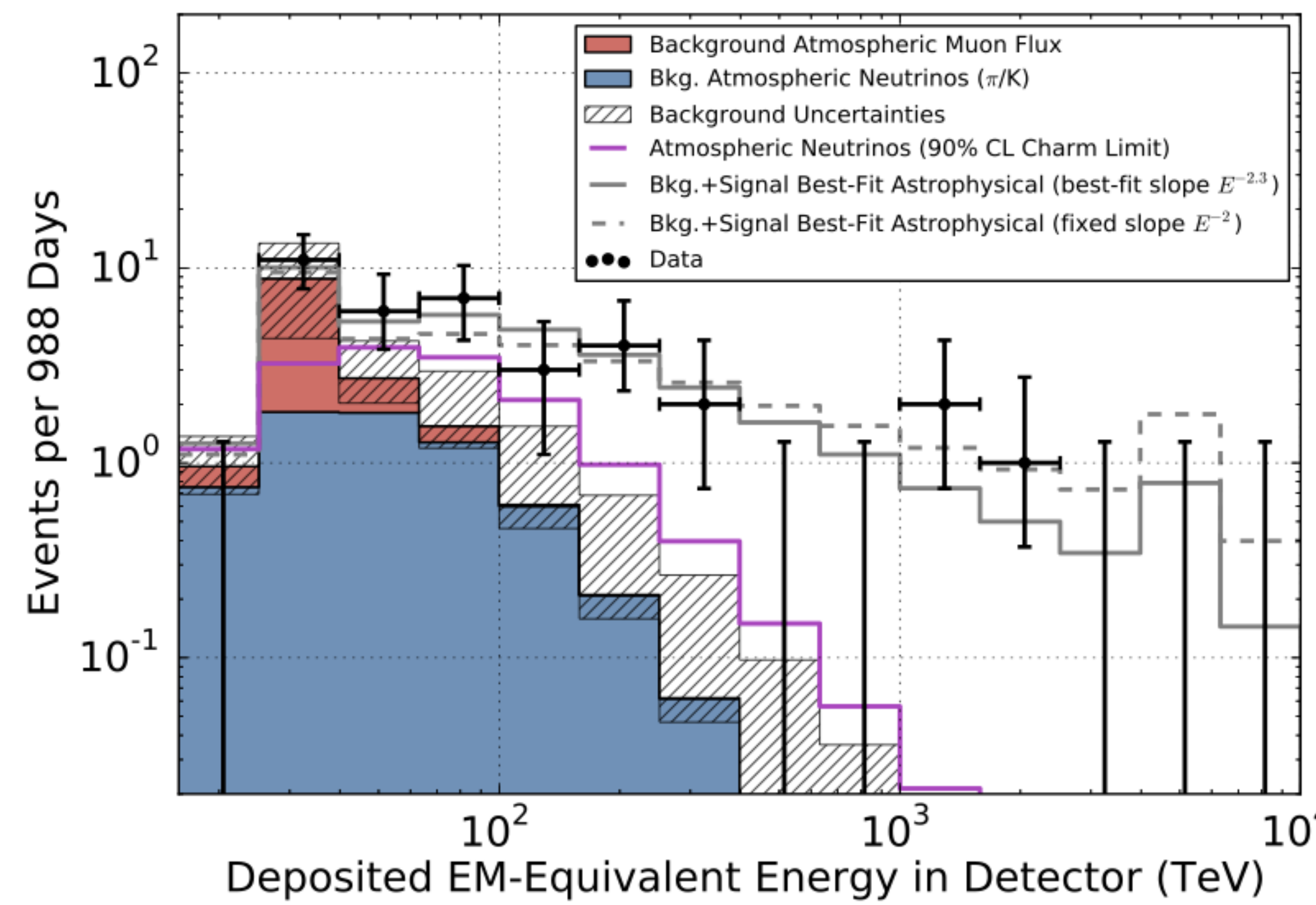
excess of events ~ 5.7σ

Source(s) not identified!

IceCube data

✓ Looking for lower energy contained events, 988 days livetime





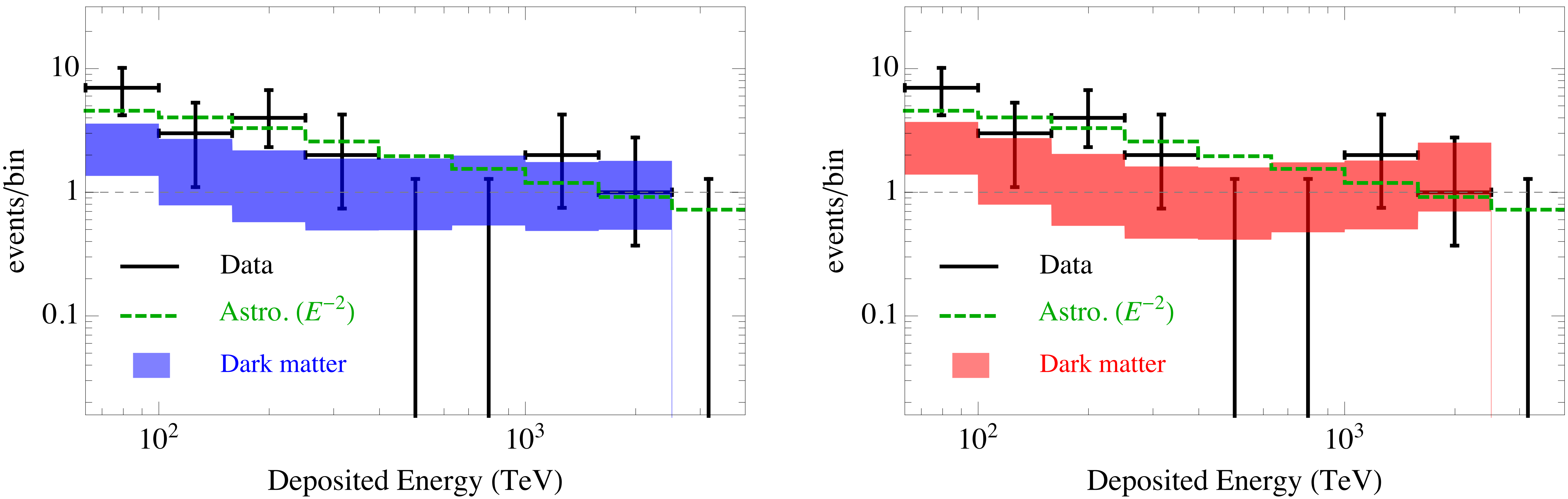
3 years of data

Confronting with energy distribution of IceCube data

3 years data set

A. E., S. K. Kang and P. Serpico,
JCAP (2014) [arXiv:1410.5979 [hep-ph]]

IH, $\tau_{DM} = 1.1 \times 10^{28}$ s $m_{DM} = 4$ PeV NH, $\tau_{DM} = 7.3 \times 10^{27}$ s



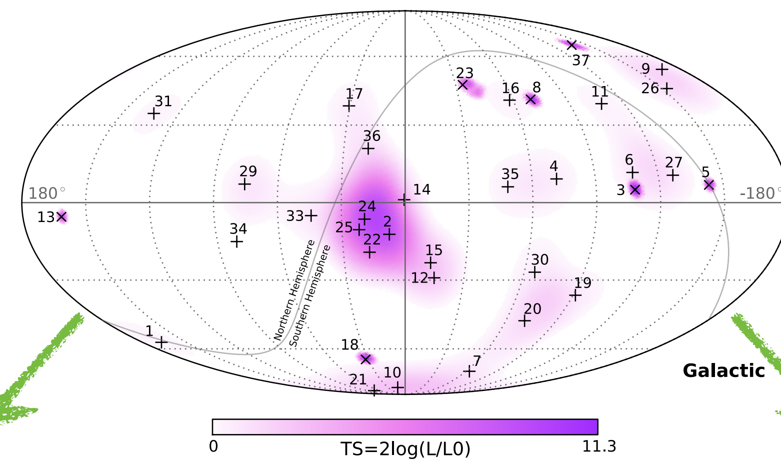
Calculation based on a model for DM: neutrino portal with dim-4 operator (heavy sterile neutrino), B-L symmetry (inflation), Leptogenesis (other sterile neutrinos), with production mechanism (either inflation decay or freeze-in mechanism)

T. Higaki, R. Kitano and R. Sato,
JHEP (2014) [1405.0013]

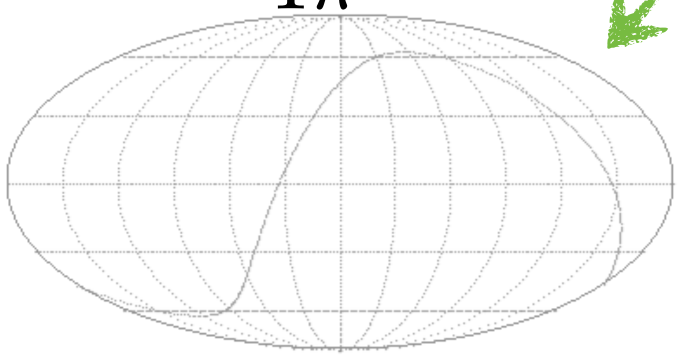
The predicted neutrino flux is fixed by the model

Angular distribution of neutrinos from decaying DM

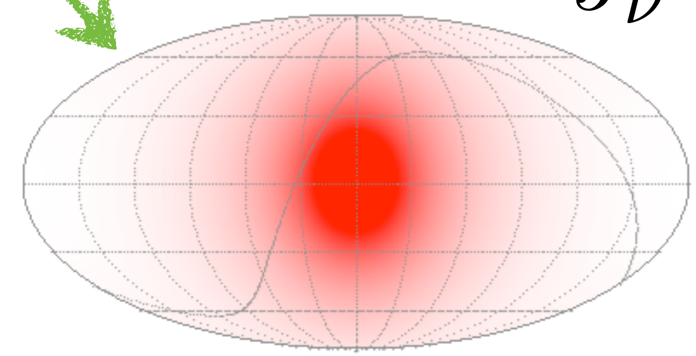
✓ We would compare



$$p^{\rm iso} = \frac{1}{4\pi}$$



$$p^{\rm DM} = \frac{1}{J_\nu}\frac{{\rm d}^2 J_\nu}{{\rm d}b\,{\rm d}l}$$



Angular distribution of neutrinos from decaying DM

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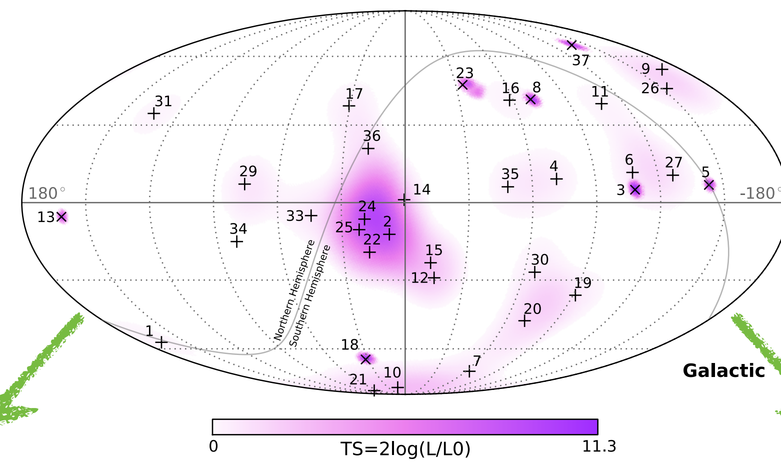

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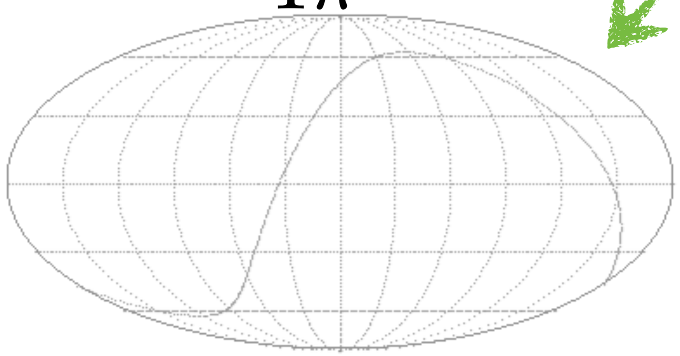

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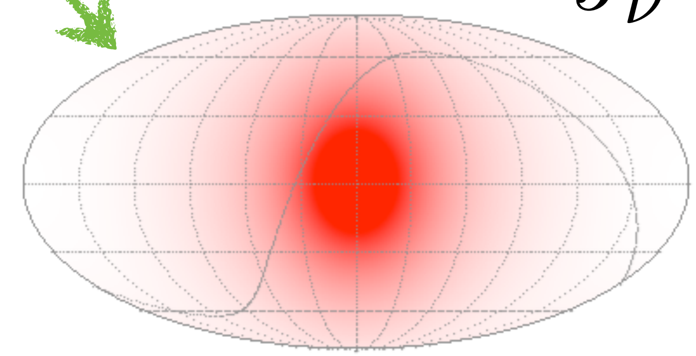

PDF of data

$$p_i(b,l) = \frac{1}{2\pi\sigma_i^2}\exp\left[-\frac{|\vec{x}-\vec{x}_i|^2}{2\sigma_i^2}\right]$$

"flat sky" approximation

PDF of isotropic dis.

$$p^{\rm iso} = \frac{1}{4\pi}$$

PDF of DM

$$p^{\rm DM}(b,l) = \frac{1}{J_\nu}\frac{{\rm d}^2 J_\nu}{{\rm d}b\,{\rm d}l} = \frac{\int_0^\infty \rho[r(s,b,l)]{\rm d}s + \Omega_{\rm DM}\rho_c\beta}{4\pi(\eta + \Omega_{\rm DM}\rho_c\beta)}$$

Angular distribution of neutrinos from decaying DM

✓ Likelihood analysis

Number of signal events

Test Statistics

$$\mathrm{TS_{like}} = 2\sum_{i=1}^{N}\left(\ln f_i - \ln p_i^{\mathrm{iso}}\right) = 2\ln\left(\prod_{i=1}^{N} f_i\right) - 2N\ln\left(\frac{1}{4\pi}\right)$$

$$f_i = \int p_i(b,l)\, p^{\mathrm{DM}}(b,l)\cos(b)\;\mathrm{d}b\;\mathrm{d}l = \frac{1}{2\pi\sigma_i^2}\int e^{-\frac{|\vec{x}_i-\vec{x}|^2}{2\sigma_i^2}}\, p^{\mathrm{DM}}(b,l)\cos(b)\;\mathrm{d}b\;\mathrm{d}l$$

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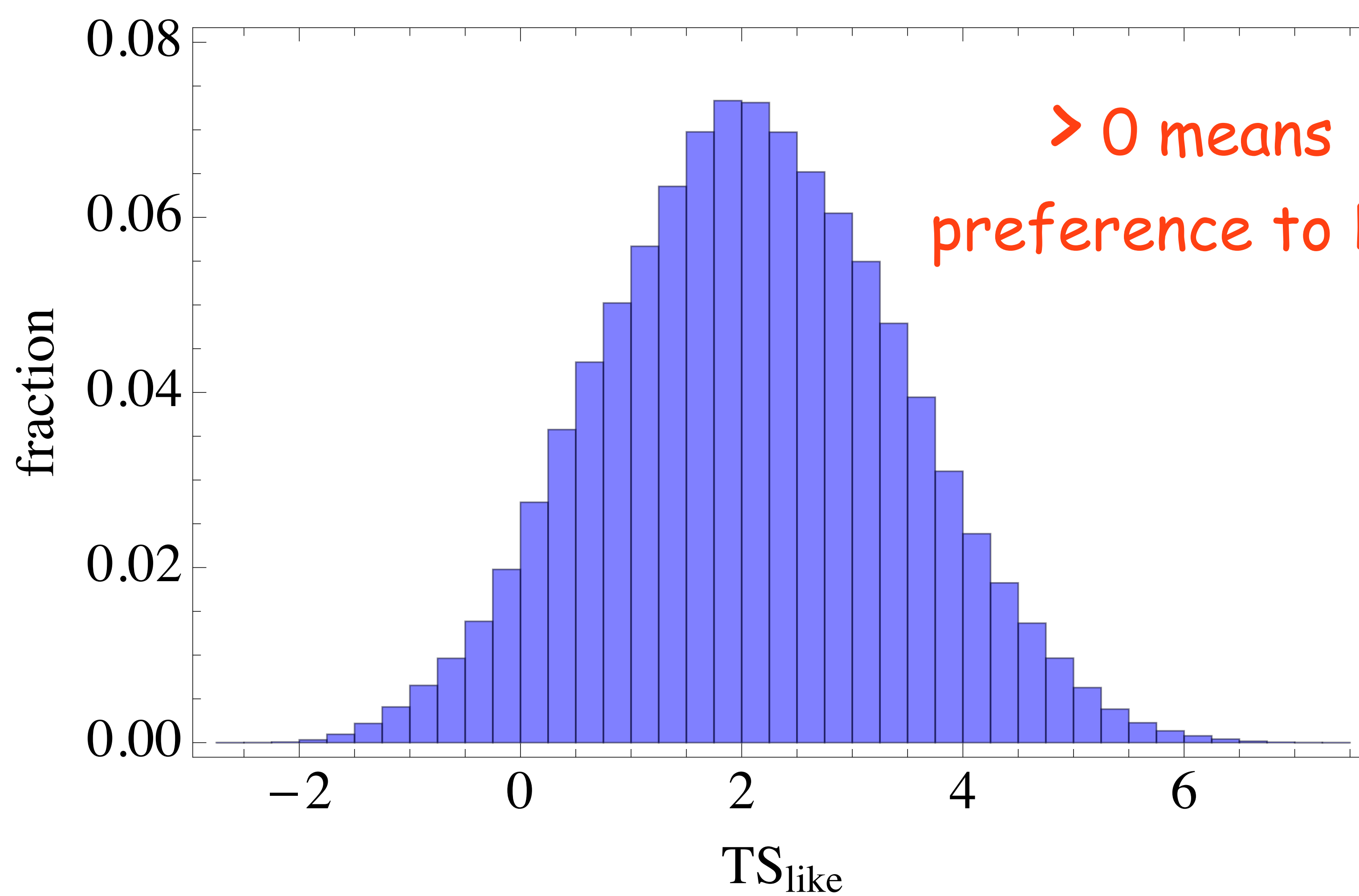
ways of selecting the bkg events among the low energy events $\binom{26}{15}$



> 0 means preference to DM

A. E., S. K. Kang and P. Serpico, JCAP (2014) [arXiv:1410.5979 [hep-ph]]

Angular distribution of neutrinos from decaying DM

✔ Likelihood analysis

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N → Number of signal events

$$f_i = \int p_i(b,l)\, p^{\mathrm{DM}}(b,l)\cos(b)\,\mathrm{d}b\,\mathrm{d}l = \frac{1}{2\pi\sigma_i^2}\int e^{-\frac{|\vec{x}_i-\vec{x}|^2}{2\sigma_i^2}}\, p^{\mathrm{DM}}(b,l)\cos(b)\,\mathrm{d}b\,\mathrm{d}l$$

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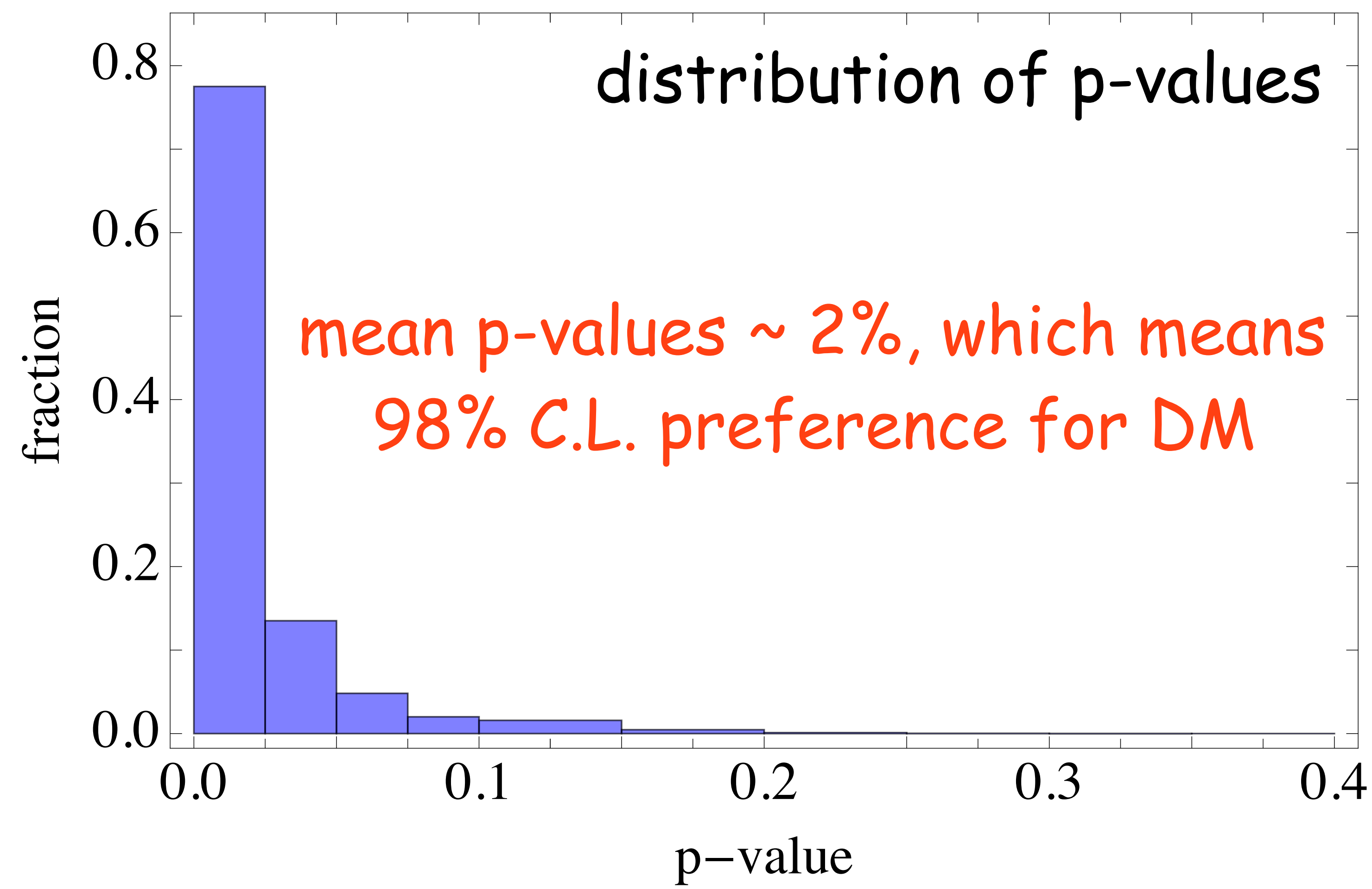
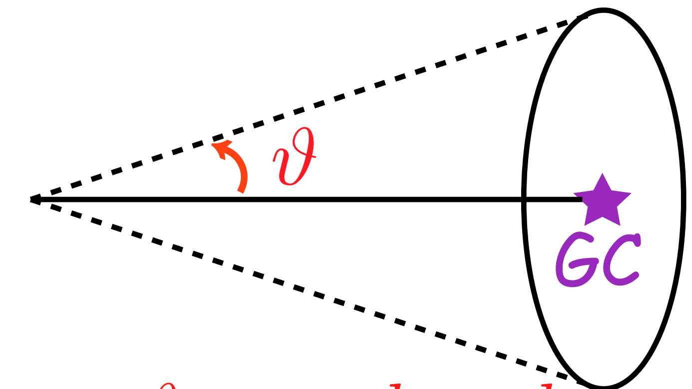


A. E., S. K. Kang and P. Serpico, JCAP (2014) [arXiv:1410.5979 [hep-ph]]

Angular distribution of neutrinos from decaying DM

✓ **Kolmogorov-Smirnov test:** a powerful non-parametric test

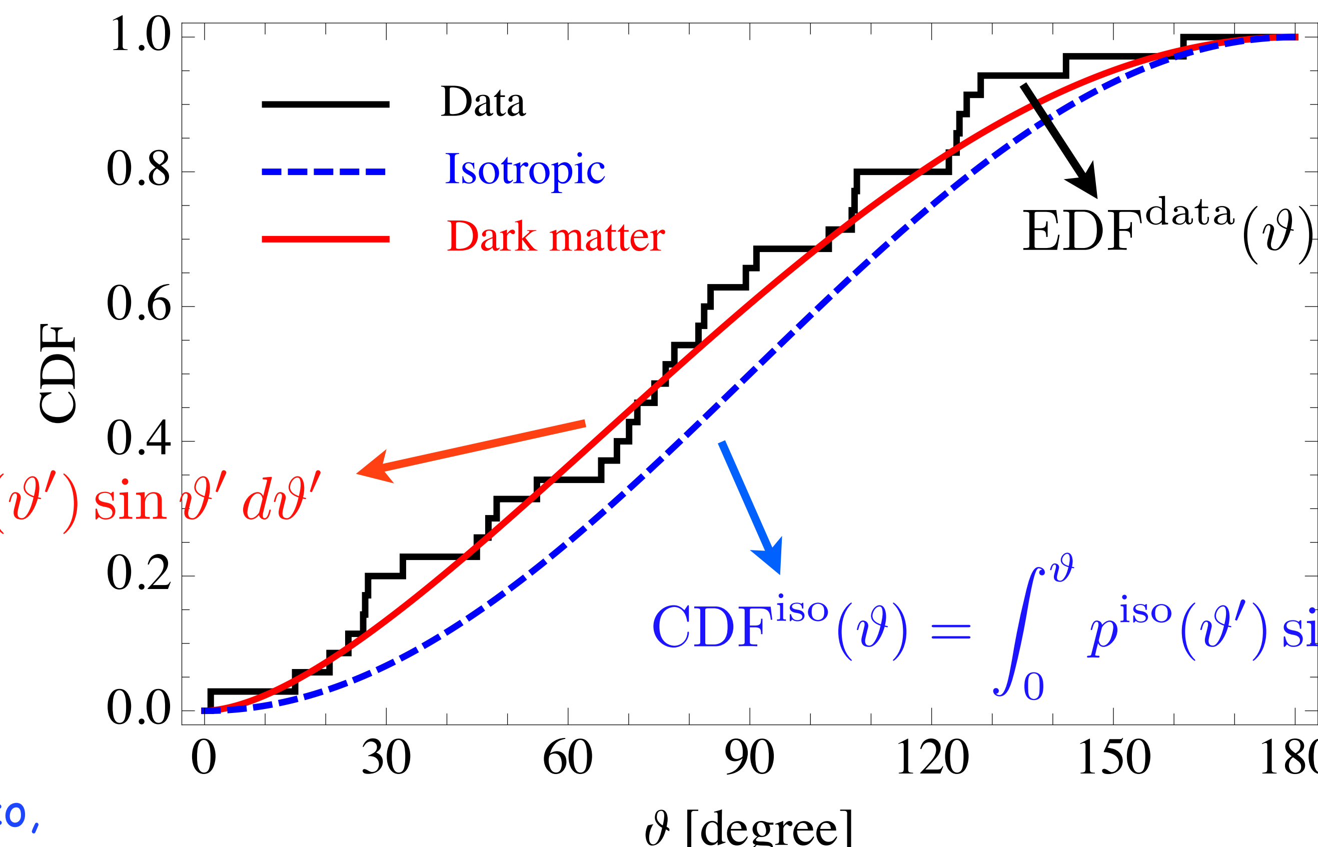
The 2-dim KS test have some ambiguities



$$\cos\vartheta = \cos b \cos l$$

$$p^{\rm iso}(\vartheta) = \int_0^{2\pi} p^{\rm iso}(\vartheta,\varphi)\,{\rm d}\varphi = \int_0^{2\pi} \frac{1}{4\pi}\,{\rm d}\varphi = \frac{1}{2}$$

$$p^{\rm DM}(\vartheta) = \int_0^{2\pi} p^{\rm DM}(\vartheta,\varphi)\,{\rm d}\varphi = \frac{\int_0^{\infty}\rho[r(s,\vartheta)]{\rm d}s + \Omega_{\rm DM}\rho_c\beta}{2(\eta + \Omega_{\rm DM}\rho_c\beta)}$$



$$\text{EDF}^{\rm data}(\vartheta) = \frac{1}{N}\sum_{i=1}^{N}\Theta(\vartheta - \vartheta_i)$$

$$\text{CDF}^{\rm DM}(\vartheta) = \int_0^{\vartheta} p^{\rm DM}(\vartheta')\sin\vartheta'\,d\vartheta'$$

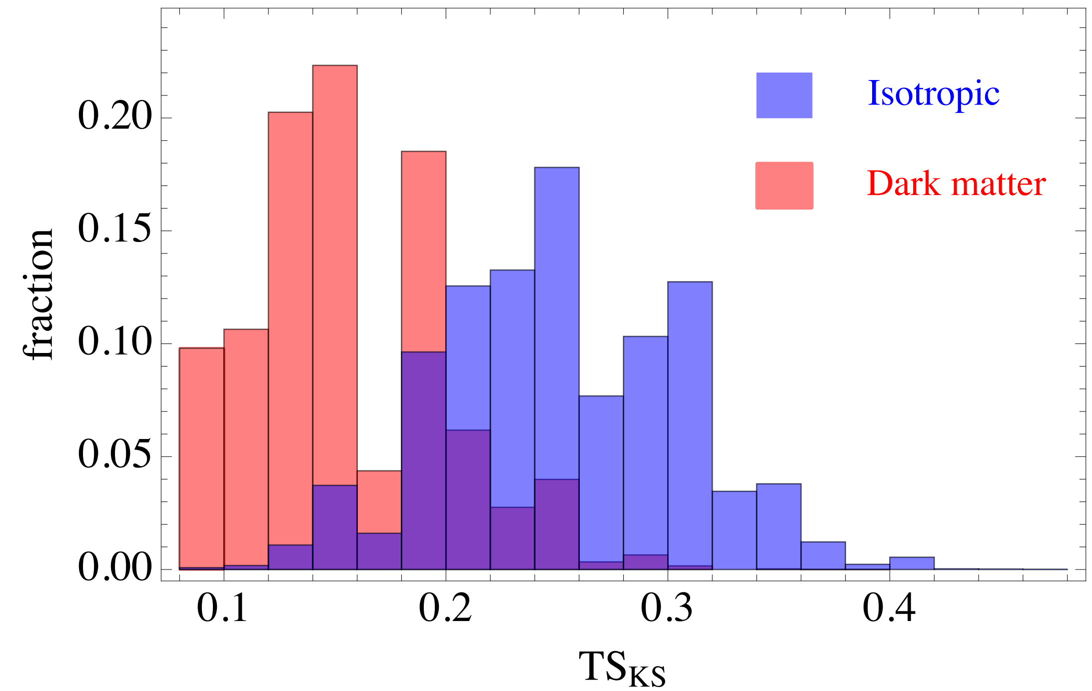
$$\text{CDF}^{\rm iso}(\vartheta) = \int_0^{\vartheta} p^{\rm iso}(\vartheta')\sin\vartheta'\,d\vartheta' = \frac{1-\cos\vartheta}{2}$$

A. E., S. K. Kang and P. Serpico,
JCAP (2014) [arXiv:1410.5979 [hep-ph]]

Angular distribution of neutrinos from decaying DM

✓ Kolmogorov-Smirnov test:

statistically larger TS for isotropic distribution



Test Statistics

$$\mathrm{TS}_{\mathrm{KS}} = \max_{1\leq i\leq N}\left\{\mathrm{CDF}^{\mathrm{DM}}(\vartheta_i) - \frac{i-1}{N}, \frac{i}{N} - \mathrm{CDF}^{\mathrm{DM}}(\vartheta_i)\right\}$$

A. E., S. K. Kang and P. Serpico, JCAP (2014) [arXiv:1410.5979 [hep-ph]]

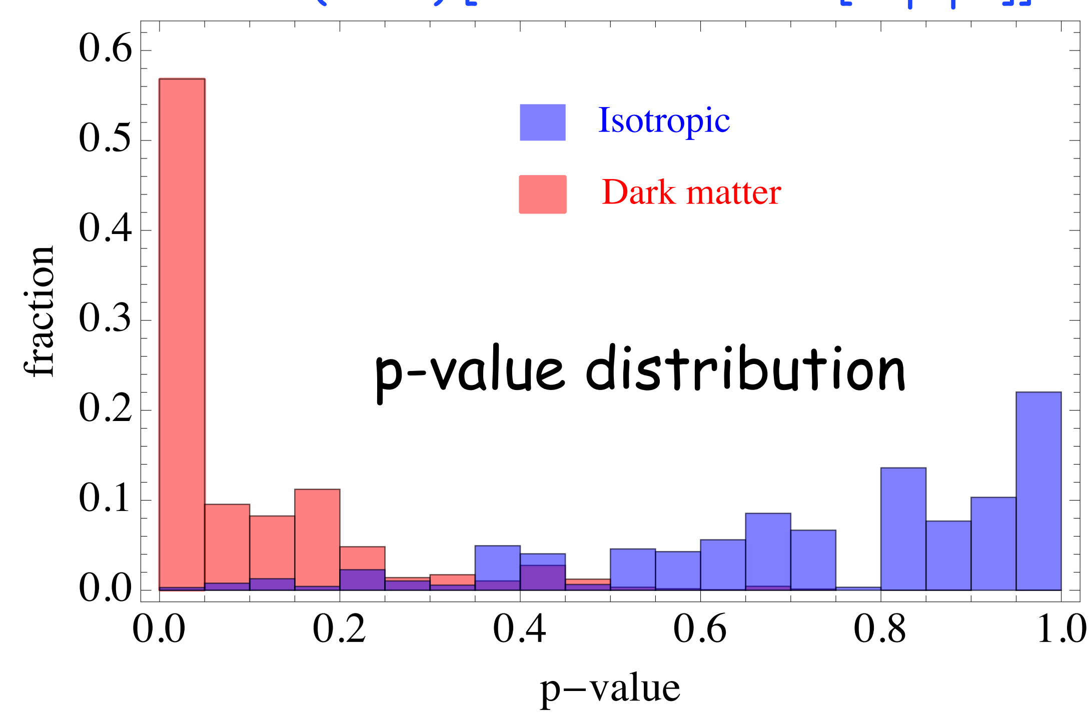
again, generating a sample (10^5) of isotropically distributed set of 20 events

↓

on the average, 10% of generated isotropic sample have smaller TS_{KS} than the values obtained for data vs DM dis.
for data vs isotropic dis. it is 73%

↓

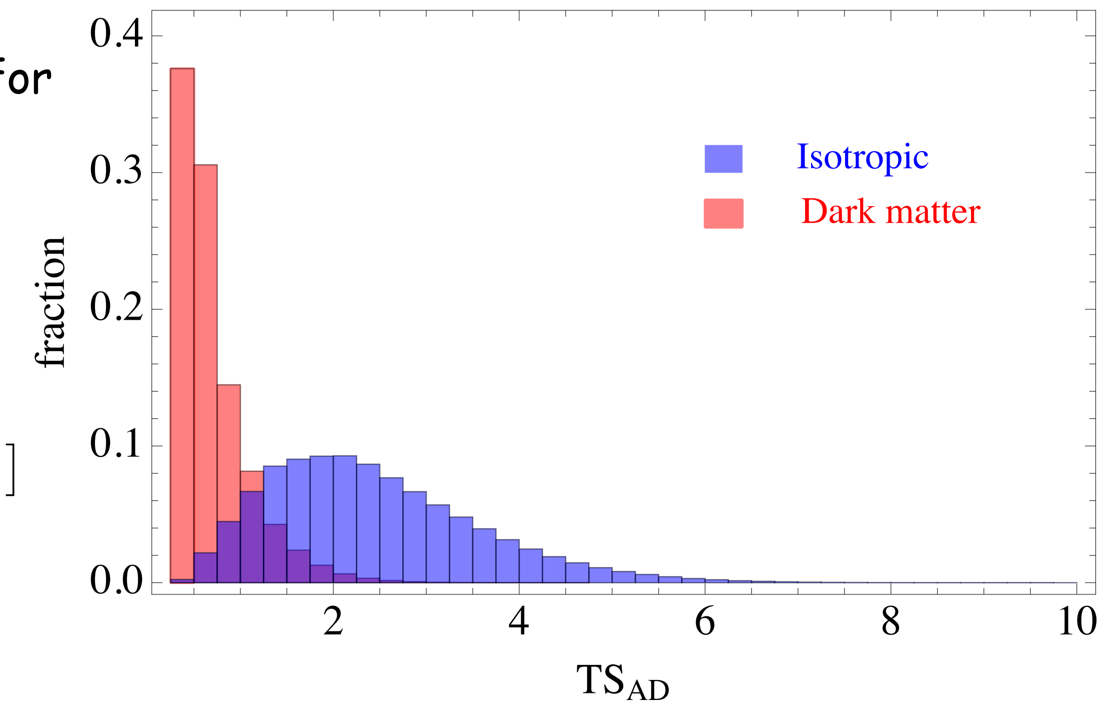
less than 2σ preference for DM dis.



Angular distribution of neutrinos from decaying DM

✓ **Anderson-Darling test:** a powerful non-parametric test, especially sensitive to the end points

statistically larger TS for isotropic distribution



Test Statistics

$$\mathrm{TS}_{\mathrm{AD}} = -N - \frac{1}{N}\sum_{i=1}^{N}(2i-1)\left[\ln\left(\mathrm{CDF}^{\mathrm{DM}}(\vartheta_i)\right) + \ln\left(1-\mathrm{CDF}^{\mathrm{DM}}(\vartheta_{N+1-i})\right)\right]$$

A. E., S. K. Kang and P. Serpico, JCAP (2014) [arXiv:1410.5979 [hep-ph]]

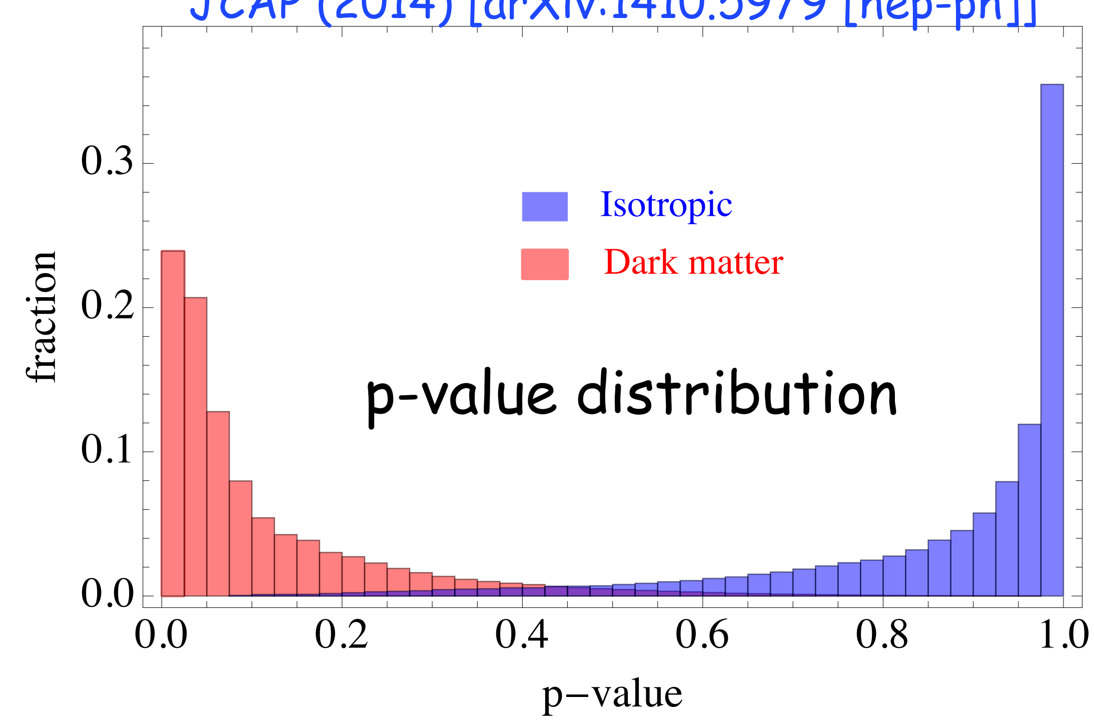
again, generating a sample (10^5) of isotropically distributed set of 20 events

↓

on the average, 11% of generated isotropic sample have smaller TS_{KS} than the values obtained for data vs DM dis.
for data vs isotropic dis. it is 86%

↓

less than 2σ preference for DM dis.

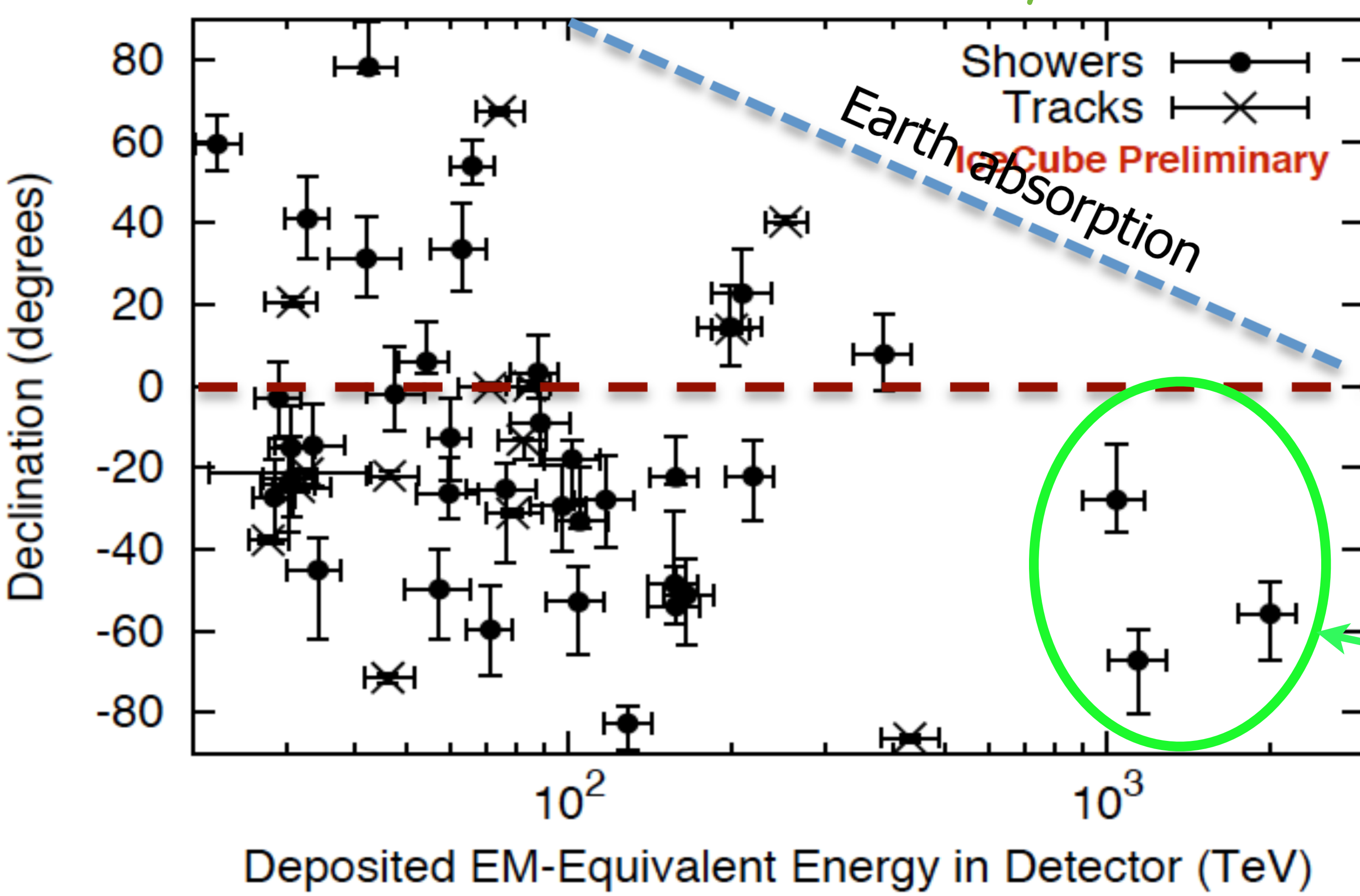


Observation of High Energy Neutrinos in IceCube

✓ Looking for lower energy contained events, 1347 days livetime

IPA 2015

The whole family!



✓ totally 54 events

✓ still three events with energy ~ PeV

$$\Phi \propto E^{-\gamma} : \gamma = 2.58 \pm 0.25$$

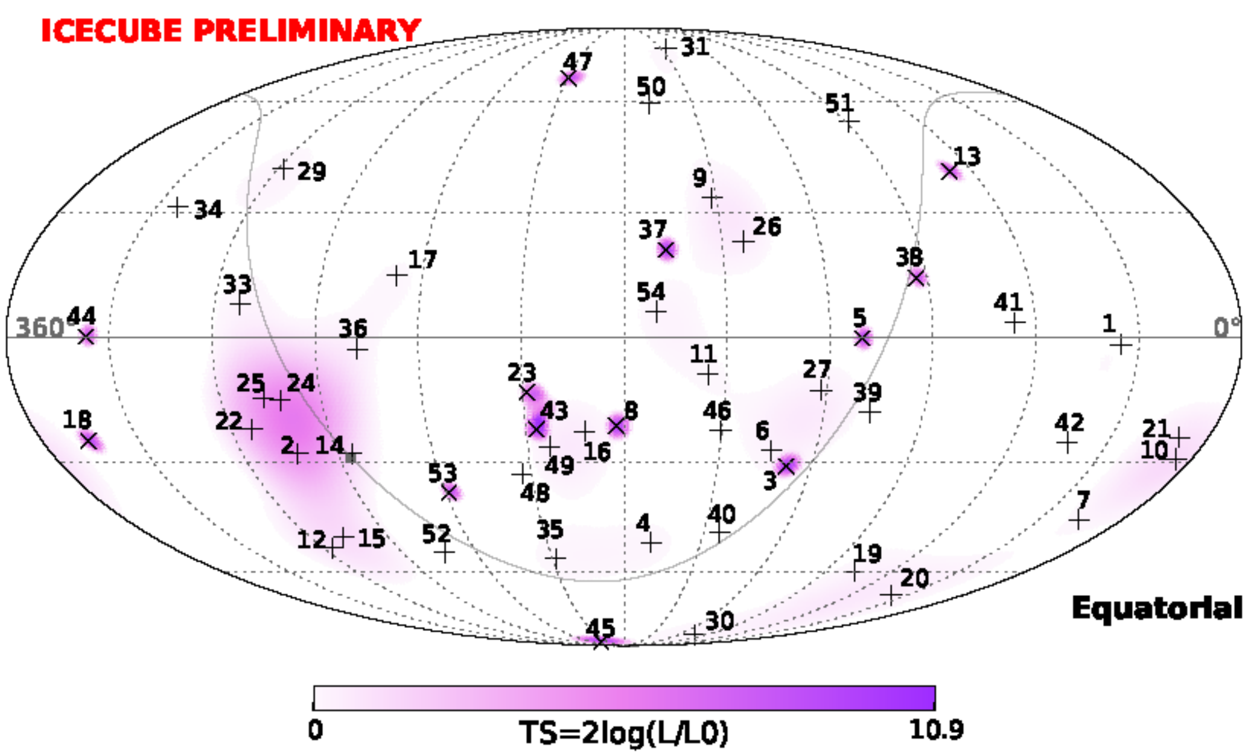
4 years of data

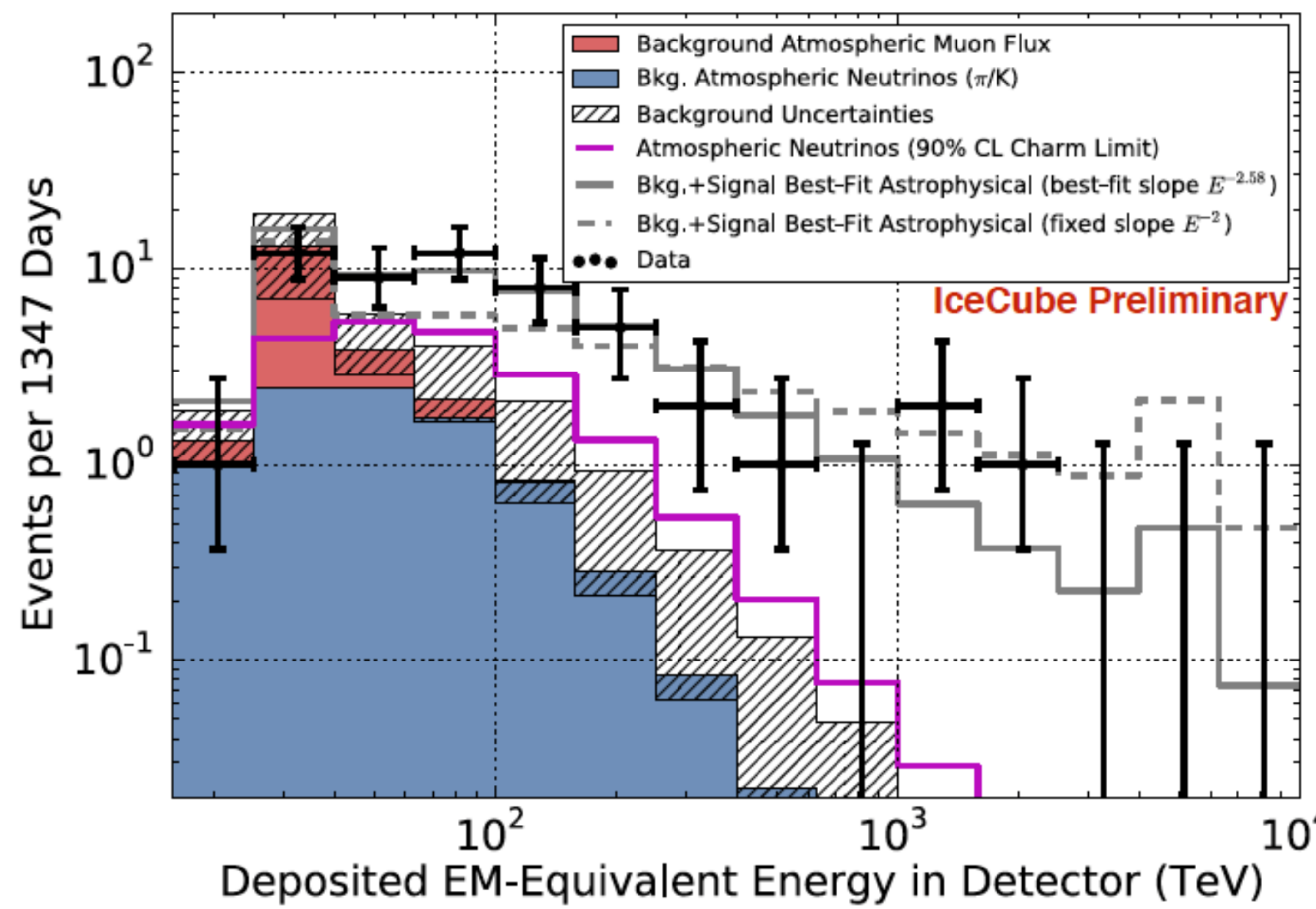
excess of events ~ 7σ

Source(s) not identified!

IceCube data

✓ Looking for lower energy contained events, 1347 days livetime





4 years of data

Confronting with energy distribution of IceCube data

4 years data set

✓ More refined analysis of the 4 years data set

$$\frac{d\Phi^c}{dE_\nu}(E_\nu;\tau_{\rm DM},m_{\rm DM},\phi_a,\gamma)=\frac{d\Phi^c_{\rm DM}}{dE_\nu}(E_\nu;\tau_{\rm DM},m_{\rm DM})+\frac{d\Phi_{\rm astro}}{dE_\nu}(E_\nu;\phi_a,\gamma)$$

single power-law astro flux

$$\left.\frac{d\Phi_{{\rm astro},\nu_\alpha}}{dE_\nu}\right|_\oplus=\phi_a\left(\frac{E_\nu}{100\ {\rm TeV}}\right)^{-\gamma}$$

fitting parameters

$$\boldsymbol{\theta}=\{\tau_{\rm DM},m_{\rm DM},\phi_a,\gamma\}$$

Confronting with energy distribution of IceCube data

4 years data set

A. Bhattacharya, A. E., S. Palomares-Ruiz, I. Sarcevic, JCAP (2017) [arXiv:1706.05746]

Best-fit values of $\boldsymbol{\theta} = \{\tau_{\rm DM}, m_{\rm DM}, \phi_a, \gamma\}$

10^{-18} [$\text{GeV}^{-1}\text{cm}^{-2}\text{s}^{-1}\text{sr}^{-1}$]

[60 TeV — 10 PeV]

Decay channel	$N_{\rm DM}(\tau_{\rm DM}[10^{28}\text{ s}])$	$m_{\rm DM}$ [TeV]	$N_{\rm astro}(\phi_{\rm astro})$	γ
$u\,\bar{u}$	10.2 (0.021)	522	16.6 (1.2)	2.42
$b\,\bar{b}$	12.9 (0.089)	1066	13.8 (0.83)	2.32
$t\,\bar{t}$	16.1 (0.58)	11134	10.7 (1.9)	3.91
$W^+\,W^-$	11.3 (1.4)	4860	15.5 (2.5)	3.66
$Z\,Z$	10.5 (1.6)	4800	16.3 (2.6)	3.61
$h\,h$	13.6 (0.17)	606	13.2 (0.76)	2.29
$e^+\,e^-$	5.0 (1.2)	4116	21.9 (3.2)	3.33
$\mu^+\,\mu^-$	6.3 (5.0)	6437	20.7 (3.2)	3.46
$\tau^+\,\tau^-$	7.6 (4.4)	6749	19.3 (3.0)	3.53
$\nu_e\,\bar{\nu}_e$	3.7 (2.6)	4041	22.7 (3.2)	3.24
$\nu_\mu\,\bar{\nu}_\mu$	6.4 (2.4)	4133	20.6 (3.2)	3.48
$\nu_\tau\,\bar{\nu}_\tau$	6.7 (2.3)	4117	20.1 (3.1)	3.50

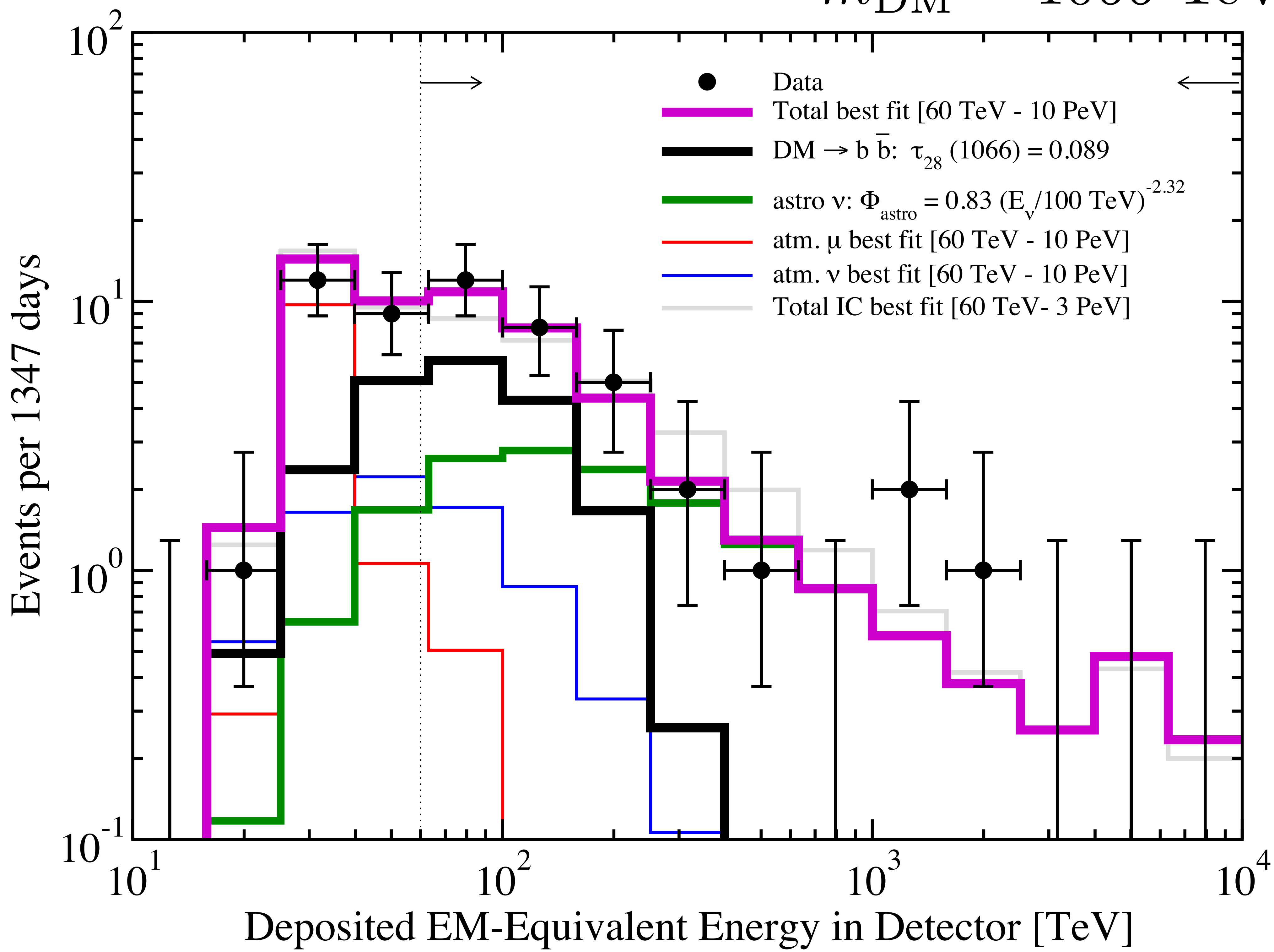
Confronting with energy distribution of IceCube data

4 years data set

Event rate:

$$\mathrm{DM} \to b\bar{b}$$

$m_{\mathrm{DM}} = 1066$ TeV



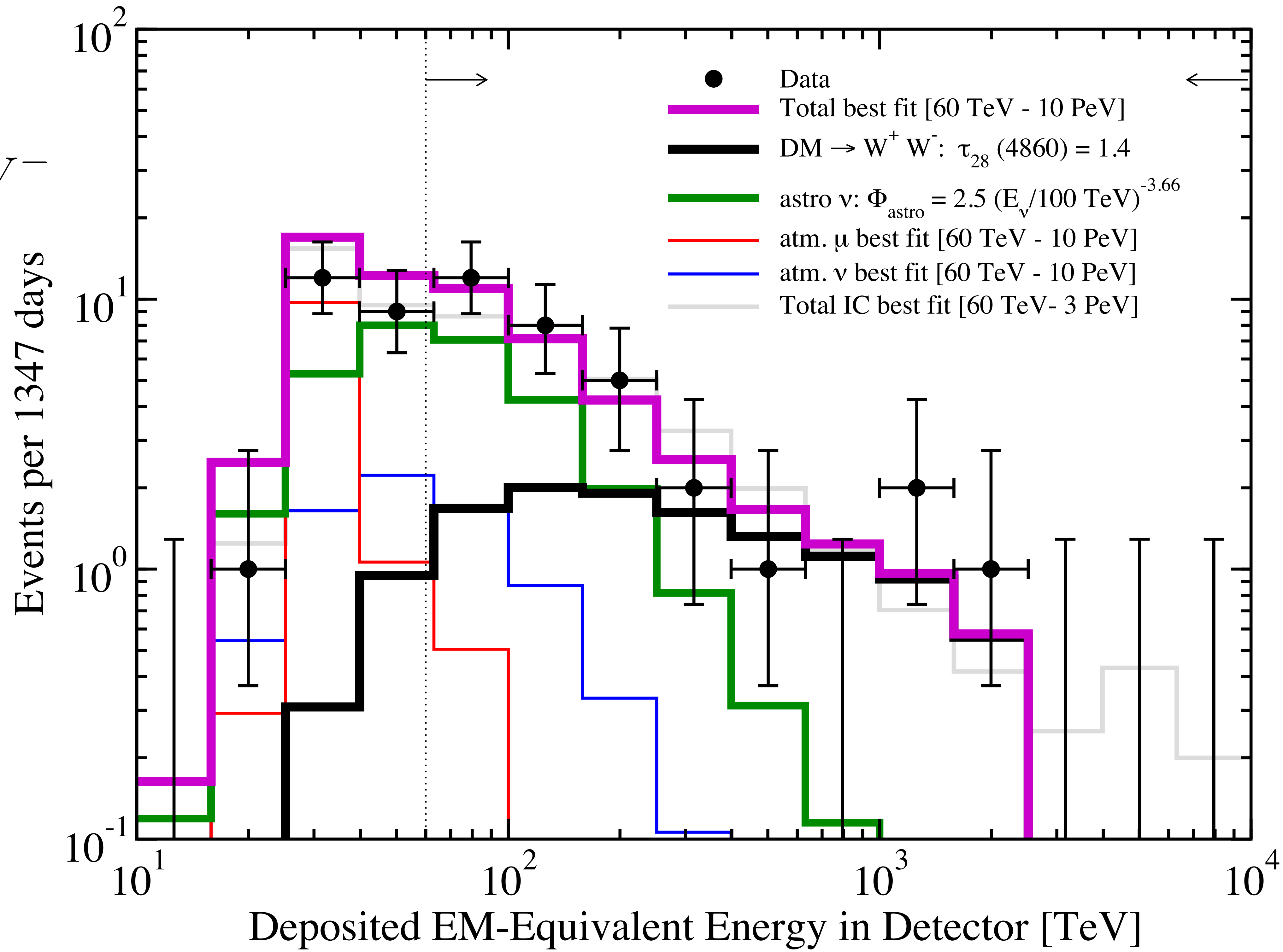
Confronting with energy distribution of IceCube data

4 years data set

$m_{\rm DM} = 4860$ TeV

Event rate:

$\mathrm{DM} \to W^+W^-$



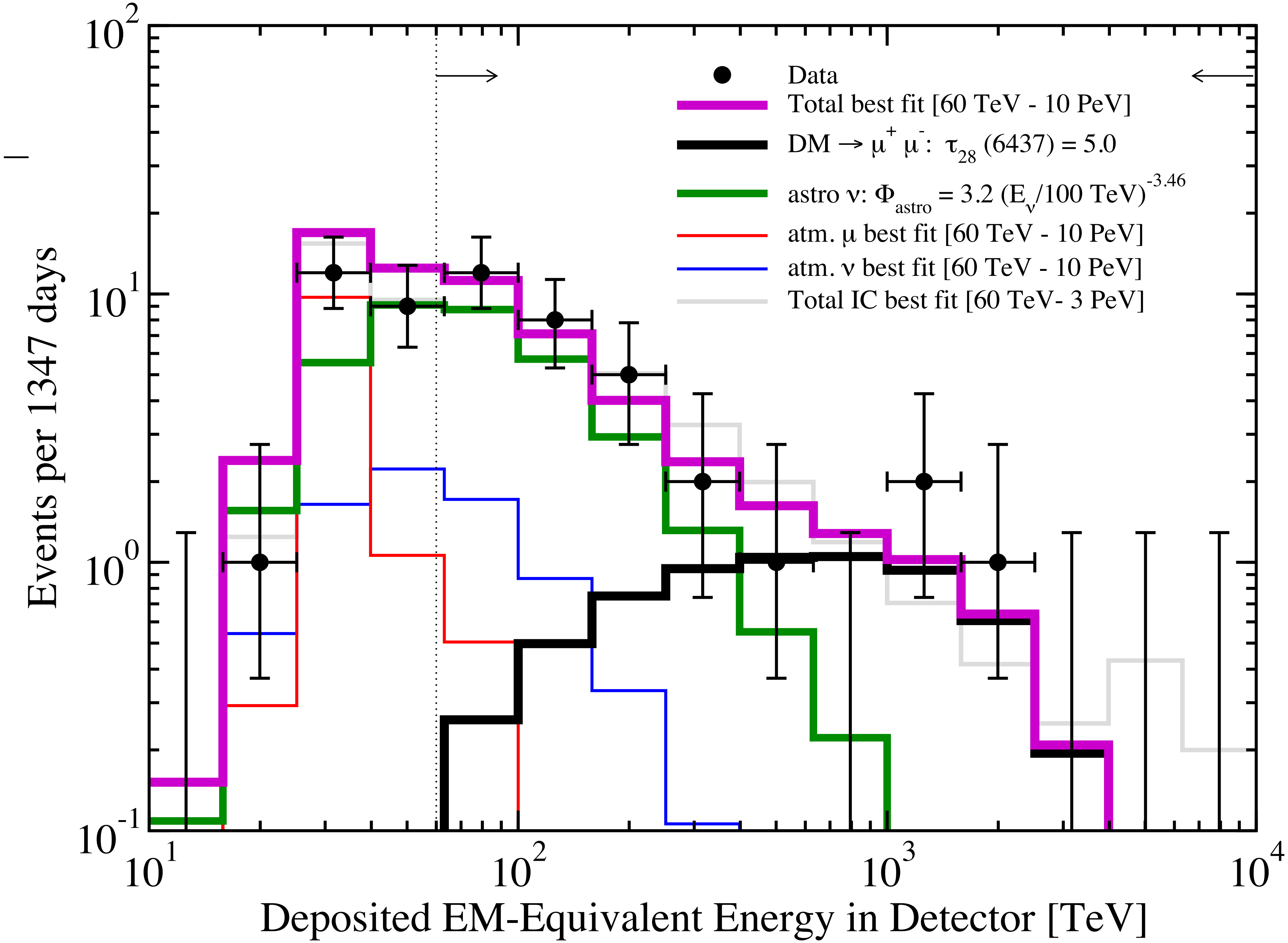
Confronting with energy distribution of IceCube data

4 years data set

$m_{\rm DM} = 6437$ TeV

Event rate:

$\mathrm{DM} \to \mu^+\mu^-$



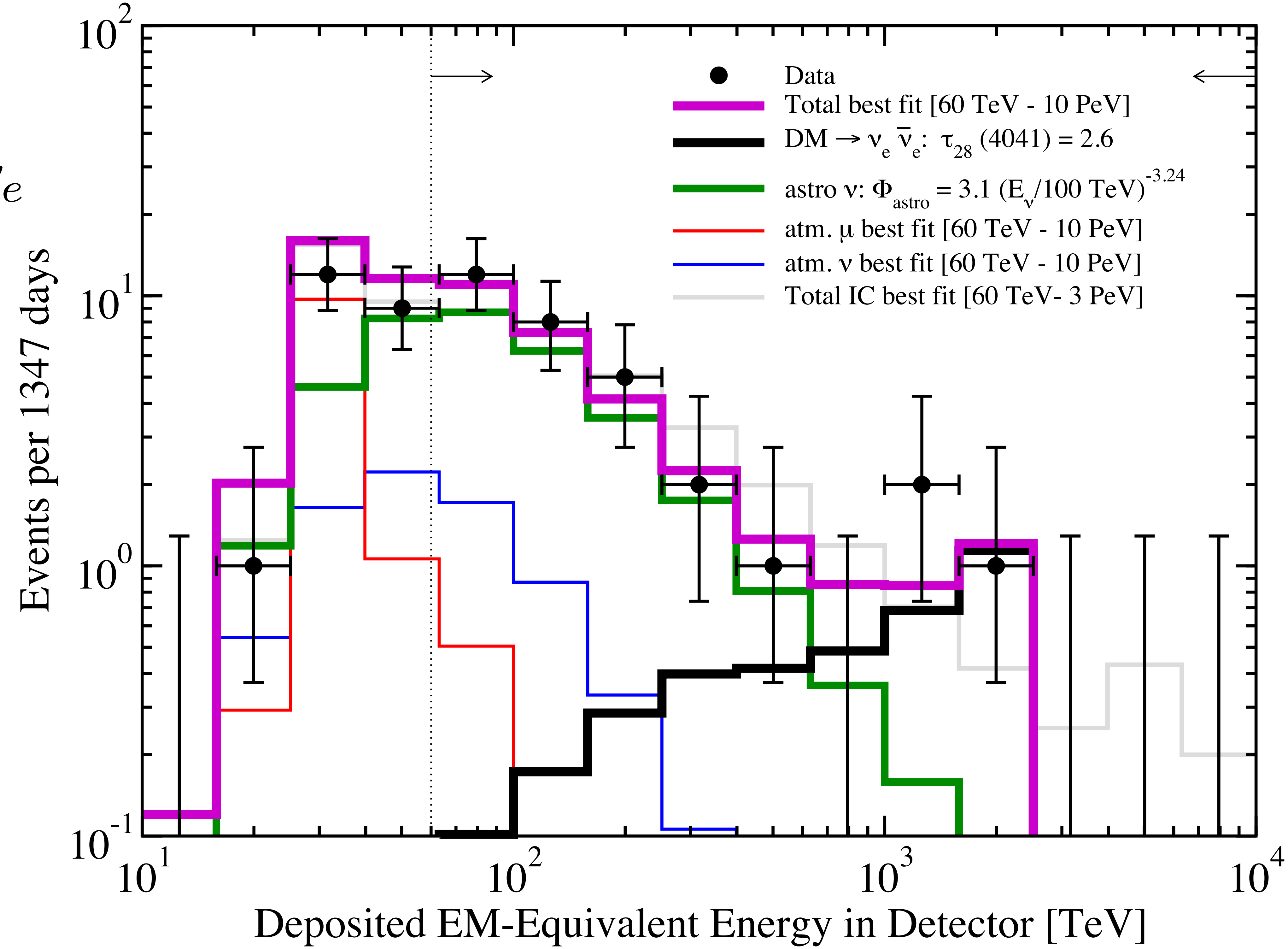
Confronting with energy distribution of IceCube data

4 years data set

$m_{\rm DM} = 4041$ TeV

Event rate:

$\mathrm{DM} \to \nu_e \bar{\nu}_e$



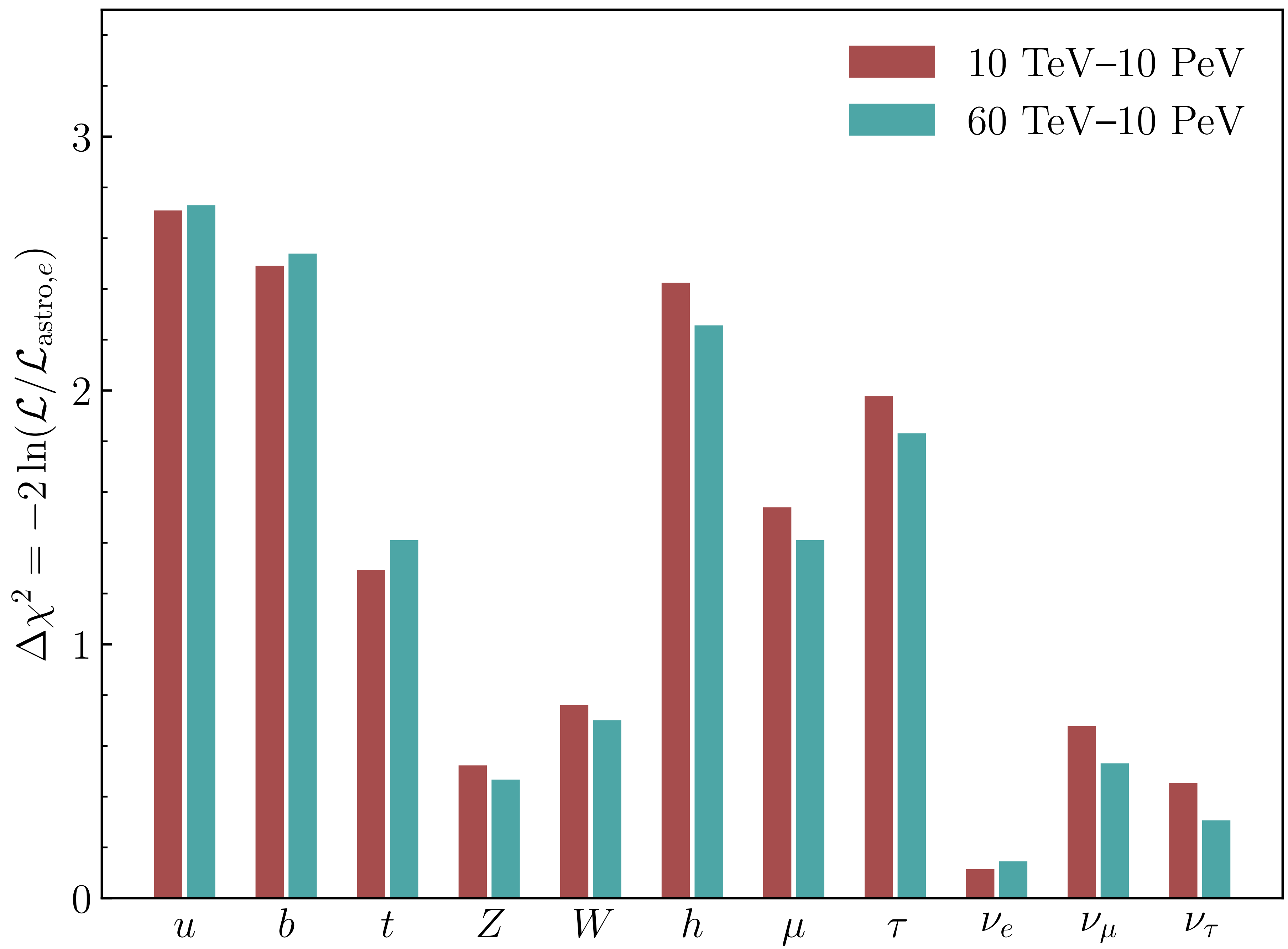
Confronting with energy distribution of IceCube data

4 years data set

All the channels: the case of astro + DM (one channel decay)

with respect to

$$\mathrm{DM} \to e^- e^+$$



Confronting with energy distribution of IceCube data

Multiple channel DM decay:

4 years data set

$$\boldsymbol{\theta}_{2c} = \{N_{\rm DM}, m_{\rm DM}, {\rm BR}\}$$

Decay channels	$N_{\rm DM}$ ($\tau_{\rm DM}$ [10^{28} s])	$m_{\rm DM}$ [TeV]	BR
$u\,\bar{u}$, $e^+\,e^-$	26.6 (0.22)	3991	0.84
$u\,\bar{u}$, $\nu_e\,\bar{\nu}_e$	26.7 (0.19)	3902	0.92
$b\,\bar{b}$, $e^+\,e^-$	26.5 (0.22)	4042	0.84
$b\,\bar{b}$, $\mu^+\,\mu^-$	26.4 (0.25)	5444	0.94
$b\,\bar{b}$, $\nu_e\,\bar{\nu}_e$	26.6 (0.19)	3933	0.92
$b\,\bar{b}$, $\nu_\mu\,\bar{\nu}_\mu$	26.6 (0.20)	4023	0.93
$b\,\bar{b}$, $\tau^+\,\tau^-$	26.5 (0.25)	5539	0.94
$t\,\bar{t}$, $\nu_\mu\,\bar{\nu}_\mu$	26.1 (0.32)	8866	1.00
$W^+\,W^-$, $\mu^+\,\mu^-$	25.3 (0.22)	4633	1.00
$W^+\,W^-$, $\nu_\mu\,\bar{\nu}_\mu$	25.3 (0.22)	4633	1.00
$h\,h$, $\mu^+\,\mu^-$	26.3 (0.28)	7031	1.00
$h\,h$, $\nu_e\,\bar{\nu}_e$	26.3 (0.20)	4103	0.92

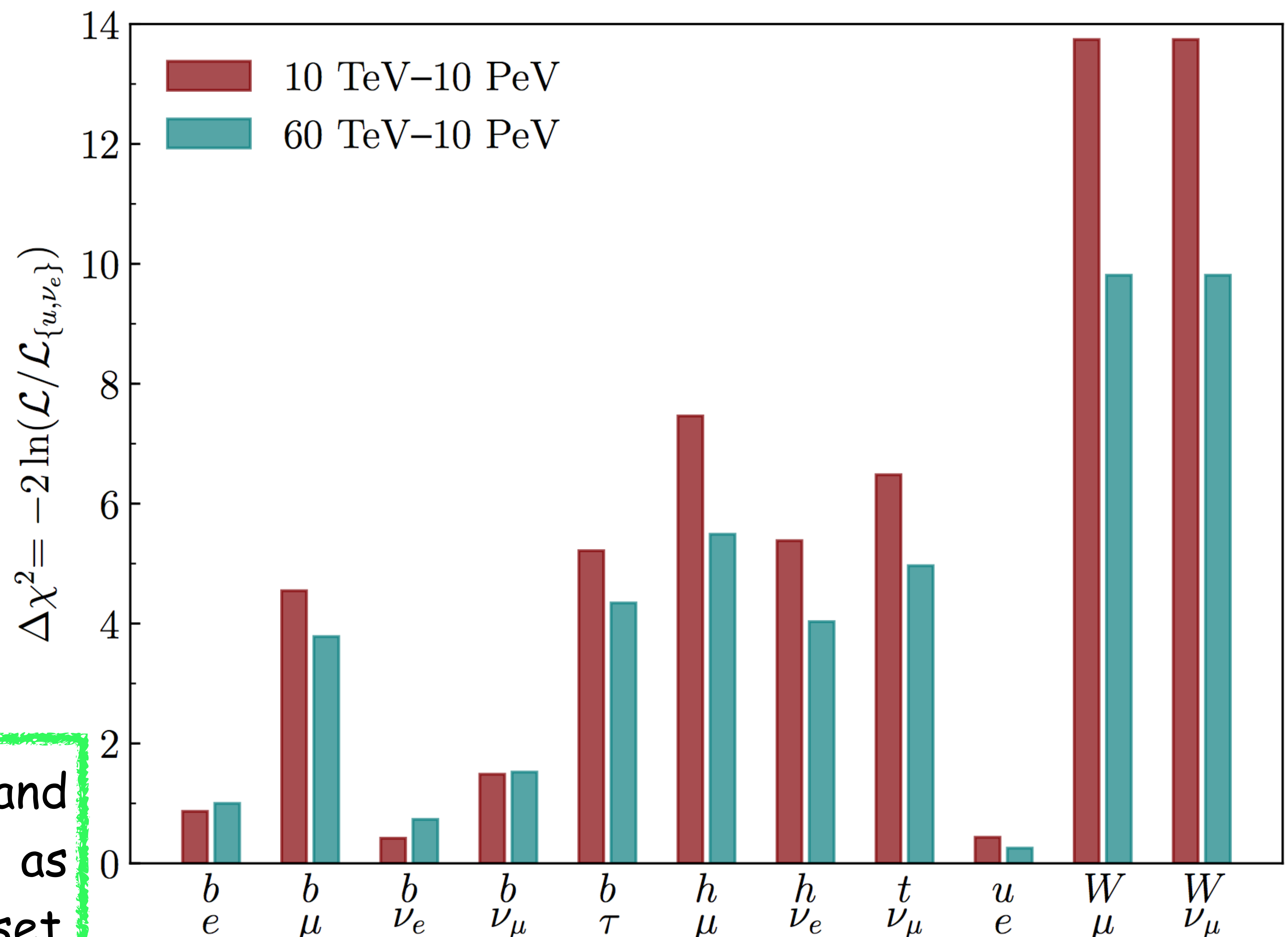
Confronting with energy distribution of IceCube data

4 years data set

Multiple channel DM decay:

with respect to

$$\mathrm{DM} \to \{u\bar{u}, \nu_e\bar{\nu}_e\}$$



The best-fit channel and DM-mass is the same as IceCube 2-years dataset

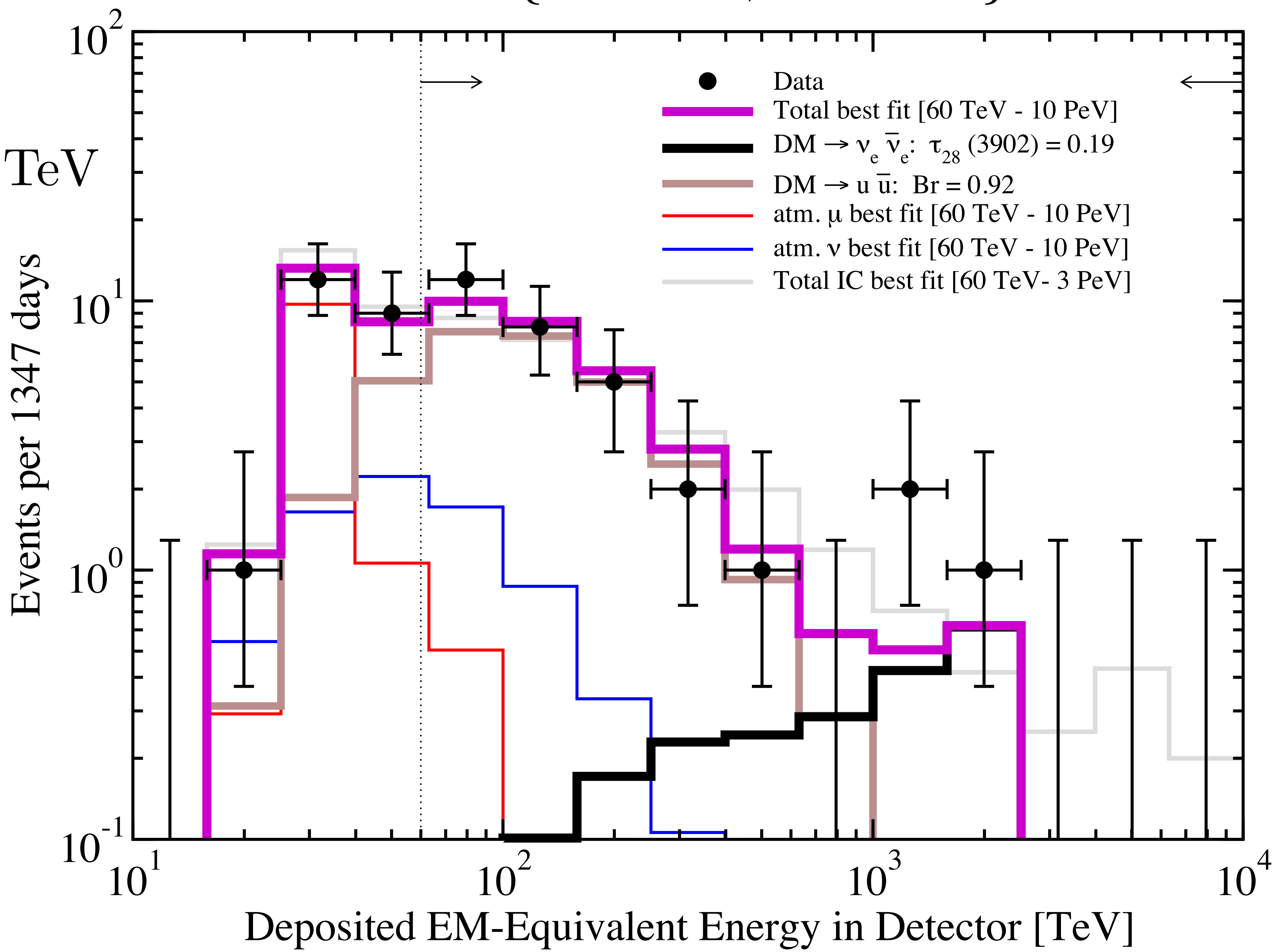
Confronting with energy distribution of IceCube data

4 years data set

Event rate:

$m_{\rm DM} = 3902$ TeV

$$\mathrm{DM} \to \{92\%\ u\bar{u}, 8\%\ \nu_e\bar{\nu}_e\}$$

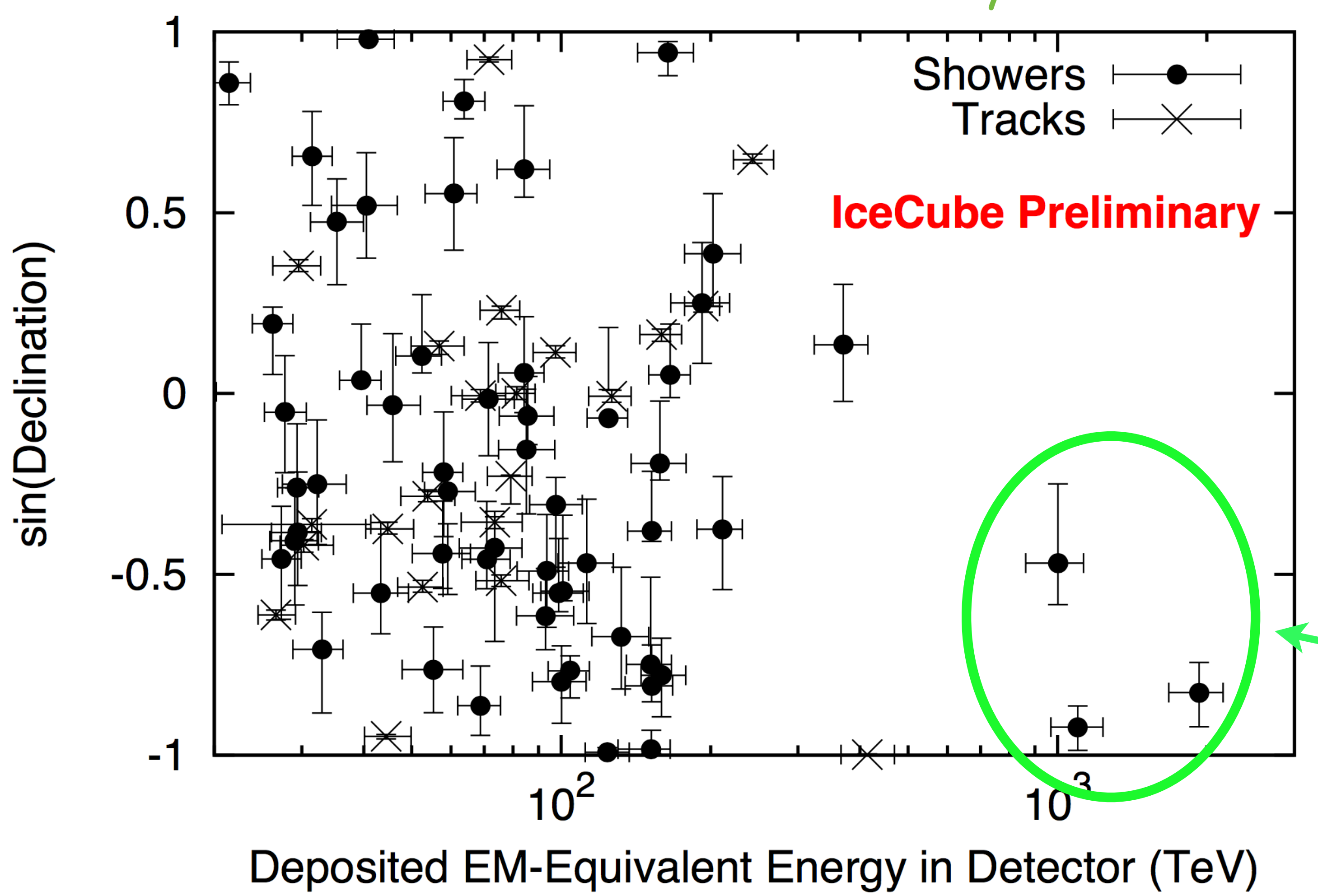


Observation of High Energy Neutrinos in IceCube

✓ Looking for lower energy contained events, 2078 days livetime

ICRC 2017

The whole family!



✓ totally 82 events

✓ still three events with energy ~ PeV

$$\Phi \propto E^{-\gamma} : \quad \gamma = 2.9 \pm 0.3$$

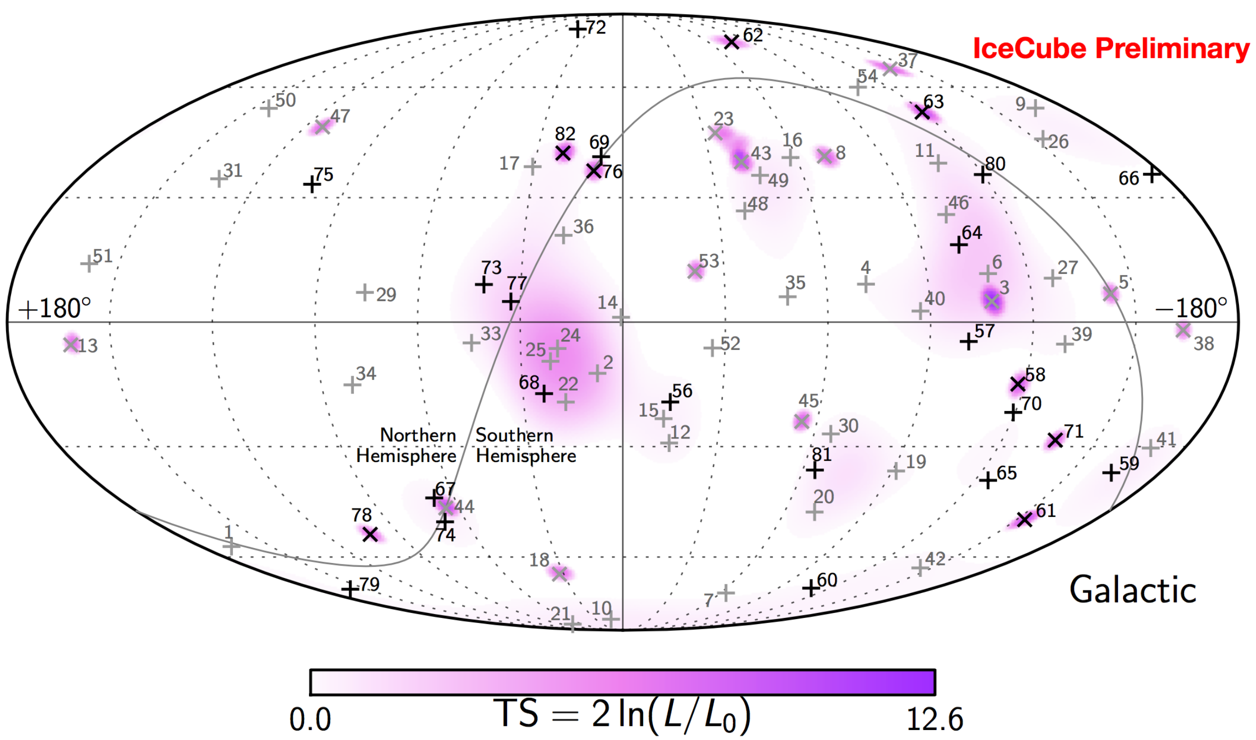
6 years of data

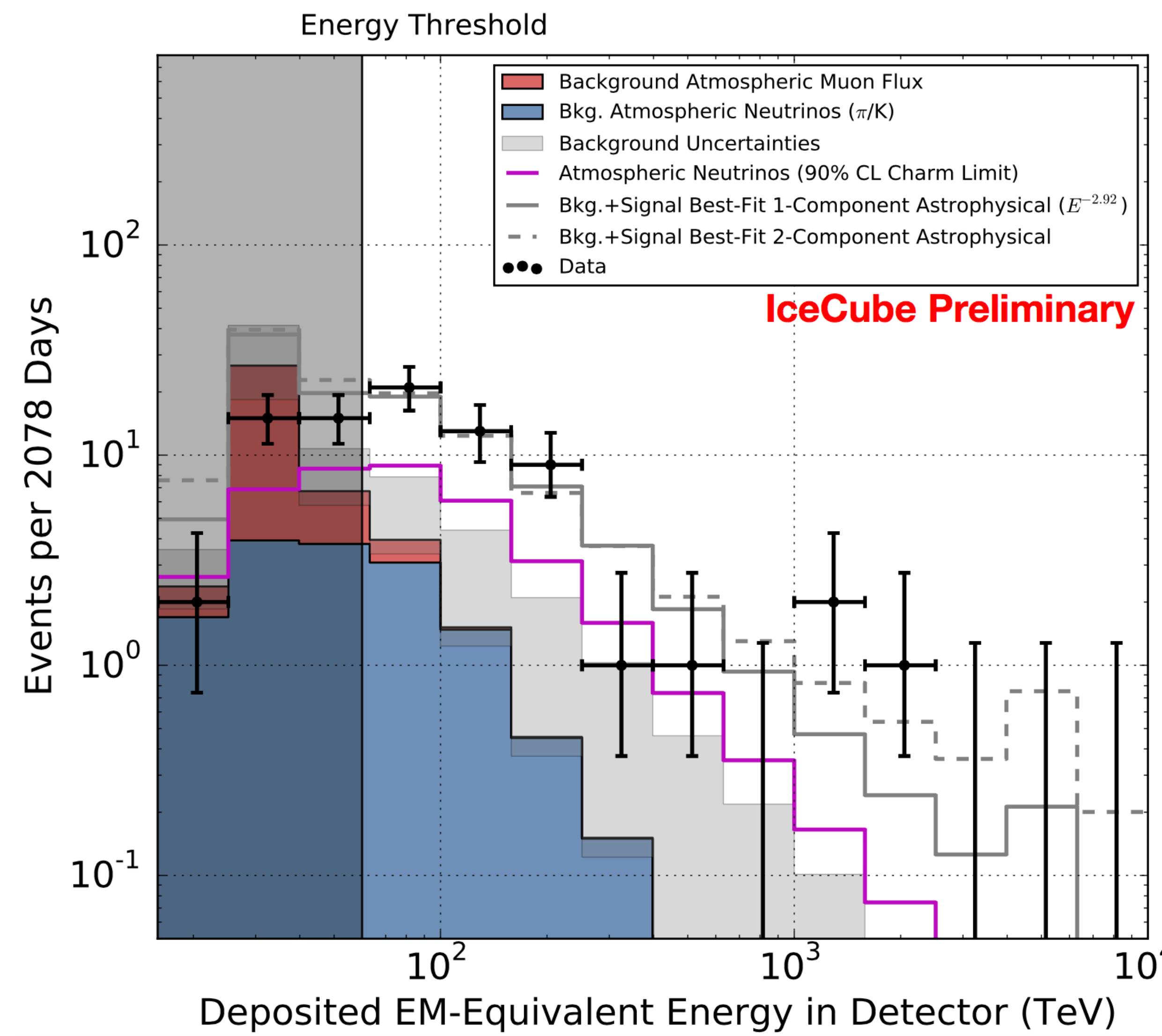
excess of events > 7σ

Source(s) not identified!

IceCube data

✓ Looking for lower energy contained events, 2078 days livetime





6 years of data

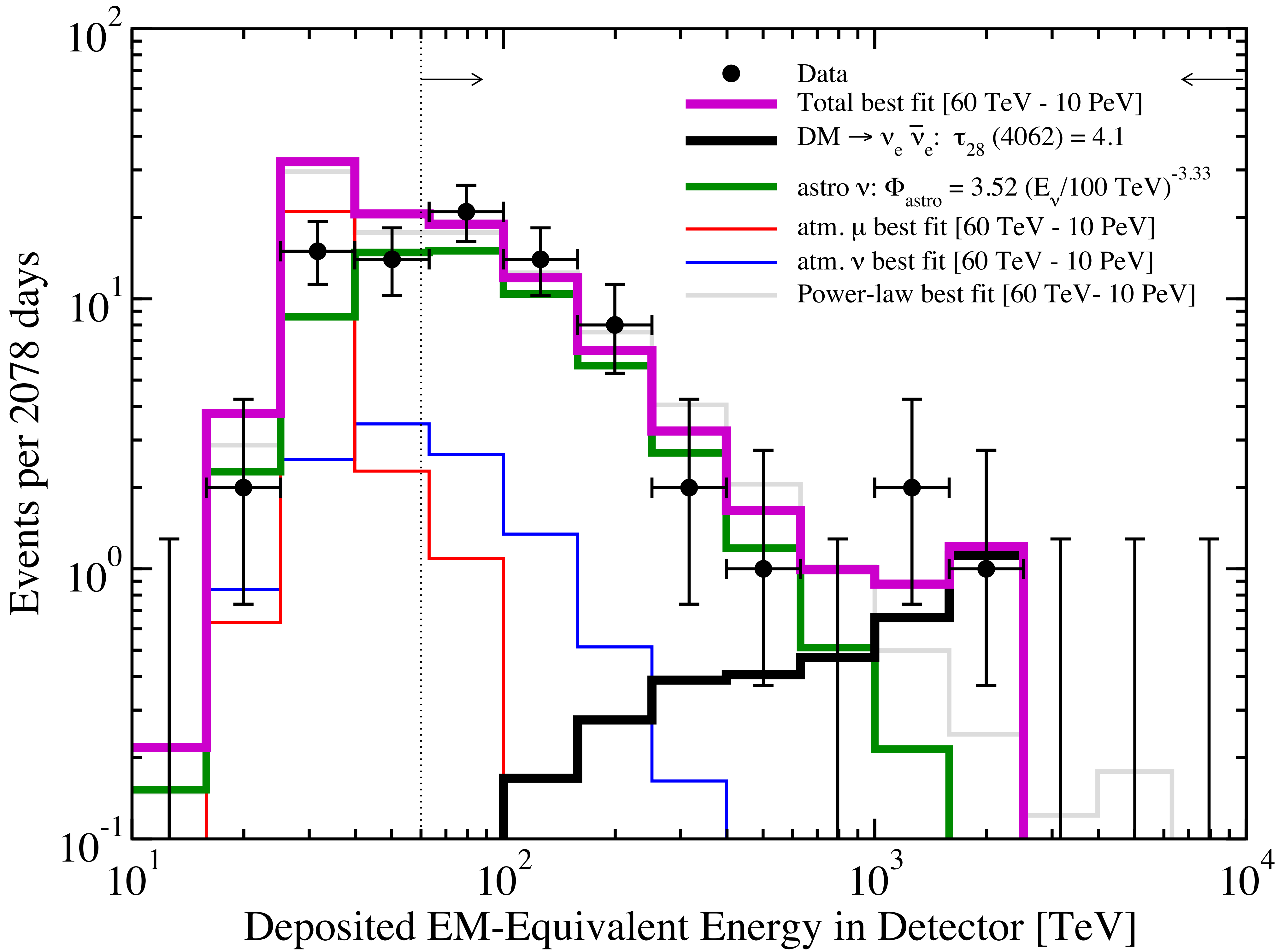
Confronting with energy distribution of IceCube data

6 years data set

$m_{\rm DM} = 4062$ TeV

Event rate:

$\mathrm{DM} \to \nu_e \bar{\nu}_e$



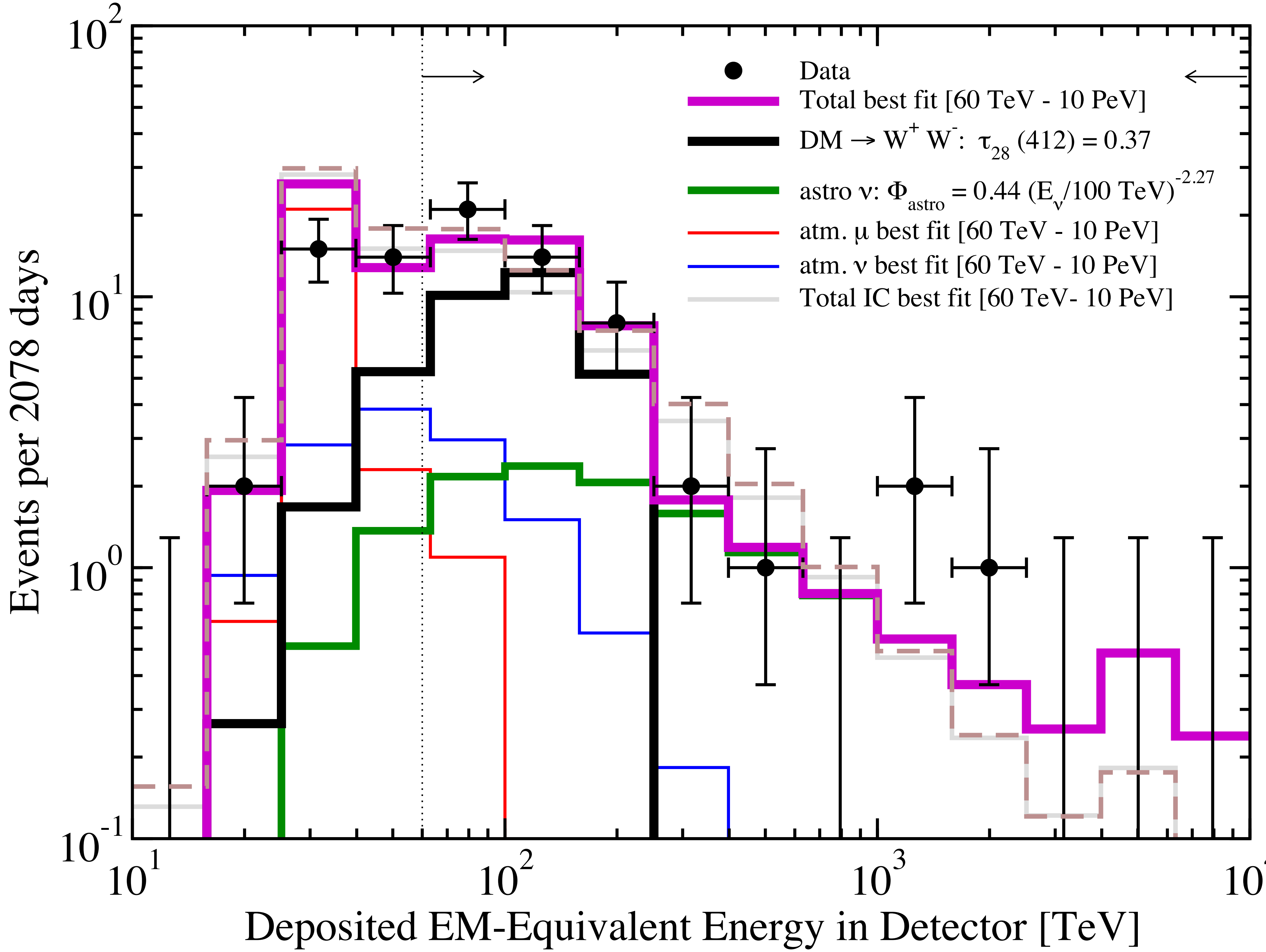
Confronting with energy distribution of IceCube data

6 years data set

$m_{\rm DM} = 412$ TeV

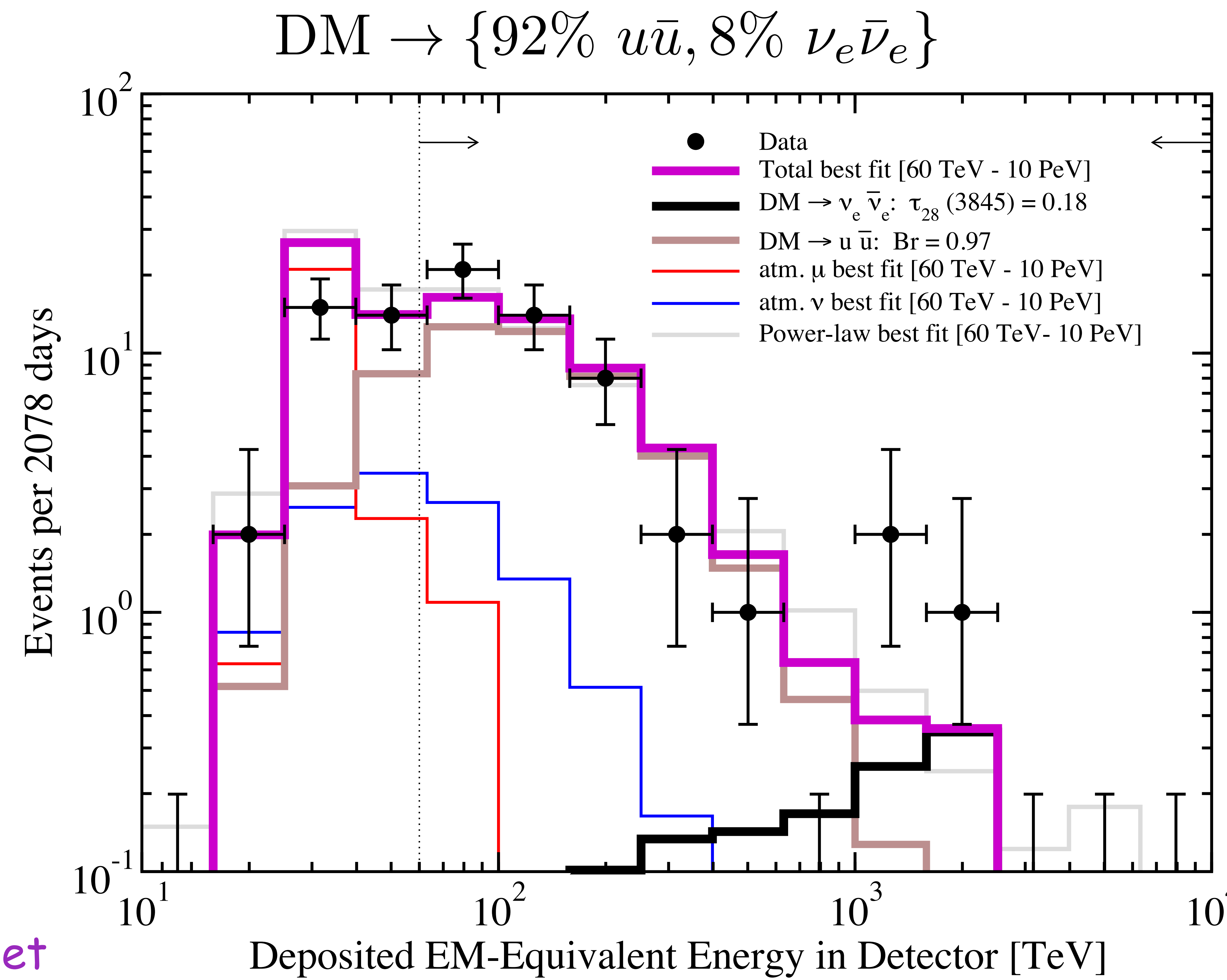
Event rate:

$\mathrm{DM} \to W^+W^-$



Confronting with energy distribution of IceCube data

Event rate:



6 years data set

Gamma ray bounds

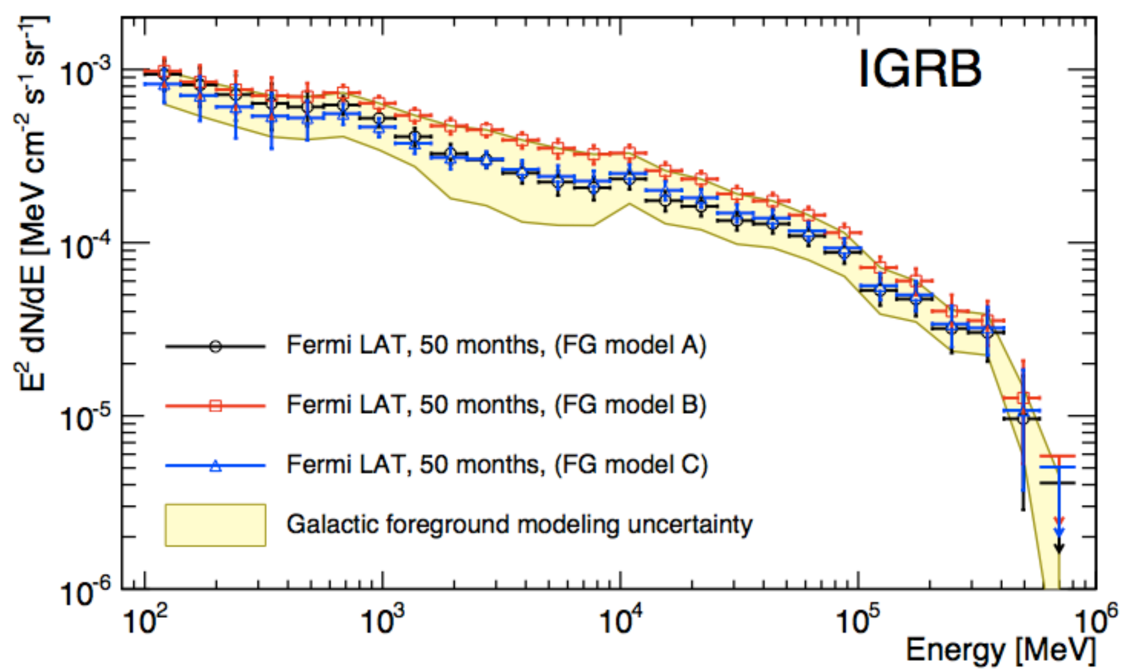
Universe is opaque for gamma-rays with E > 1 TeV → cascades develop: gamma-ray interaction with interstellar radiation field and CMB → gamma-rays populate at lower energies < $10^{(2-3)}$ GeV

Gamma ray bounds

Universe is opaque for gamma-rays with E > 1 TeV → cascades develop: gamma-ray interaction with interstellar radiation field and CMB → gamma-rays populate at lower energies < $10^{(2-3)}$ GeV

✓ Isotropic diffuse gamma-ray background by Fermi-LAT

M. Ackermann et al. [The Fermi LAT Collaboration], arXiv:1410.3696 [astro-ph.HE].



integrated energy density

$$\omega_\gamma = \frac{4\pi}{c}\int_{E_1}^{E_2} E_\gamma \frac{d\varphi_\gamma}{dE_\gamma}\, dE_\gamma \lesssim 4.4\times 10^{-7}\ \mathrm{eV/cm^3}$$

$E_1 \sim \mathcal{O}(1)$ GeV $\qquad E_2 \sim \mathcal{O}(100)$ GeV

total electromagnetic energy budget (NH case)

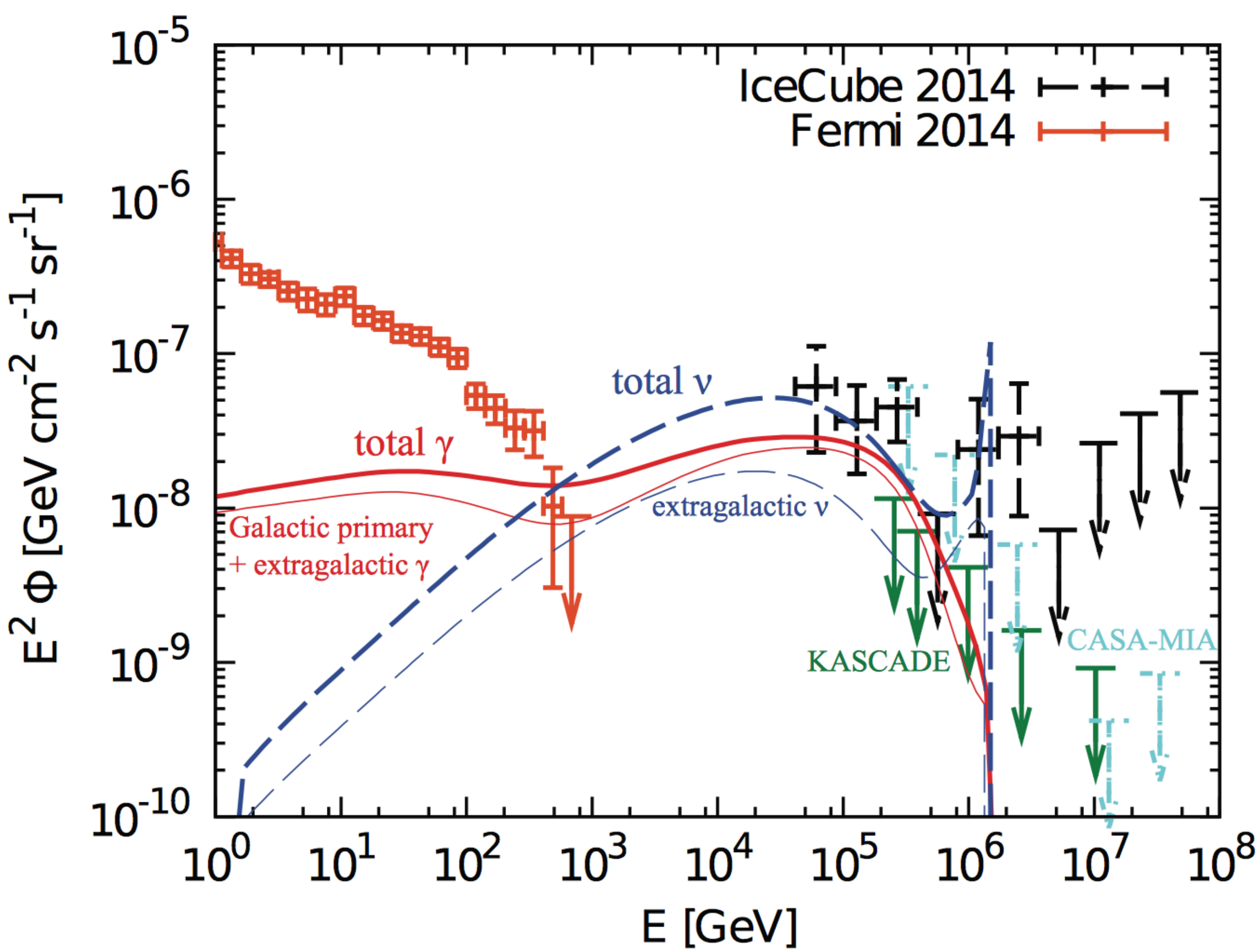
$$\frac{4\pi}{c}\int \sum_{\mathrm{i=gal,extragal}} \left[E_\gamma \left.\frac{d\varphi_\gamma}{dE_\gamma}\right)^{\mathrm{i}} + E_e \left.\frac{d\varphi_{e^\pm}}{dE_e}\right)^{\mathrm{i}} \right] dE \simeq 5.2\times 10^{-8}\ \mathrm{eV/cm^3}$$

✓

Gamma ray bounds

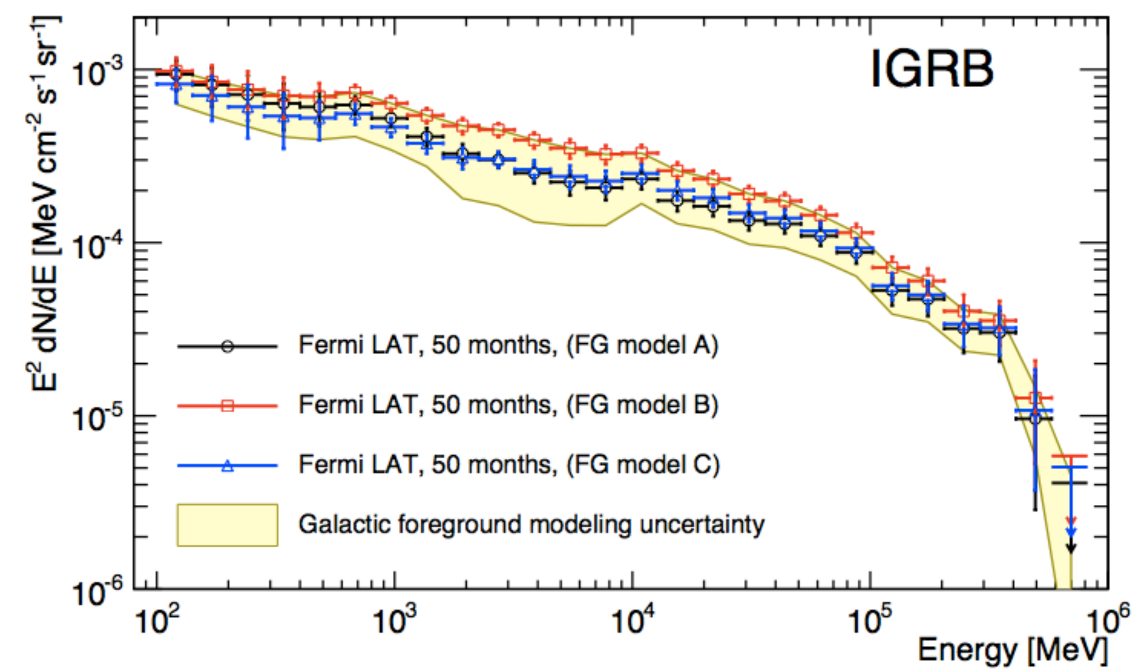
Universe is opaque for gamma-rays with E > 1 TeV → cascades develop: gamma-ray interaction with interstellar radiation field and CMB → gamma-rays populate at lower energies < $10^{(2\text{-}3)}$ GeV

✓ Isotropic diffuse gamma-ray background by Fermi-LAT



Murase, Laha, Ando, Ahlers, arXiv:1503.04663

M. Ackermann et al. [The Fermi LAT Collaboration], arXiv:1410.3696 [astro-ph.HE].



Gamma ray bounds

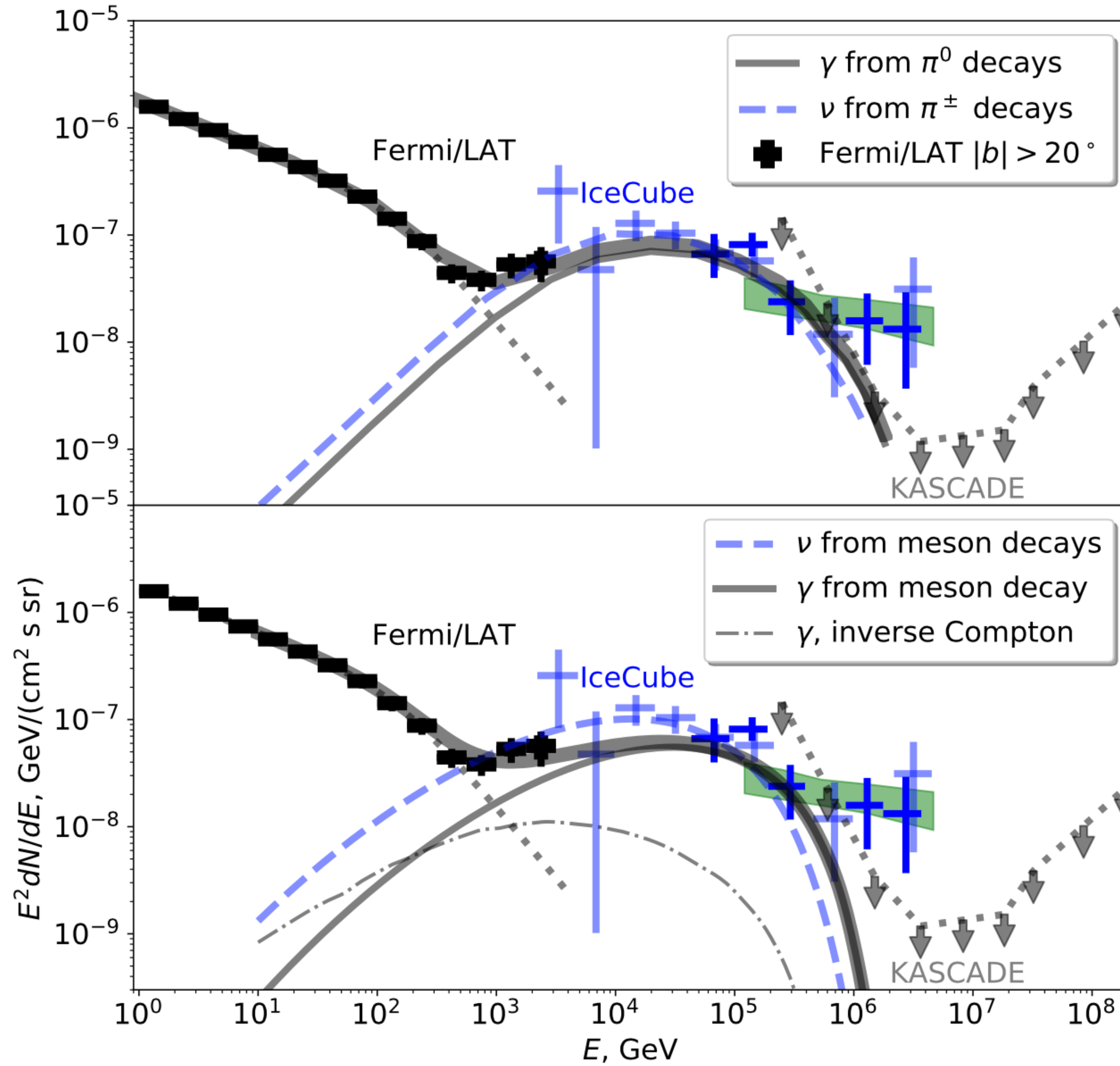
arXiv:1802.09983

✓ Multi-TeV high Galactic latitude diffuse gamma-ray flux

1) Injected cosmic ray by a recent nearby PeVatron

2) Cosmic ray interaction in large scale halo around the Milky Way

3) Decay of the dark matter particles

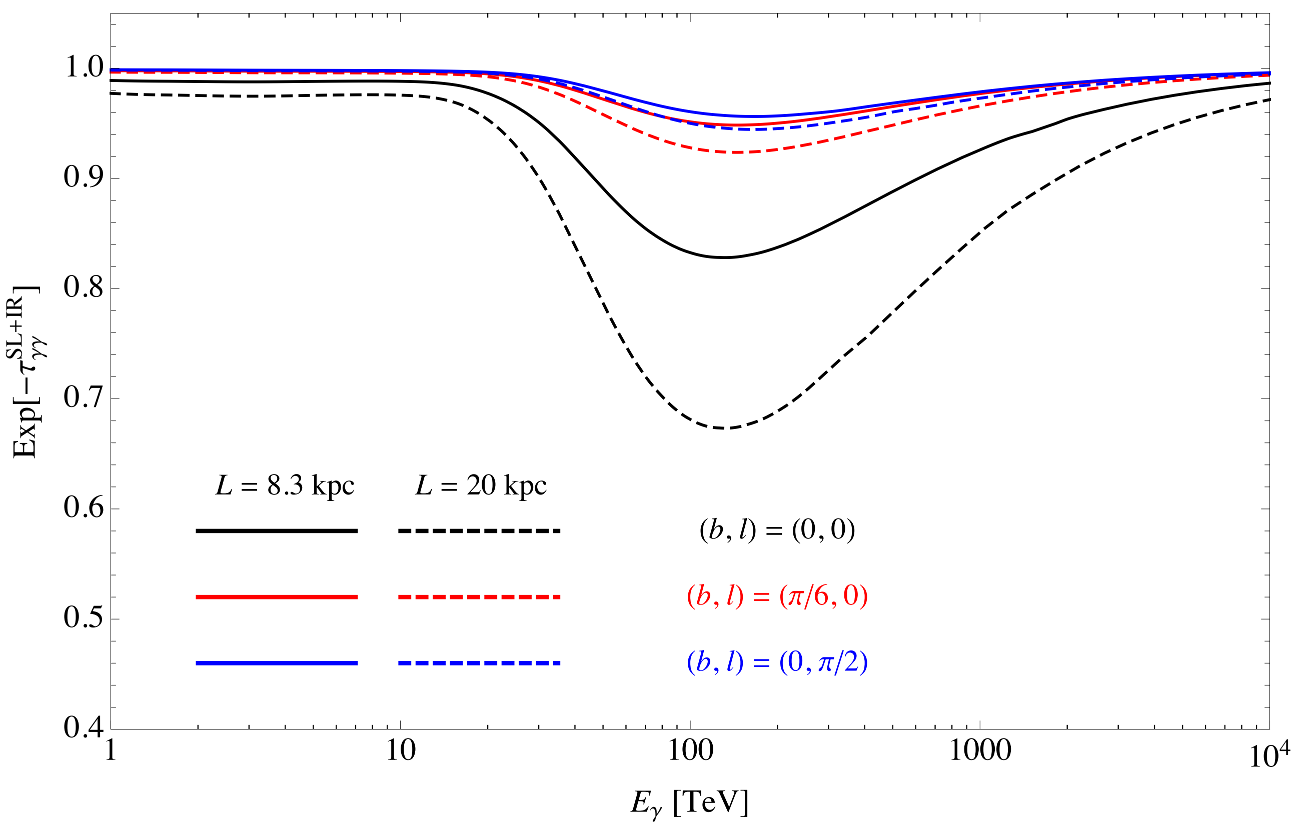


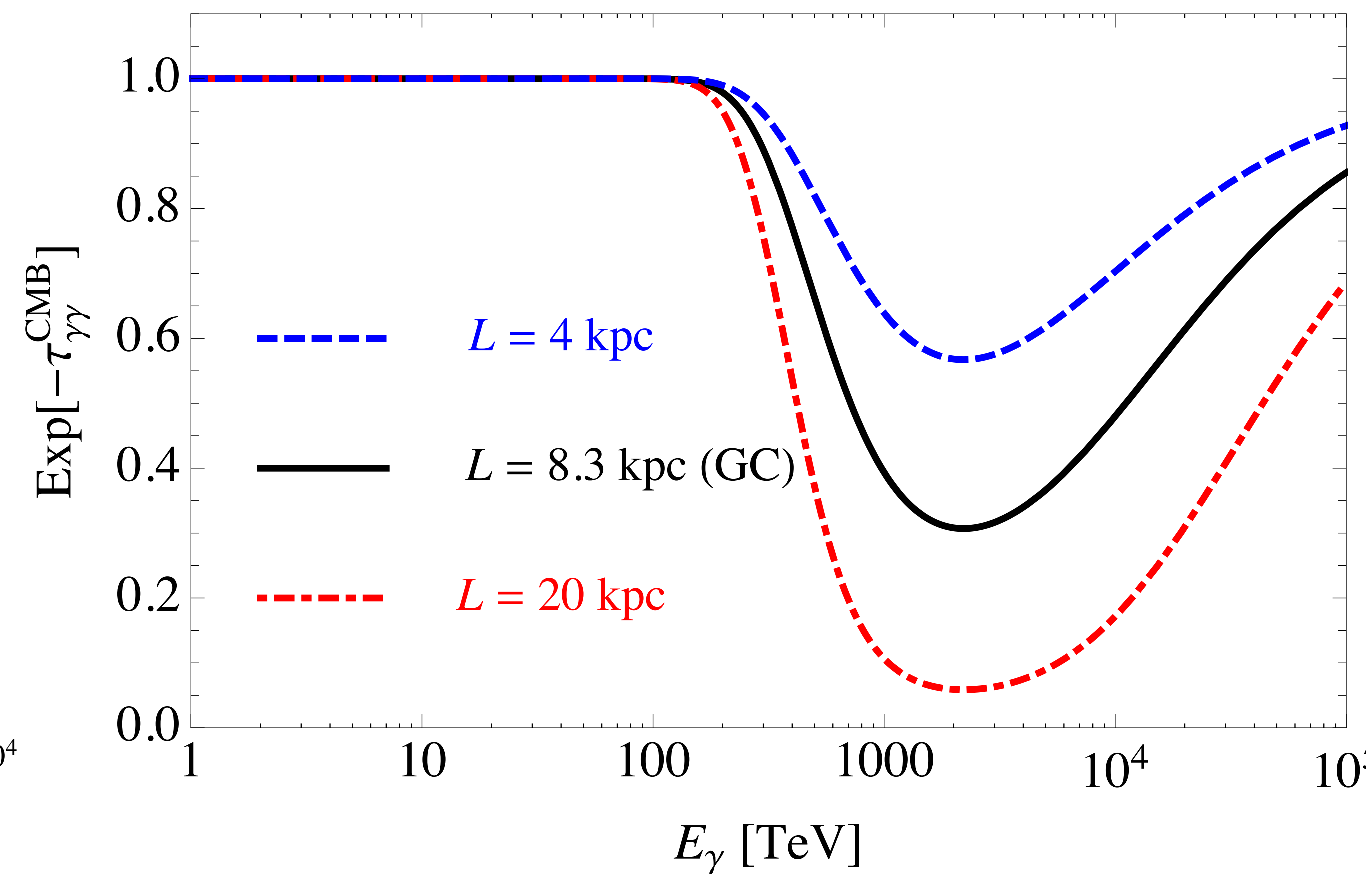
Gamma ray bounds

✓ Galactic component

at ~ PeV, the absorption length of gamma-rays are comparable to Galactic distances

A. E. and P. Serpico, JCAP (2015), arXiv:1505.06486





Absorption at ~ 100 TeV

Absorption due to pair production on SL+IR photons

Absorption at ~ PeV

Absorption due to pair production on CMB photons

Gamma ray bounds

✓ Galactic component

at ~ PeV, the absorption length of gamma-rays are comparable to Galactic distances

Prompt component

$$\frac{d\Phi_\gamma}{dE_\gamma}(E_\gamma, b, l) = \frac{1}{4\pi\, m_{\rm DM}\, \tau_{\rm DM}} \frac{dN_\gamma}{dE_\gamma}(E_\gamma) \int_0^\infty \rho_{\rm h}[\varrho(s,b,l)]\, e^{-\tau_{\gamma\gamma}(E_\gamma,s,b,l)}\, ds$$

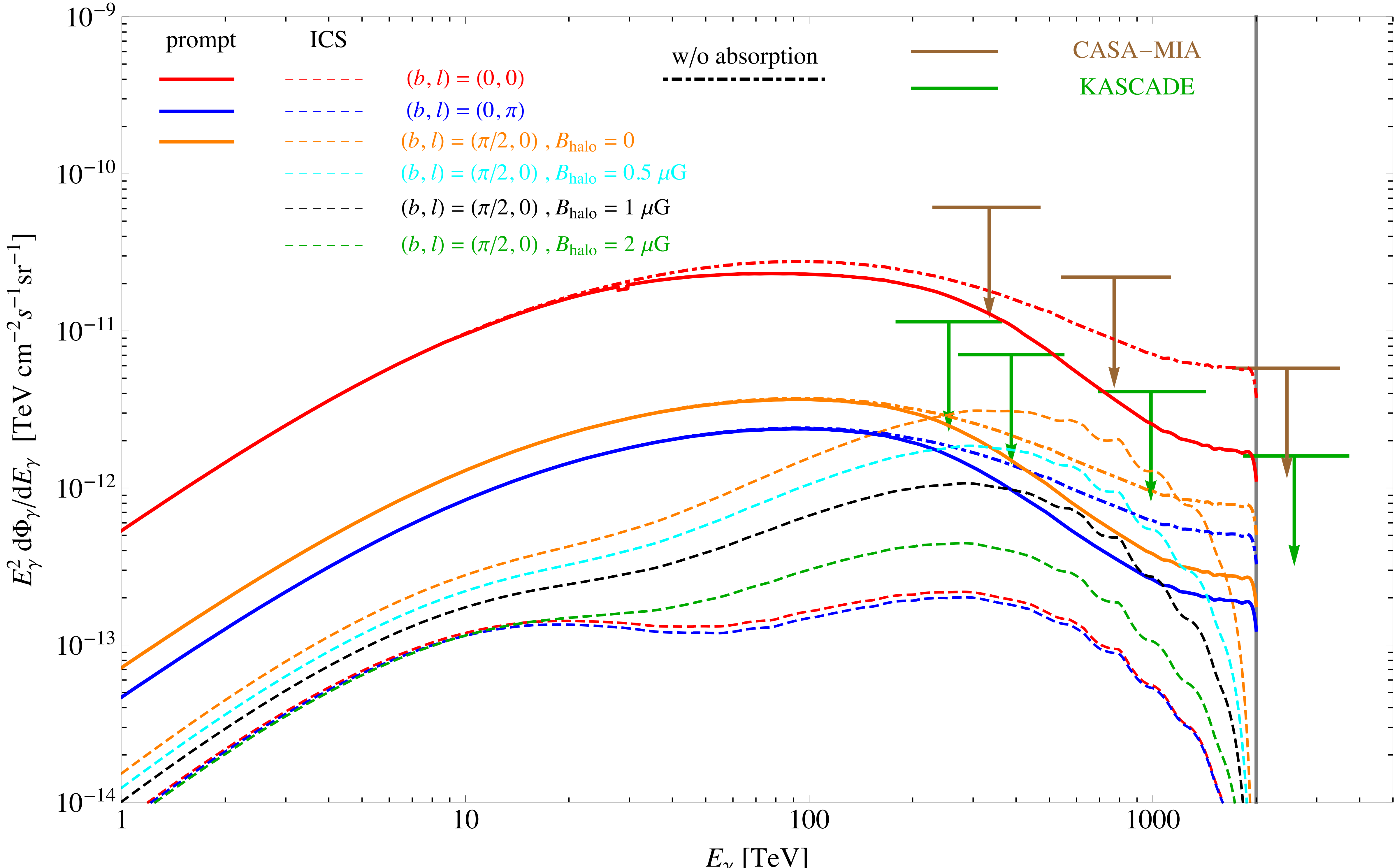
inverse-Compton component

$$\frac{d\Phi_{\rm IC}}{dE_\gamma}(E_\gamma, b, l) = \frac{1}{4\pi E_\gamma} \int_0^\infty ds\, e^{-\tau_{\gamma\gamma}(E_\gamma,s,b,l)} \int_{m_e}^{m_{\rm DM}/2} dE_e \frac{dn_e}{dE_e}(E_e, \varrho)\, P_{\rm IC}(E_e, E_\gamma, \varrho)$$

Gamma ray bounds

✓ Galactic component

$\tau_{DM} = 10^{28}$ s and $m_{DM} = 4$ PeV

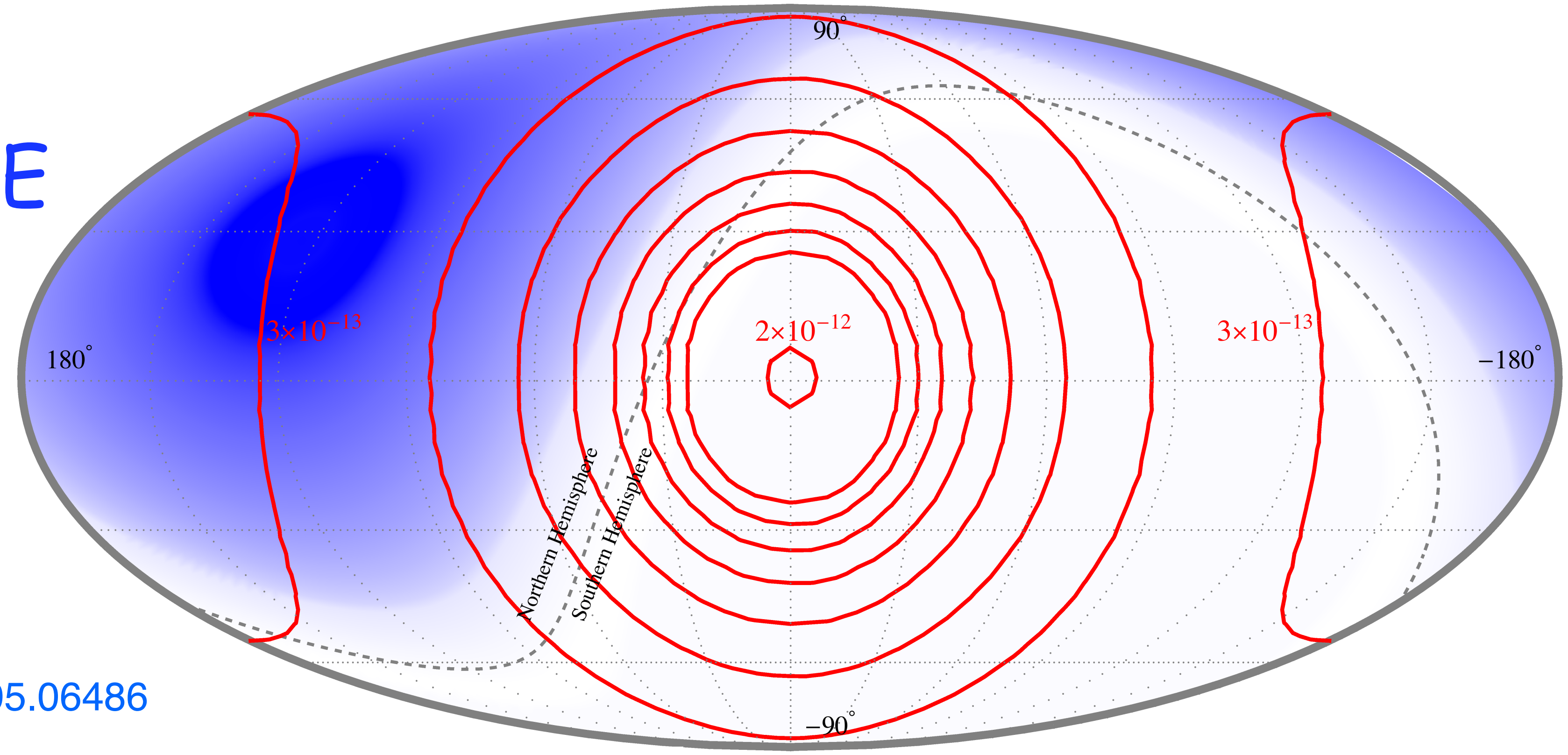


A. E. and P. Serpico, JCAP (2015), arXiv:1505.06486

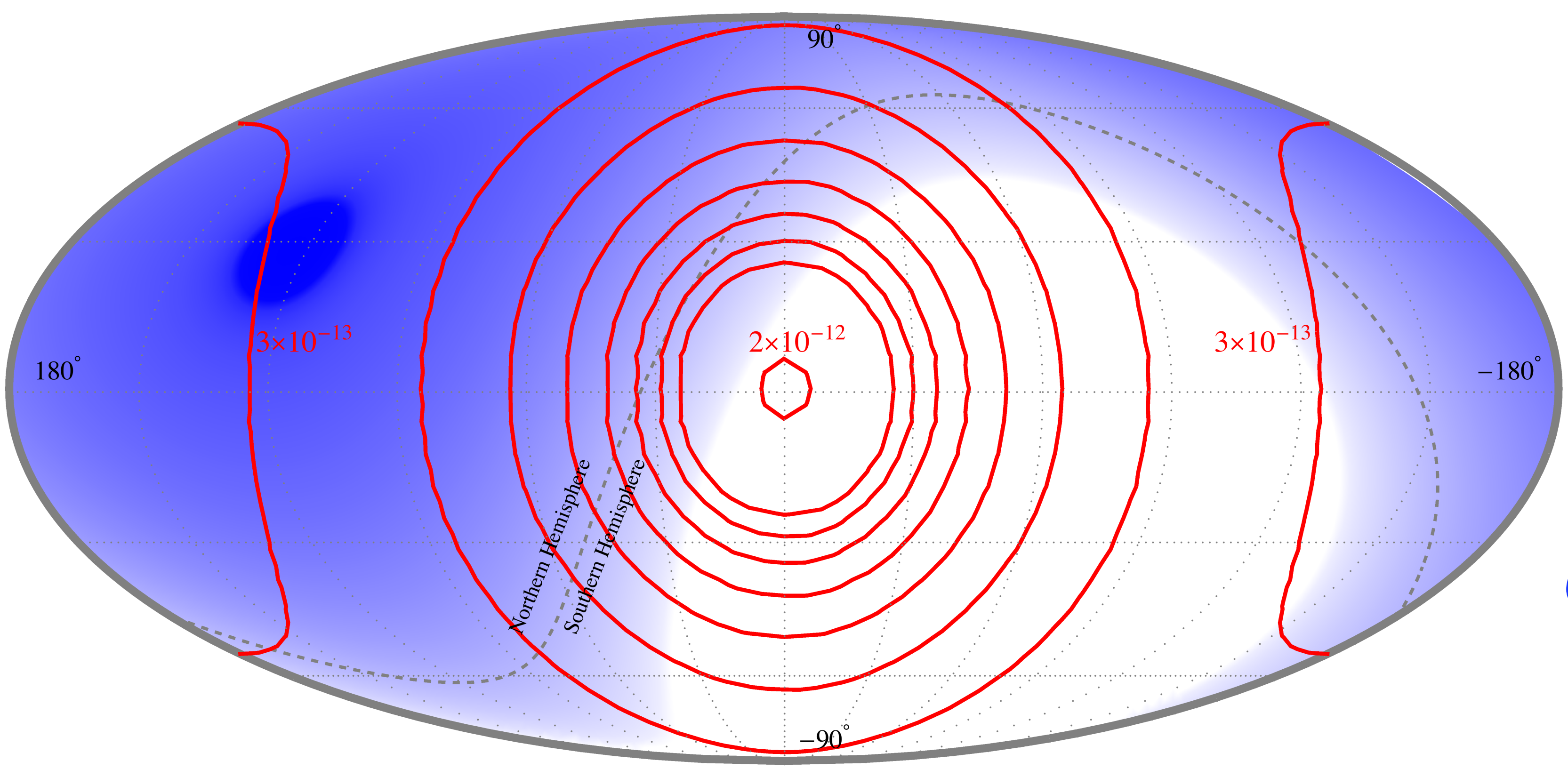
Gamma ray bounds

✓ Galactic component

KASCADE



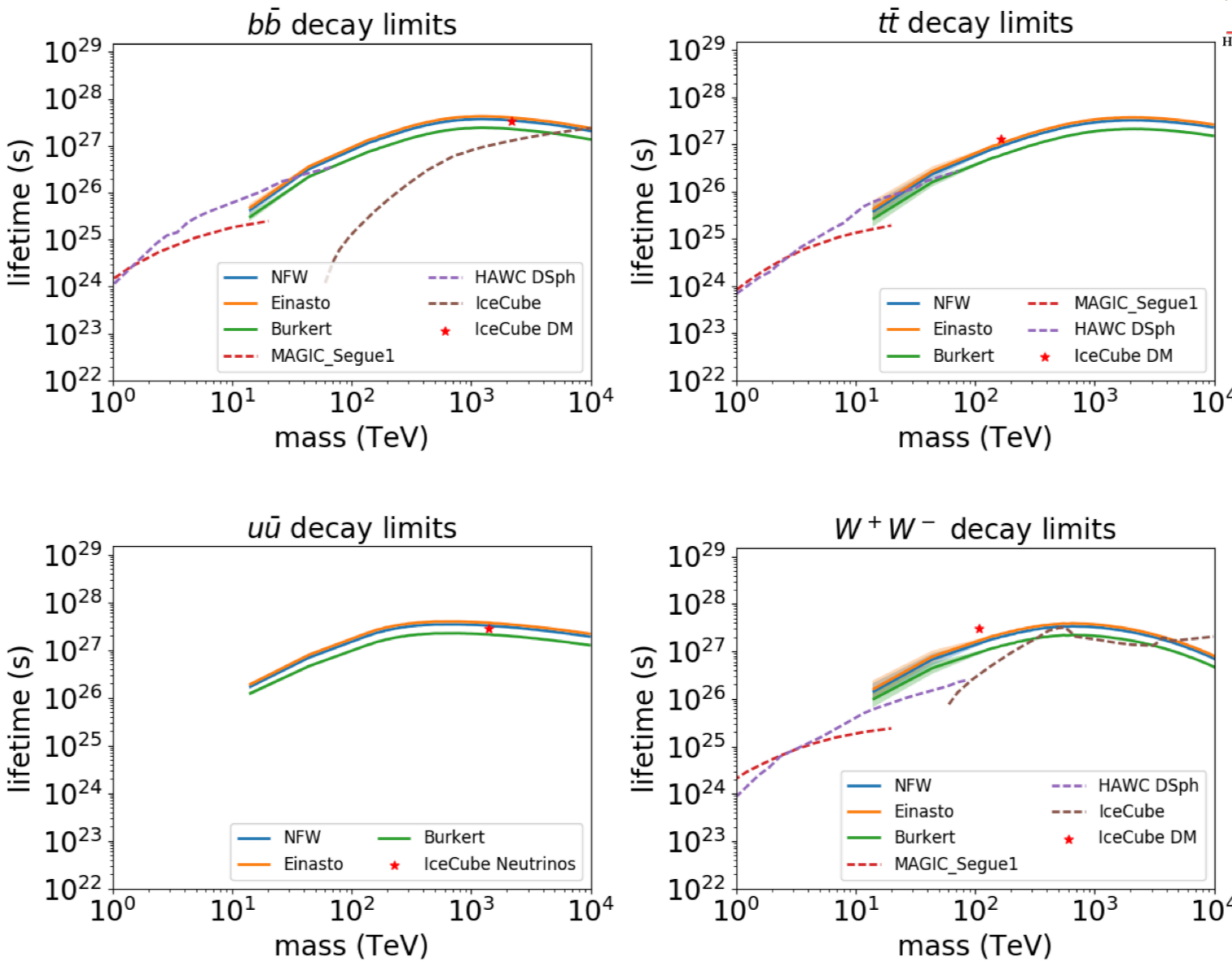
A. E. and P. Serpico, JCAP (2015), arXiv:1505.06486



CASA-MIA

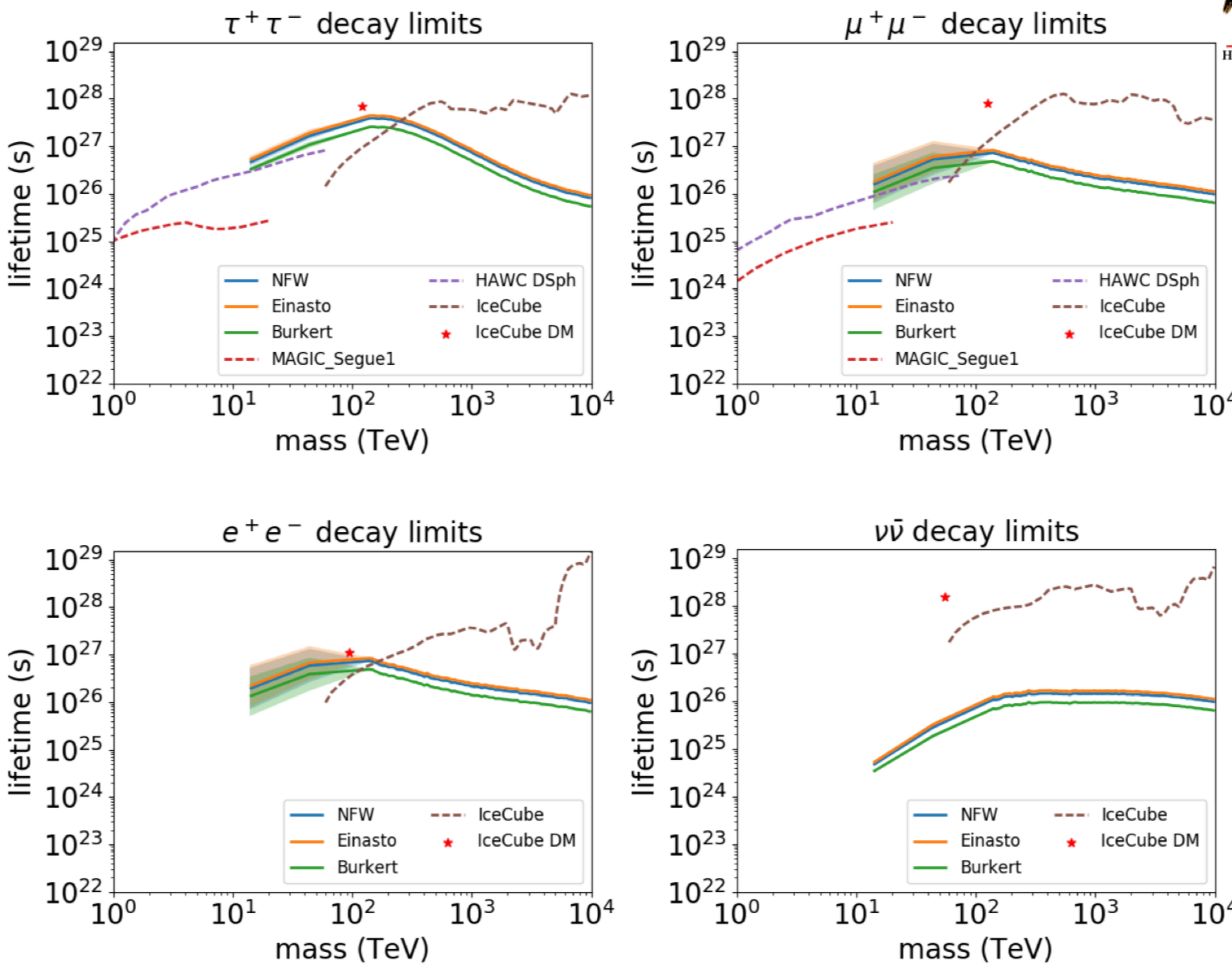
Gamma ray bounds

✓ Galactic component



Gamma ray bounds

✓ Galactic component

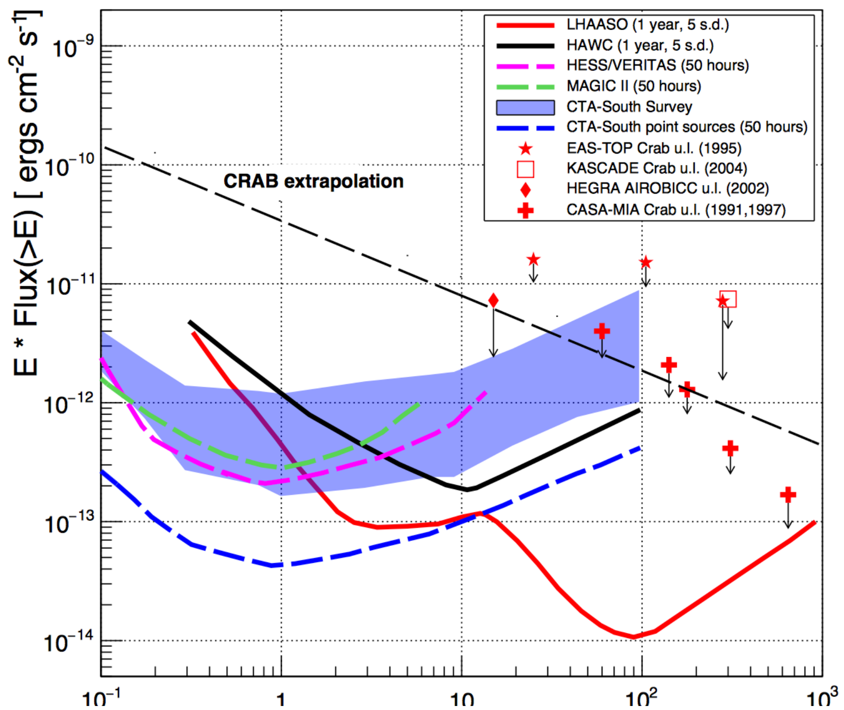


Gamma ray bounds

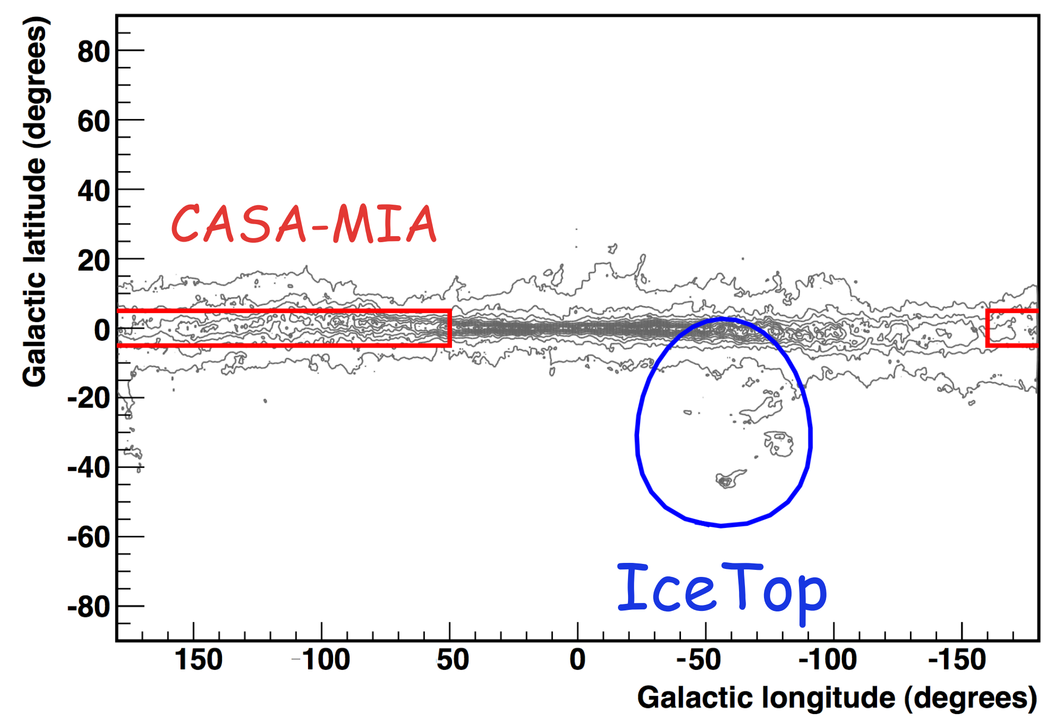
✓ Galactic component

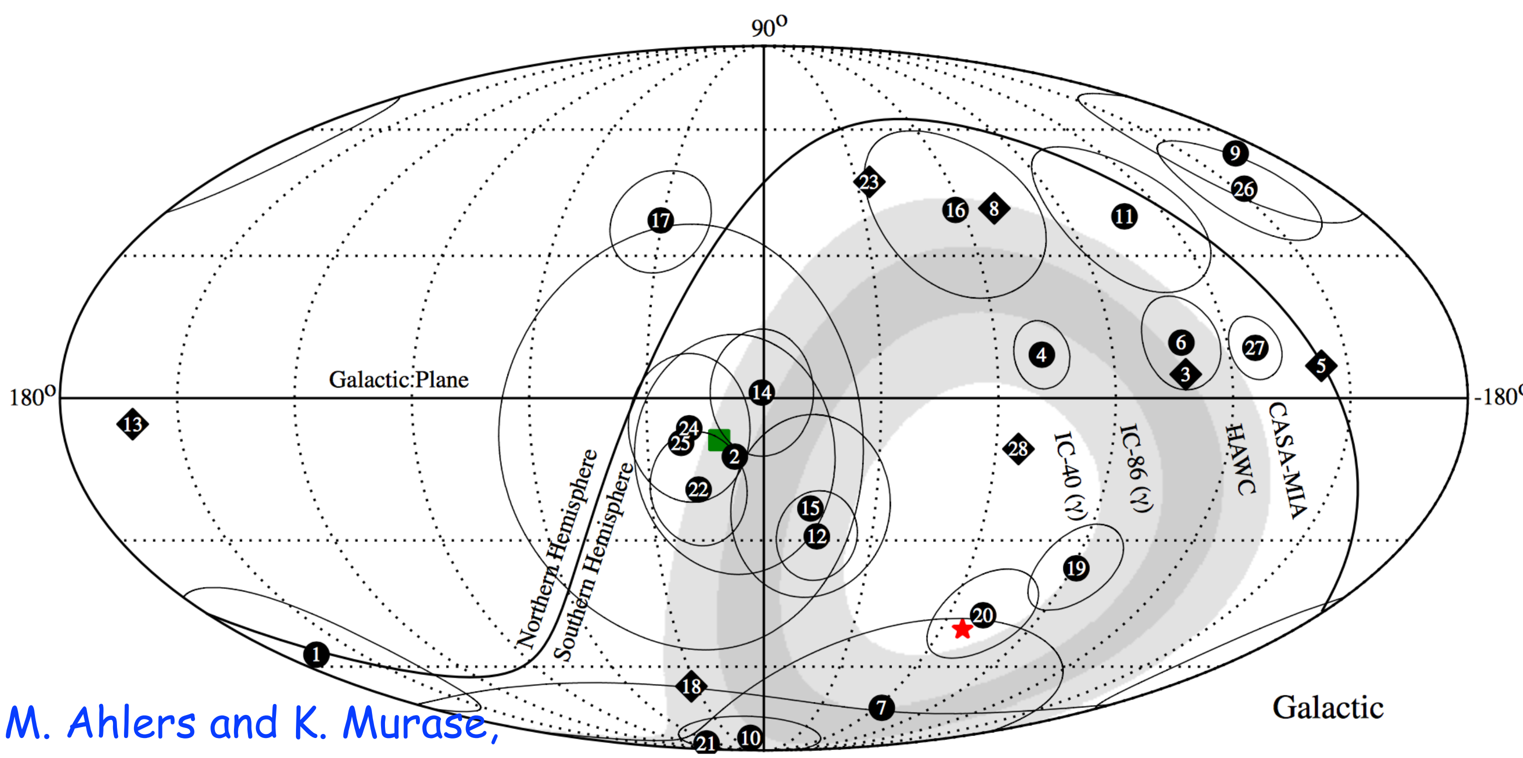
Future experiments

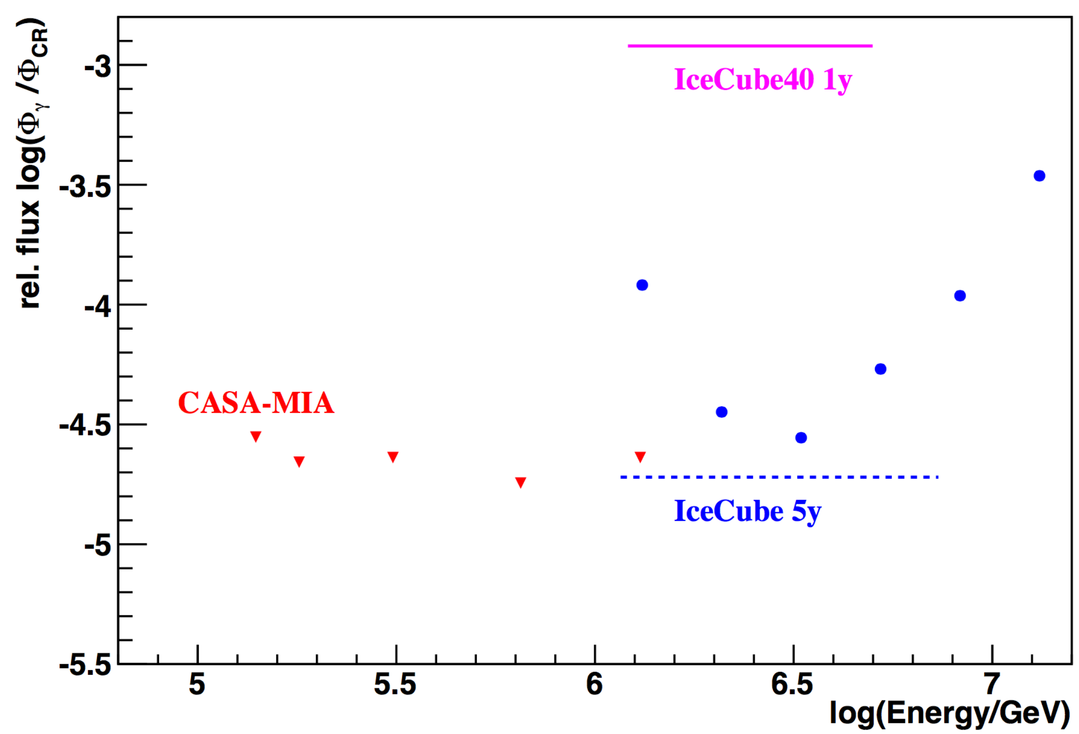
LHAASO



IceTop







M. Ahlers and K. Murase, PRD (2014), arXiv:1309.4077

Gamma ray bounds

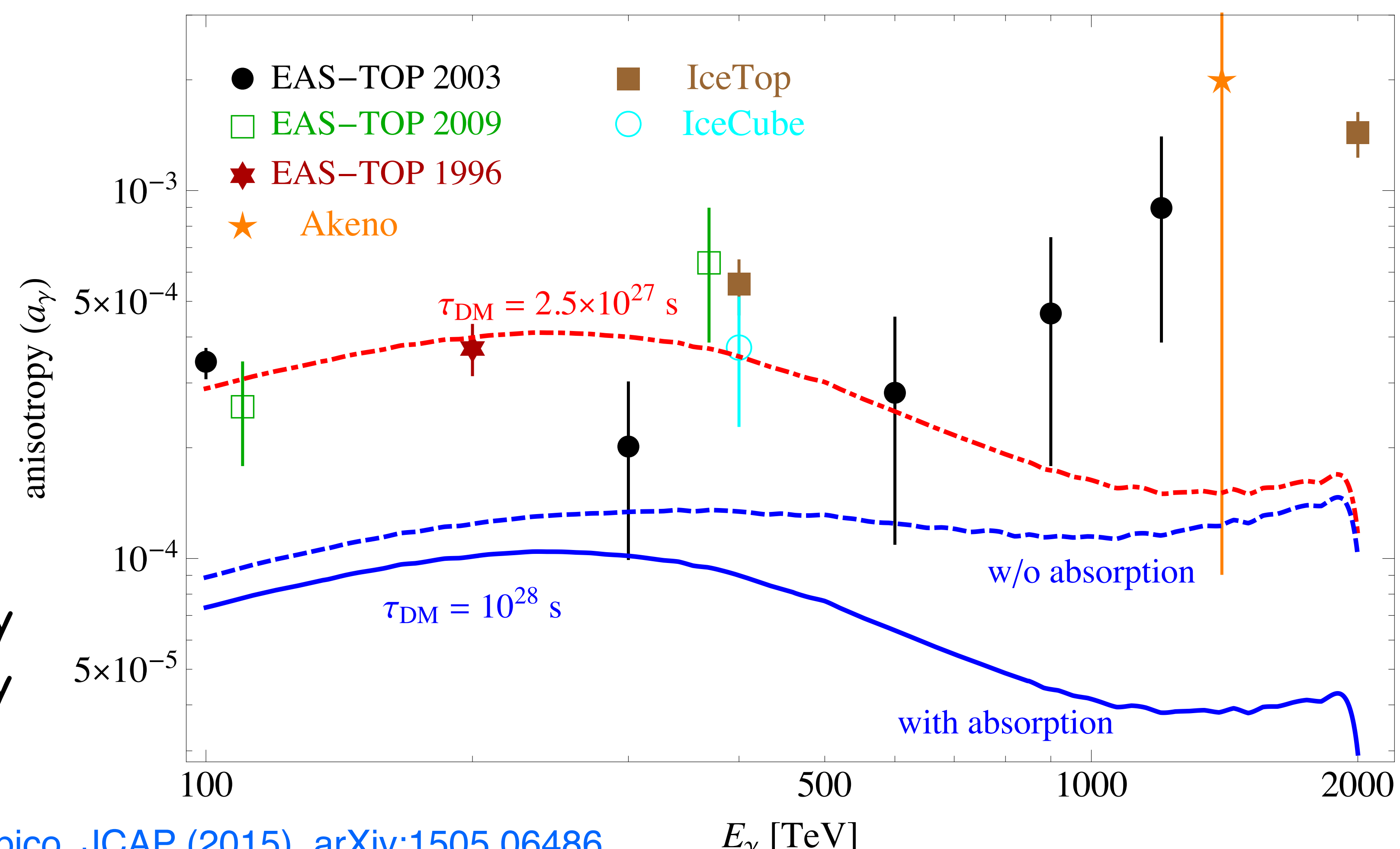
✓ Galactic component

Anisotropy

$$a_\gamma = \frac{\left.\frac{d\Phi_\gamma}{dE_\gamma}\right|_{GC} - \left.\frac{d\Phi_\gamma}{dE_\gamma}\right|_{anti-GC}}{\frac{d\Phi_{CR}}{dE}}$$

→ Total CR flux

- ✓ No need to γ/hadron discrimination
- ✓ Absorption suppress the anisotropy
- ✓ The bound 2.5×10^{27} s can be set
- ✓ Adding the phase info of anisotropy would improve the limits significantly



A. E. and P. Serpico, JCAP (2015), arXiv:1505.06486

conclusions

- ✓ The excess of events observed by IceCube in the energy range ~ 30 TeV - 2 PeV is an evidence for astrophysical flux or other "New Physics" induced fluxes

- ✓ Several features of the observed events motivate us for a DM interpretation: cut-off at ~ 2 PeV, a mild dip in the (400 - 1000) TeV and anisotropy.

- ✓ We argued that a PeV-scale decaying DM, with generic decay channels, can naturally explain these features. The required lifetime is allowed by the current limits. Both the energy and angular distributions mildly prefer DM interpretation.

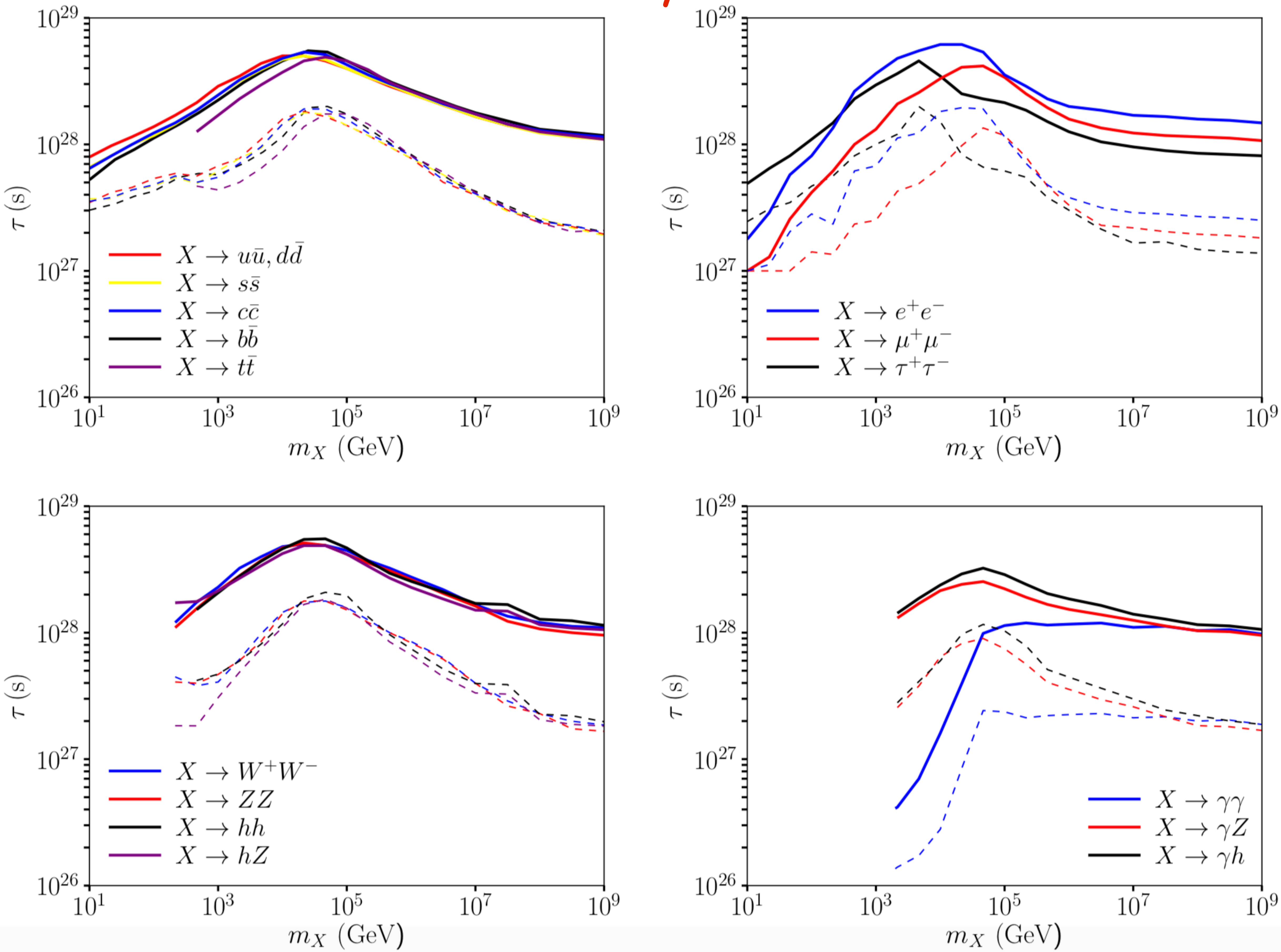
- ✓ With more statistics in the next few years, the DM interpretation of IceCube events can be tested. The gamma-ray flux expected in this scenario can be detected by the next generation of EAS detectors. Also, anisotropy measurements in the CR flux would be constraining.

conclusions

Thank you !

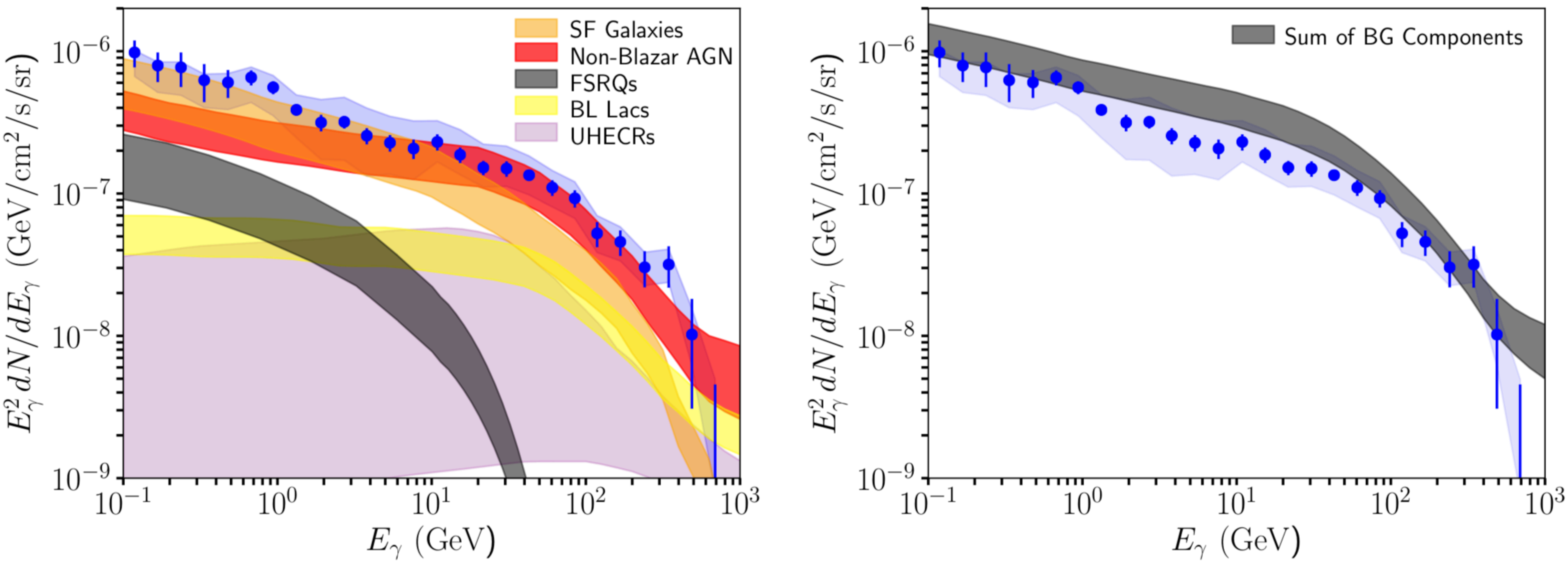
D. Hooper, C. Blanco, arXiv:1811.05988

Gamma ray bounds

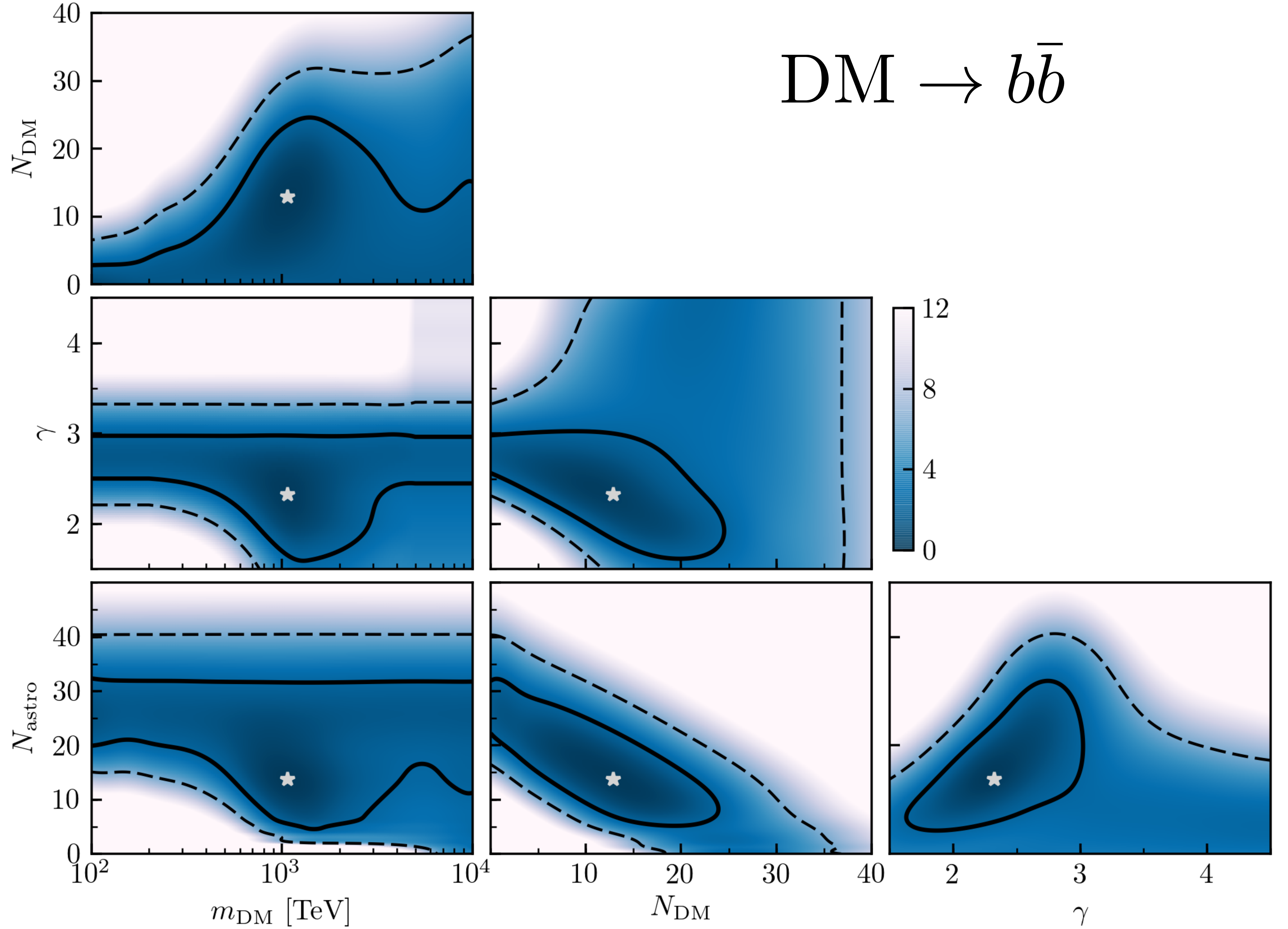


Gamma ray bounds

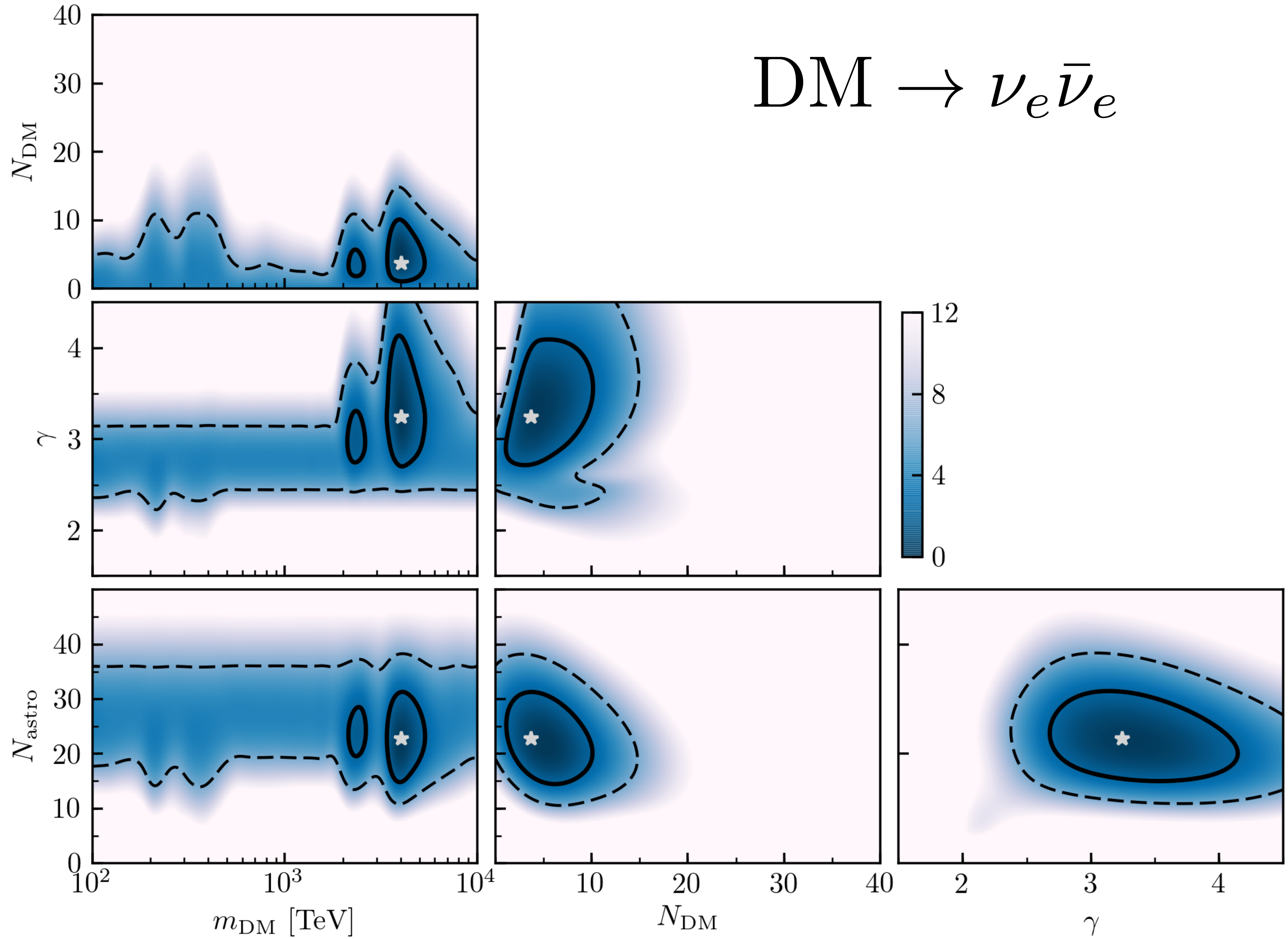
D. Hooper, C. Blanco, arXiv:1811.05988



Parameter correlations



Parameter correlations



Constraining DM properties

✓ DM lifetime

contribution of DM to the events in each bin should be smaller than $N_{\rm limit}$

bin #	$\log_{10}(E_\nu/\text{TeV})$	$N_{\rm astro}(E_\nu^{-2} \div E_\nu^{-2.3})$	$N_{\rm data}$	$N_{\rm limit}\ (E_\nu^{-2} \div E_\nu^{-2.3})$	$N_{\rm limit}$
#1	1.4 – 1.6	9.46 ÷ 10	11	7.8 ÷ 7.46	16.6
#2	1.6 – 1.8	4.31 ÷ 5.3	6	6.53 ÷ 5.87	10.5
#3	1.8 – 2.0	4.55 ÷ 5.68	7	7.41 ÷ 6.58	11.8
#4	2.0 – 2.2	3.97 ÷ 4.82	3	3.98 ÷ 3.73	6.68
#5	2.2 – 2.4	3.32 ÷ 3.56	4	5.15 ÷ 5.01	8.00
#6	2.4 – 2.6	2.59 ÷ 2.42	2	3.65 ÷ 3.71	5.32
#7	2.6 – 2.8	1.96 ÷ 1.62	0	2.3 ÷ 2.3	2.3
#8	2.8 – 3.0	1.55 ÷ 1.1	0	2.3 ÷ 2.3	2.3
#9	3.0 – 3.2	1.2 ÷ 0.74	2	4.31 ÷ 4.64	5.32
#10	3.2 – 3.4	0.92 ÷ 0.5	1	3.3 ÷ 3.51	3.89
#11	3.4 – 3.6	0.73 ÷ 0.35	0	2.3 ÷ 2.3	2.3
#12	3.6 – 3.8	1.72 ÷ 0.76	0	2.3 ÷ 2.3	2.3

Poisson statistics: at q% C.L.

$$\frac{q}{100} = \frac{\int_0^{N^i_{\rm limit}} L(N^i_{\rm data}, N)\, {\rm d}N}{\int_0^{\infty} L(N^i_{\rm data}, N)\, {\rm d}N}$$

$$L(N^i_{\rm data}, N) = \frac{(N + N^i_{\rm astro})^{N^i_{\rm data}}}{N^i_{\rm data}!} e^{-(N+N^i_{\rm astro})}$$

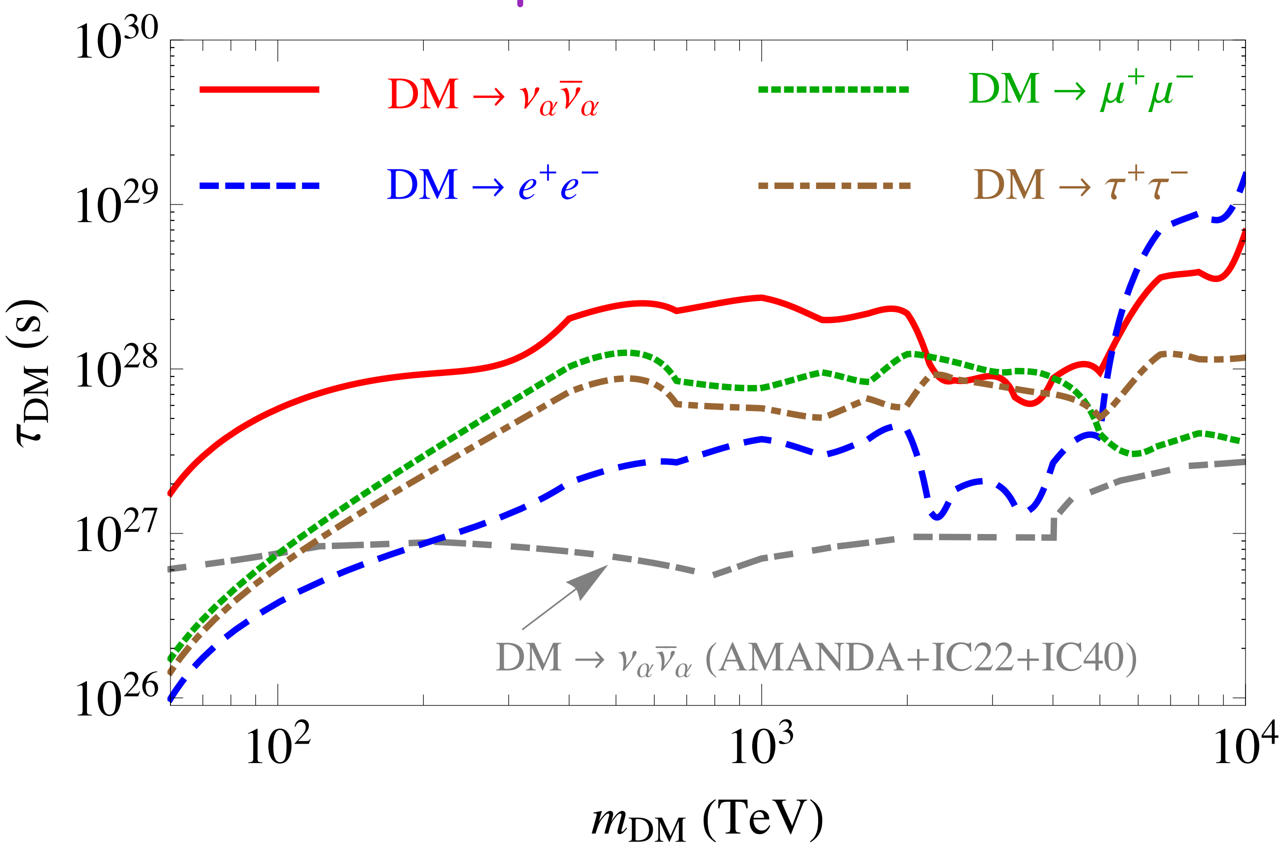
or

$$L(N^i_{\rm data}, N) = \frac{(N)^{N^i_{\rm data}}}{N^i_{\rm data}!} e^{-N}$$

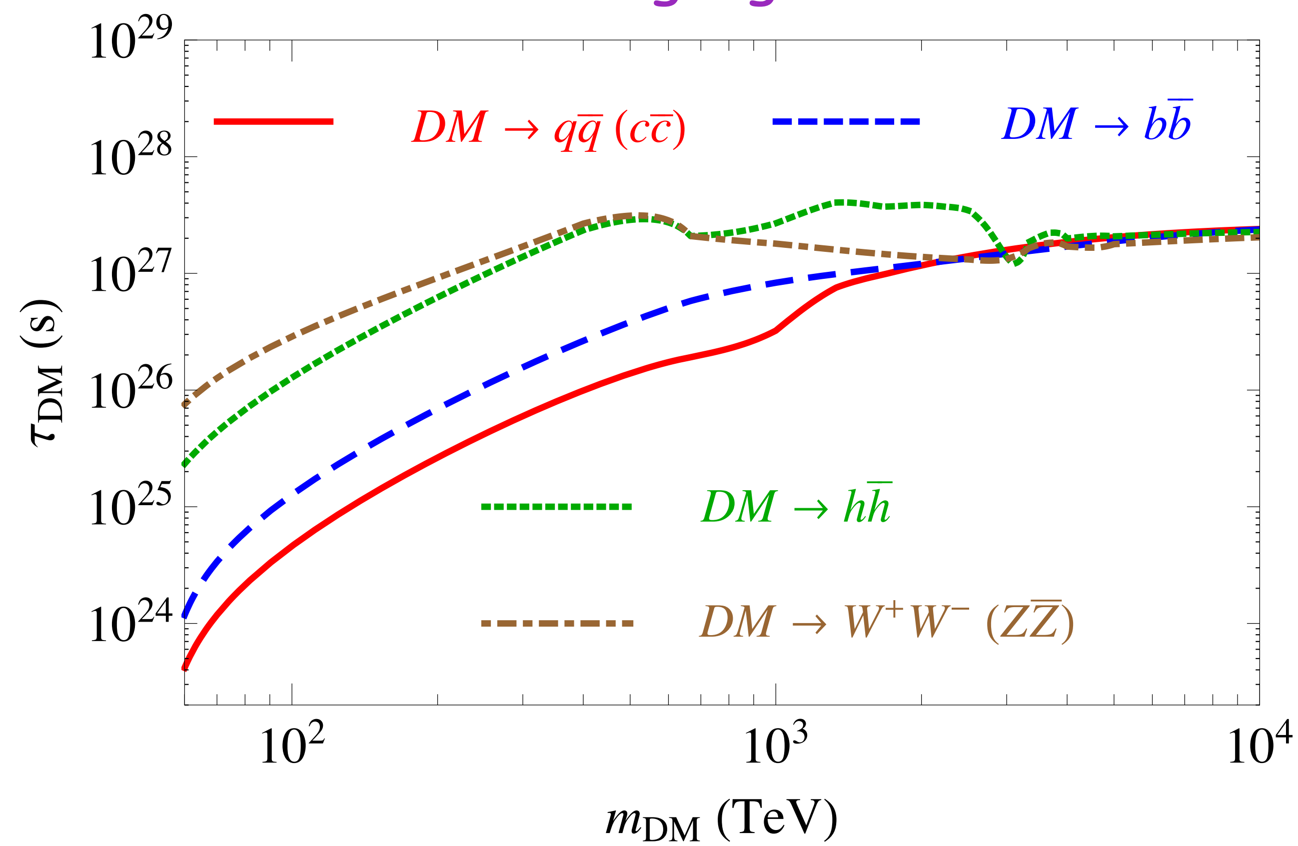
Constraining DM properties

✓ limits on DM lifetime (90% C.L.)

leptonic channels



hadronic/gauge channels



✓ at least one order of magnitude stronger lower limit on the DM lifetime, in the relevant DM mass range

✓ for a specific model, different channels should be scaled according to the corresponding branching ratios

Constraining DM properties

✓ Annihilation cross section

The lower part (< 100 TeV) of the observed spectrum can be used to probe <σv>

The isotropic components of neutrino flux from DM annihilation:

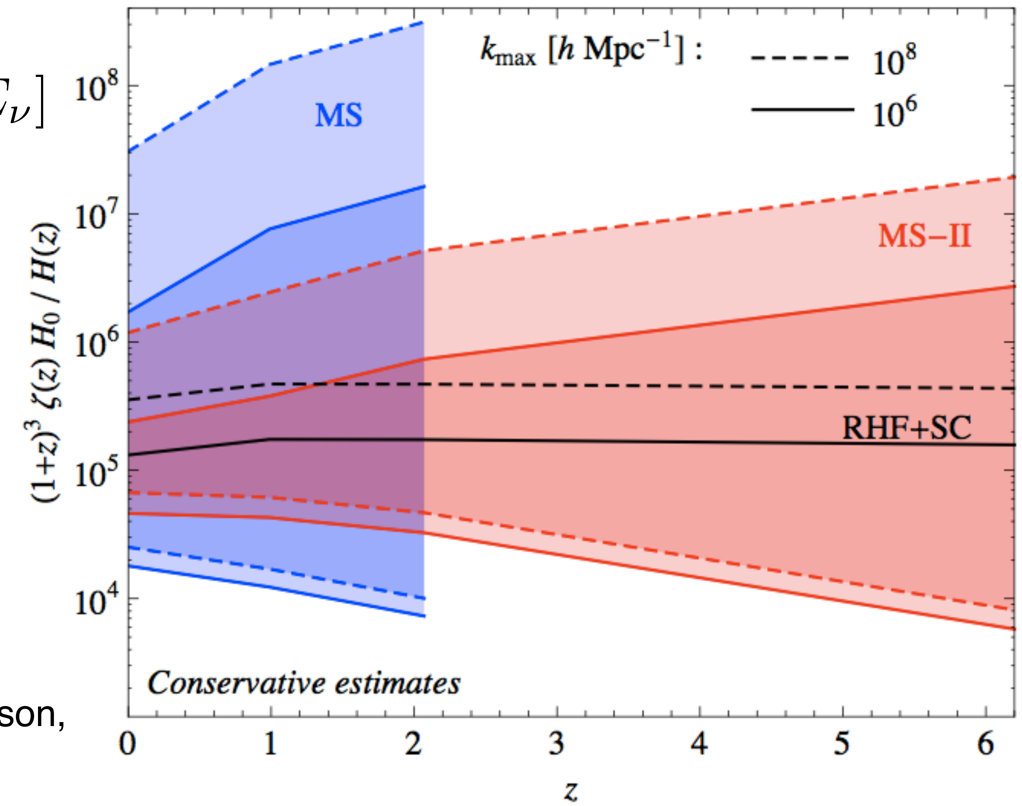
The residual isotropic flux from the Galactic halo (anti-GC direction)

$$\frac{\mathrm{d}J_{\rm iso}^{\rm ann}}{\mathrm{d}E_\nu} = \frac{\langle\sigma v\rangle}{2}\frac{1}{4\pi m_{\rm DM}^2}\frac{\mathrm{d}N}{\mathrm{d}E_\nu}(\mathrm{l.o.s.})_{\rm anti-GC} \quad \text{where } (\mathrm{l.o.s.})_{\rm anti-GC} = \int_0^\infty \rho^2[r(s,b=0,l=\pi)]\,\mathrm{d}s$$

The cosmic flux from all redshift

$$\frac{\mathrm{d}J_{\rm cos}^{\rm ann}}{\mathrm{d}E_\nu} = \frac{\langle\sigma v\rangle}{2}\frac{\Omega_{\rm DM}^2\rho_c^2}{4\pi m_{\rm DM}^2}\frac{c}{H_0}\int_0^\infty \frac{(1+z)^3\zeta(z)\mathrm{d}z}{\sqrt{\Omega_m(1+z)^3+\Omega_\Lambda}}\frac{\mathrm{d}N}{\mathrm{d}E_\nu}[(1+z)\,E_\nu]$$

$\zeta(z)$ flux multiplier (DM clustering)



E. Sefusatti, G. Zaharijas, P. D. Serpico, D. Theurel and M. Gustafsson, Mon. Not. Roy. Astron. Soc. (2014) [arXiv:1401.2117].

Constraining DM properties

✓ upper limits on annihilation cross section ⟨σv⟩ (90% C.L.)

minimum ÷ maximum value used for ζ(z) — unit of ⟨σv⟩ is 10^{-22} cm^{3}s^{-1}

DM + DM → \ m_{DM}	100 TeV	50 TeV	30 TeV
$\nu_\alpha\bar{\nu}_\alpha$	1.39 ÷ 0.22	1.21 ÷ 0.36	2.44 ÷ 0.88
$q\bar{q}$	489 ÷ 84.5	1427 ÷ 299	9934 ÷ 4603
$b\bar{b}$	185 ÷ 30.4	517 ÷ 106	3514 ÷ 1621
$c\bar{c}$	592 ÷ 100	1708 ÷ 348	11218 ÷ 5215
e^+e^-	14.7 ÷ 2.38	17.8 ÷ 5.06	41.3 ÷ 14.2
$\mu^+\mu^-$	4.47 ÷ 0.65	9.06 ÷ 1.6	23.7 ÷ 9.23
$\tau^+\tau^-$	5.84 ÷ 0.93	10.9 ÷ 2.3	28.5 ÷ 10.8
$h\bar{h}$	21.2 ÷ 3.36	53.4 ÷ 9.49	177 ÷ 76.5
$Z\bar{Z}$	11.9 ÷ 2.05	18.1 ÷ 4.09	40.7 ÷ 16.3
W^+W^-	14.4 ÷ 2.4	23.7 ÷ 4.96	54.5 ÷ 22.3

✓ for some final states (neutrinos, charged leptons) the limit is a bit stronger than the unitary bound

A note on Dark Matter

DM exist!

What We Do Not Know?

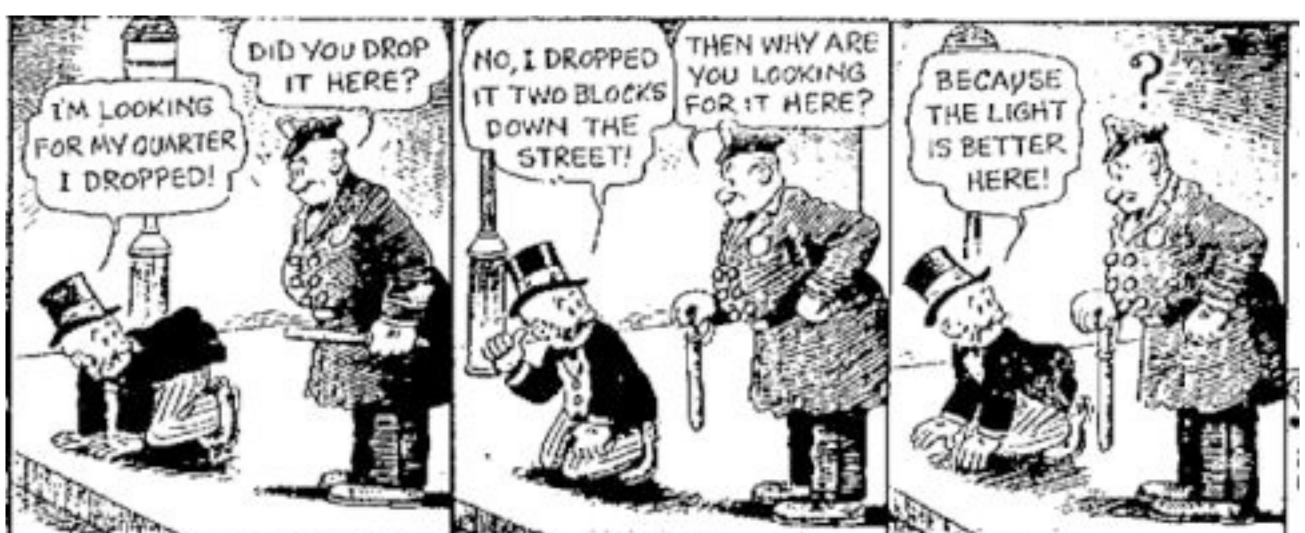
✓ Mass

axion	Sterile ν	WIMP	Wimpzilla
10^{-6} eV	~KeV	~100 GeV	M_{GUT} (10^{16} GeV)

⚠ "WIMP" paradigm ?

Note that WIMP paradigm is a "particle physics" conjecture, needs to be validated at colliders

caution: streetlight effect



Mulla Nasreddin

A note on Dark Matter

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✓ Lifetime: stable (∞) or

$\tau_{DM} > 4.3 \times 10^{17}$ s (age of Universe)

$\tau_{DM} > 2.2 \times 10^{19}$ s (CMB) Y. Gong and X. Chen, PRD77 (2008), arXiv:0802.2296

✓ Possible decay and/or annihilation channels

✓ ...

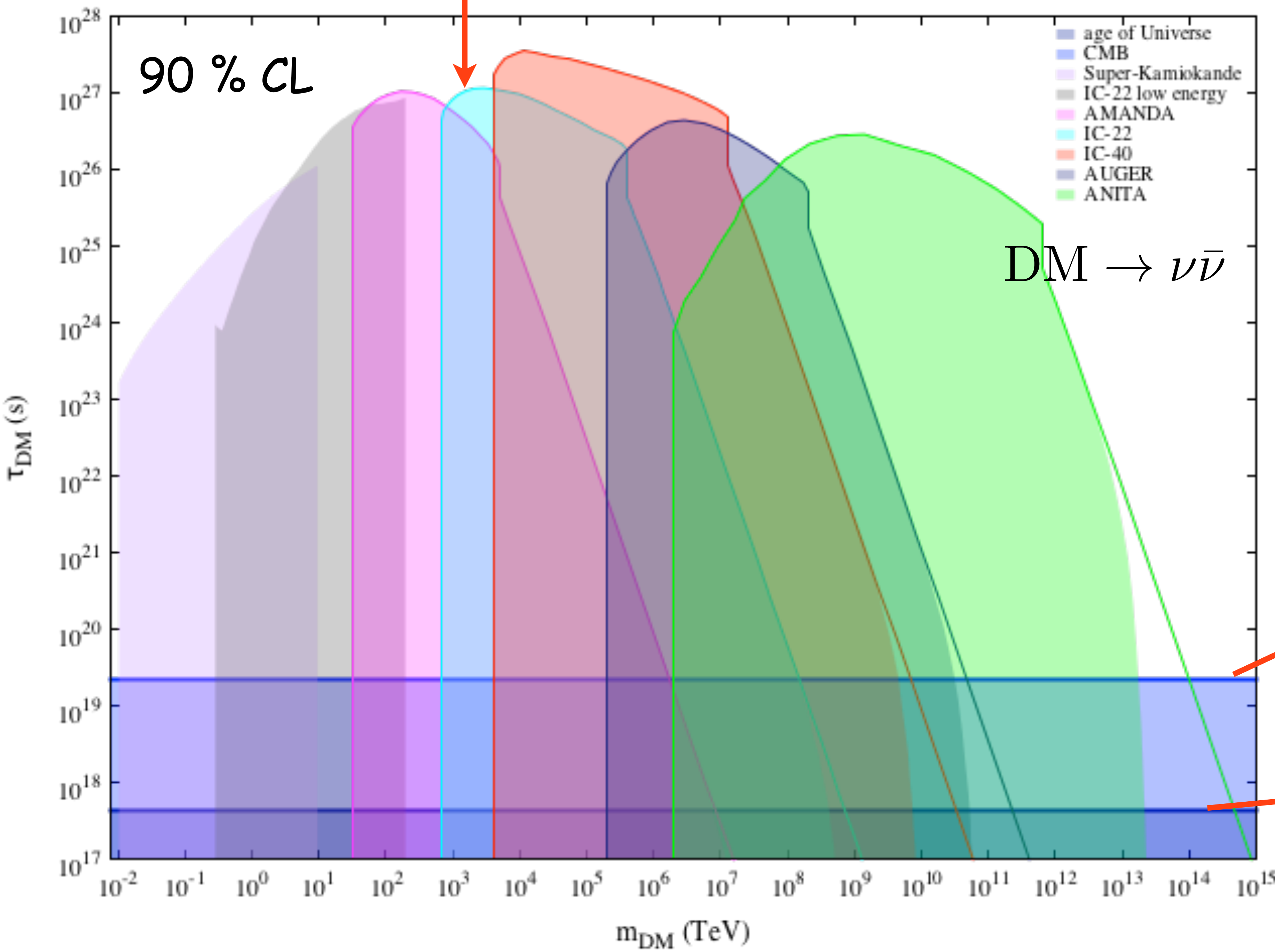
Limits on lifetime from neutrino experiments

before recent IceCube data

✓ Lifetime: stable (∞) or

this talk

A.E., Alejandro Ibarra and Orlando L. G. Peres
JCAP (2012) [arXiv: 1205.5281]



Using AMANDA, IceCube, Auger and ANITA data

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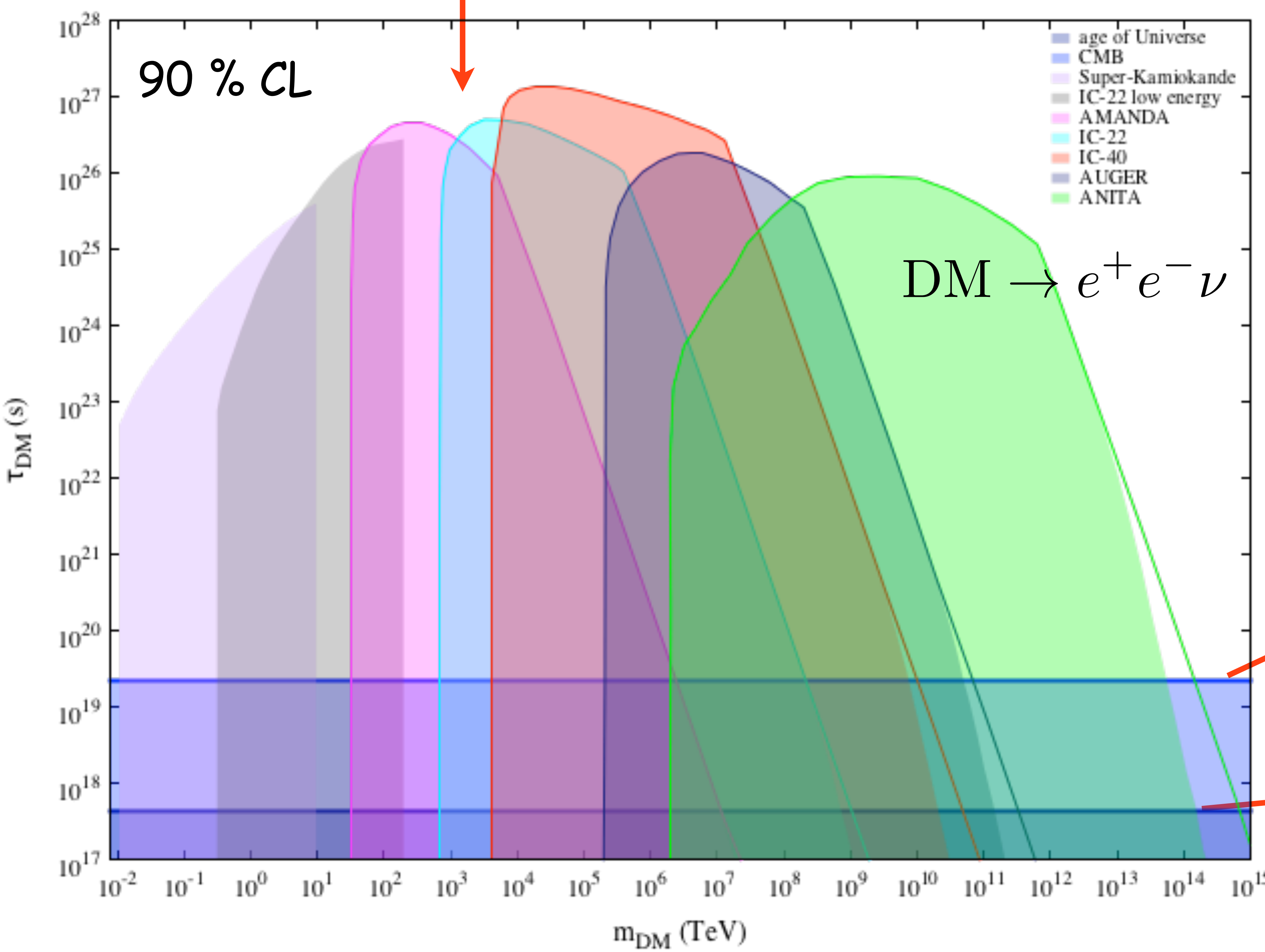

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Confronting with energy distribution of IceCube data

three years data set

SM sector ⟷ Dark sector

portal type: $$\mathcal{L}_{\rm protal} = \frac{\mathcal{O}_{\rm SM}\mathcal{O}_{\rm DM}}{\Lambda^{d-4}}$$

"neutrino" portal: $\mathcal{O}_{\rm SM} \to HL$

A. Falkowski, J. Juknevich and J. Shelton
arXiv:0908.1790

✓ d = 4 : $\mathcal{O}_{\rm DM} \to N$

heavy sterile neutrino, DM candidate

T. Higaki, R. Kitano and R. Sato, JHEP (2014)
arXiv:1405.0013

UV completion: $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$

$m_\phi \sim 10^{13}$ GeV

"Higgs" field ϕ_{B-L} plays the role of inflaton

$T_R \sim 10^7$ GeV

Confronting with energy distribution of IceCube data

three years data set

Leptogenesis: $\phi \to N_2 N_2$ $\quad M_2 \sim 10^{12}\ \text{GeV} \longrightarrow \frac{n_B}{s} \sim 10^{-10}$

DM abundance: $$\Omega_{N_1} \simeq 0.2 \left(\frac{M_1}{4\ \text{PeV}} \right)^3 \left(\frac{T_R}{3 \times 10^7\ \text{GeV}} \right)^{-1}$$

DM lifetime: $$\tau_{N_1} \simeq 8 \times 10^{28}\ \text{s} \left(\frac{M_1}{1\ \text{PeV}} \right)^{-1} \left(\frac{10^{-29}}{|y_N|^2} \right)$$

DM decay channels:

$$\text{Br}(\ell^\pm W^\mp) = 2\text{Br}(\nu_\ell Z) = 2\text{Br}(\nu_\ell h) = |U_{\ell 1}|^2 \qquad \text{NH}$$

$$\text{Br}(\ell^\pm W^\mp) = 2\text{Br}(\nu_\ell Z) = 2\text{Br}(\nu_\ell h) = |U_{\ell 3}|^2 \qquad \text{IH}$$

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A. Falkowski, J. Juknevich and J. Shelton arXiv:0908.1790 [hep-ph].

✓ d =4: $$\mathcal{O}_{\rm DM} \to N$$

production mechanism:

$m_\phi \gg m_N$ inflaton decay

$m_\phi \ll m_N$ freeze-in

$g\phi NN \ , \ g \simeq 10^{-6}$

Confronting with energy distribution of IceCube data

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SM sector ⟷ Dark sector

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"neutrino" portal: $$\mathcal{O}_{\rm SM} \to HL$$

A. Falkowski, J. Juknevich and J. Shelton arXiv:0908.1790 [hep-ph].

✓ d = 5 : $$\mathcal{O}_{\rm DM} \to \chi\phi$$

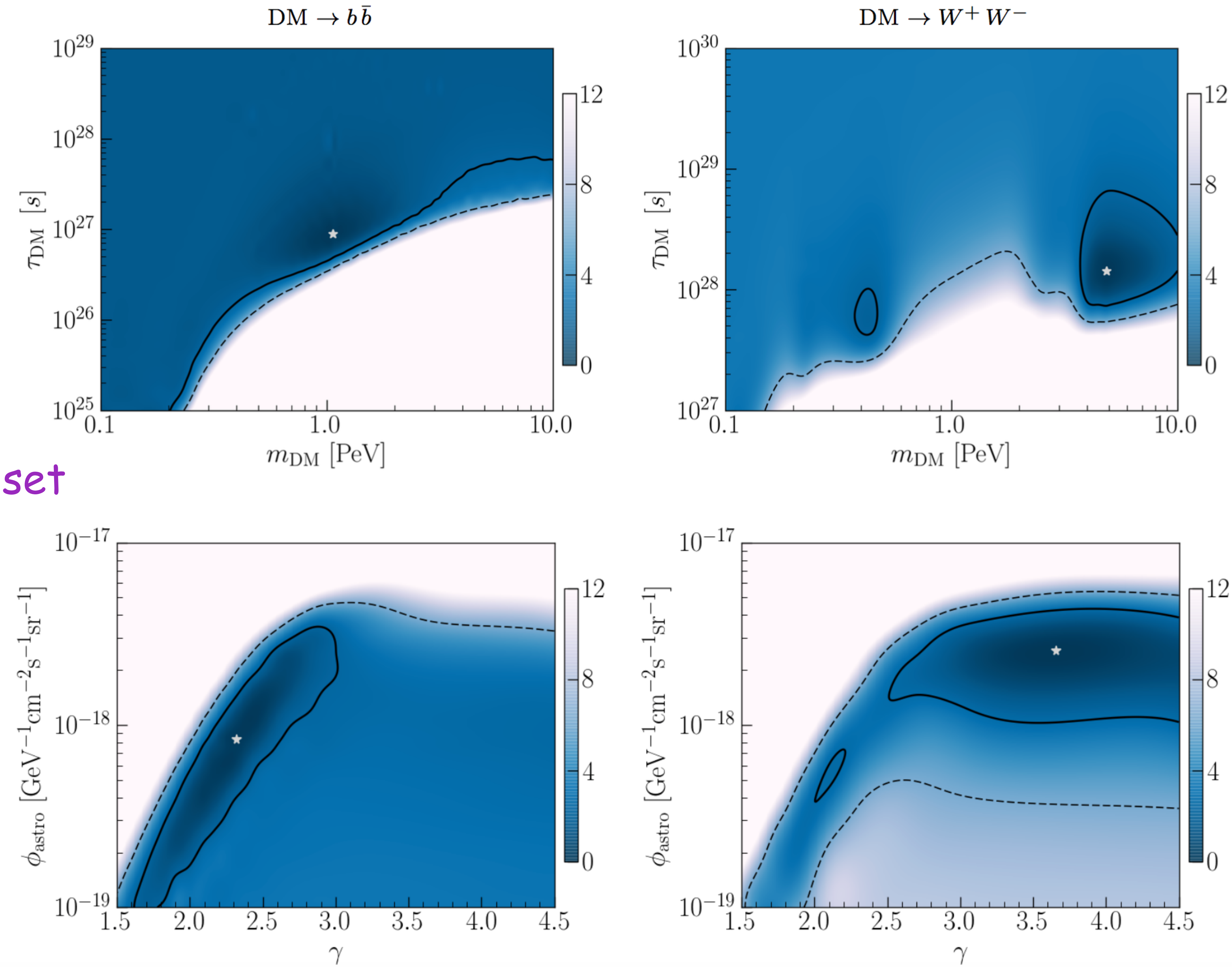
singlet fermion and scalar
(Asymmetric DM)

✓ d = 6 : other portals

✓ For d > 4 there are more freedom in branching ratios. We have shown that for the most constrained model (d=4) a good fit to the data can be obtained. Obviously better fits can be achieved for d > 4.

Confronting with energy distribution of IceCube data

preferred regions:

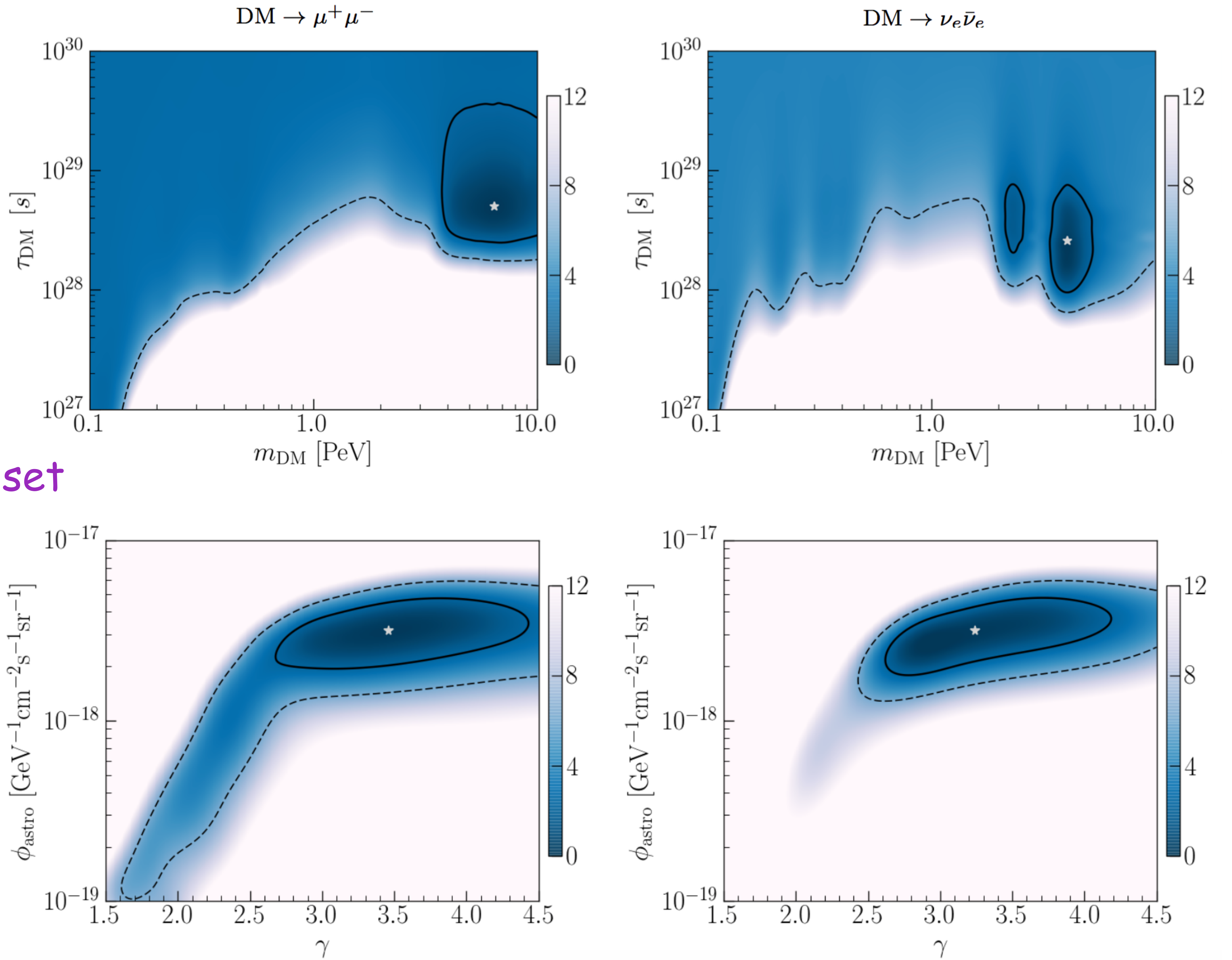


4 years data set

Confronting with energy distribution of IceCube data

preferred regions:

4 years data set

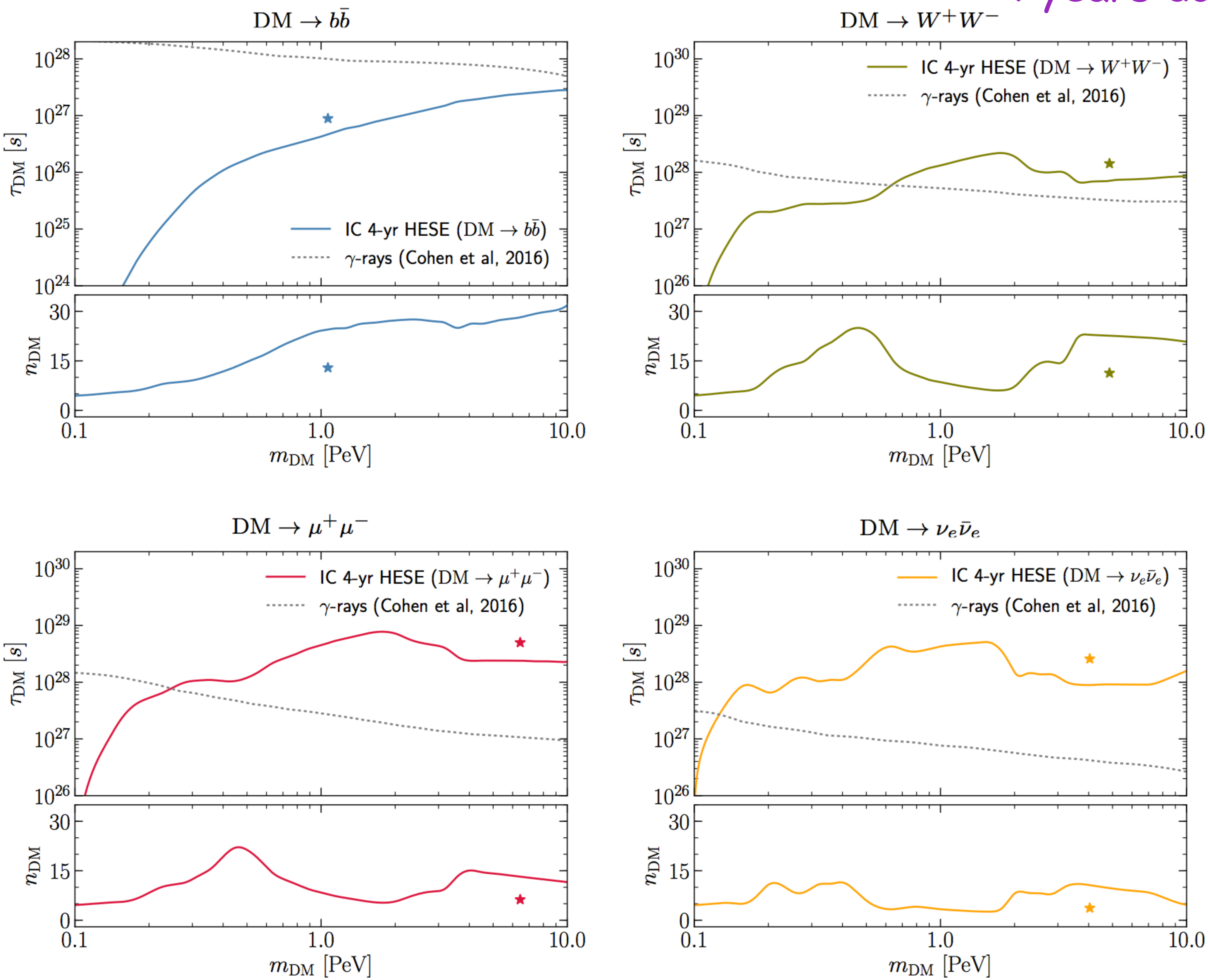


limits on DM from IceCube data

T. Cohen, K. Murase, N. L. Rodd, B. R. Safdi and Y. Soreq,

arXiv:1612.05638

4 years data set



Confronting with energy distribution of IceCube data

4 years data set

fitting parameters

✓ Likelihood analysis, taking into account the angular (up-going / down-going) and energy distribution simultaneously, tau regeneration, etc.

$$\mathcal{L}^c(\boldsymbol{\theta}) = \frac{e^{-N_{\rm DM}-N_{\rm astro}-N_\nu-N_\mu}}{N_{\rm obs}!} \prod_{i=1}^{N_{\rm obs}} \mathcal{L}_i^c(\boldsymbol{\theta})$$

$$\mathcal{L}_i^c(\boldsymbol{\theta}) = N_{\rm DM}\,\mathcal{P}^c_{{\rm DM},i}(m_{\rm DM}) + N_{\rm astro}\,\mathcal{P}_{{\rm astro},i}(\gamma) + N_\nu\,\mathcal{P}_{\nu,i} + N_\mu\,\mathcal{P}_{\mu,i}$$

Energy range [10TeV,10PeV] : $N_\nu = 9.0$ and $N_\mu = 12.6$

Energy range [60TeV,10PeV] : $N_\nu = 3.3$ and $N_\mu = 0.6$

Confronting with energy distribution of IceCube data

4 years data set

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$$\mathcal{P}^c_{{\rm DM},i}(m_{\rm DM}) = \frac{1}{\sum_{\ell,H',T'} \int_{E_{\rm min}}^{E_{\rm max}} dE_{\rm dep}\, \frac{d\left(N^c_{\rm DM}\right)^{T'}_{\ell,H'}}{dE_{\rm dep}}} \sum_\ell \frac{d\left(N^c_{\rm DM}\right)^{T_i}_{\ell,H_i}}{dE_{{\rm dep},i}}$$

$$\mathrm{TS}^c_{\rm 2D}(\boldsymbol{\theta}_{\rm \bf test}) = -2\,\ln \frac{\mathcal{L}^c(\boldsymbol{\theta}_{\rm test}, \widehat{\widehat{\boldsymbol{\nu}}}(\boldsymbol{\theta}_{\rm \bf test}))}{\mathcal{L}^c(\widehat{\boldsymbol{\theta}})}$$