

Experimental Hunt for Axions

Andreas Ringwald

DARKWIN

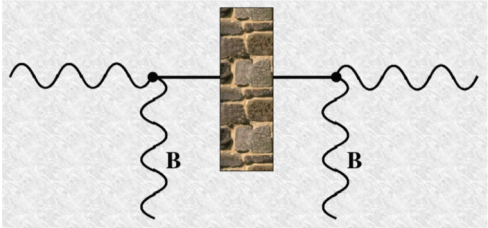
Natal, Brazil

5 September 2019

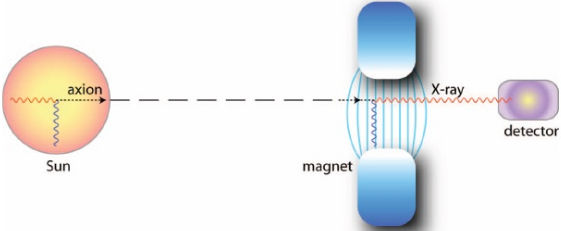


Experimental Axion Search Methods

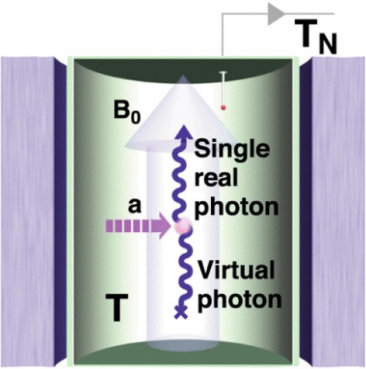
- Light-shining-through-a-wall (LSW) Searches



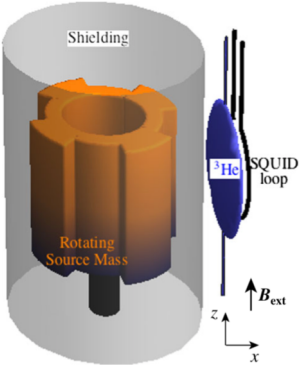
- Searches for Solar Axions



- Searches for Dark Matter Axions



- Searches for Axion-Mediated Forces



Light-Shining-through-a-Wall Searches

Searching for home-made axions

$$\mathcal{L} \supset -\frac{g_{a\gamma}}{4} a F_{\mu\nu} \tilde{F}^{\mu\nu} \equiv g_{a\gamma} a \mathbf{E} \cdot \mathbf{B}$$

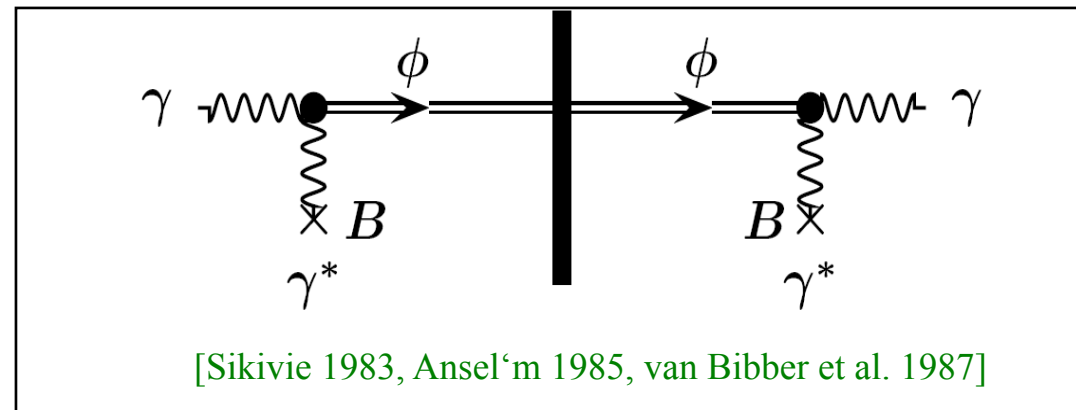
- Axion experiences mixing with photon in an external electromagnetic field
- Probability, that photon converted in axion/ALP after having traversed a distance L_B in magnetic field:

$$P(a \leftrightarrow \gamma) = 4 \frac{(g_{a\gamma} \omega B)^2}{m_a^4} \sin^2 \left(\frac{m_a^2}{4\omega} L_B \right)$$

- For very light axion:

$$P(\gamma \leftrightarrow a) \simeq \frac{1}{4} (g_{a\gamma} B L_B)^2$$

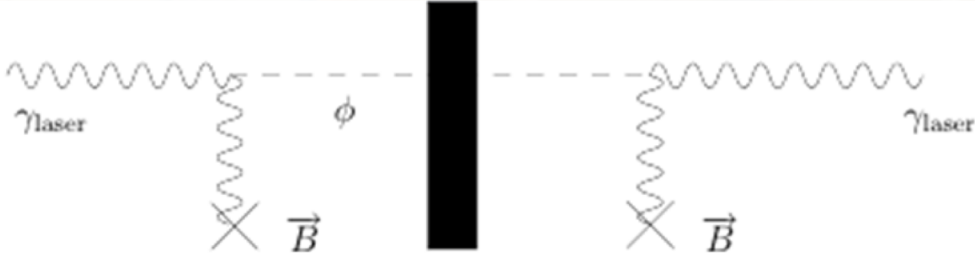
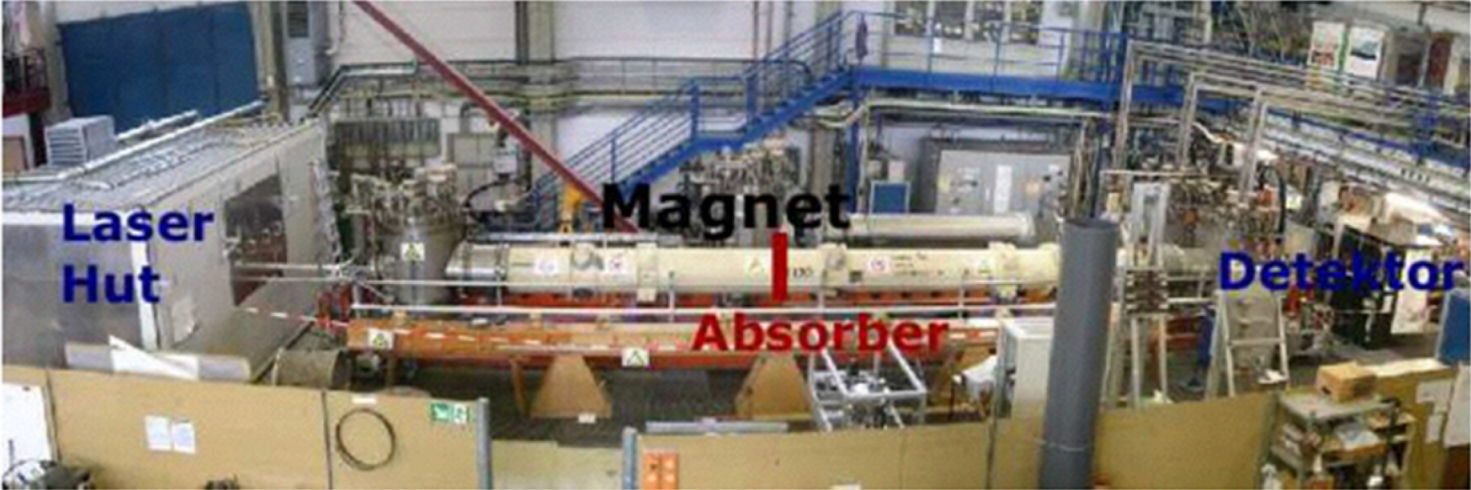
- Light-shining-through a wall:



Light-Shining-through-a-Wall Searches

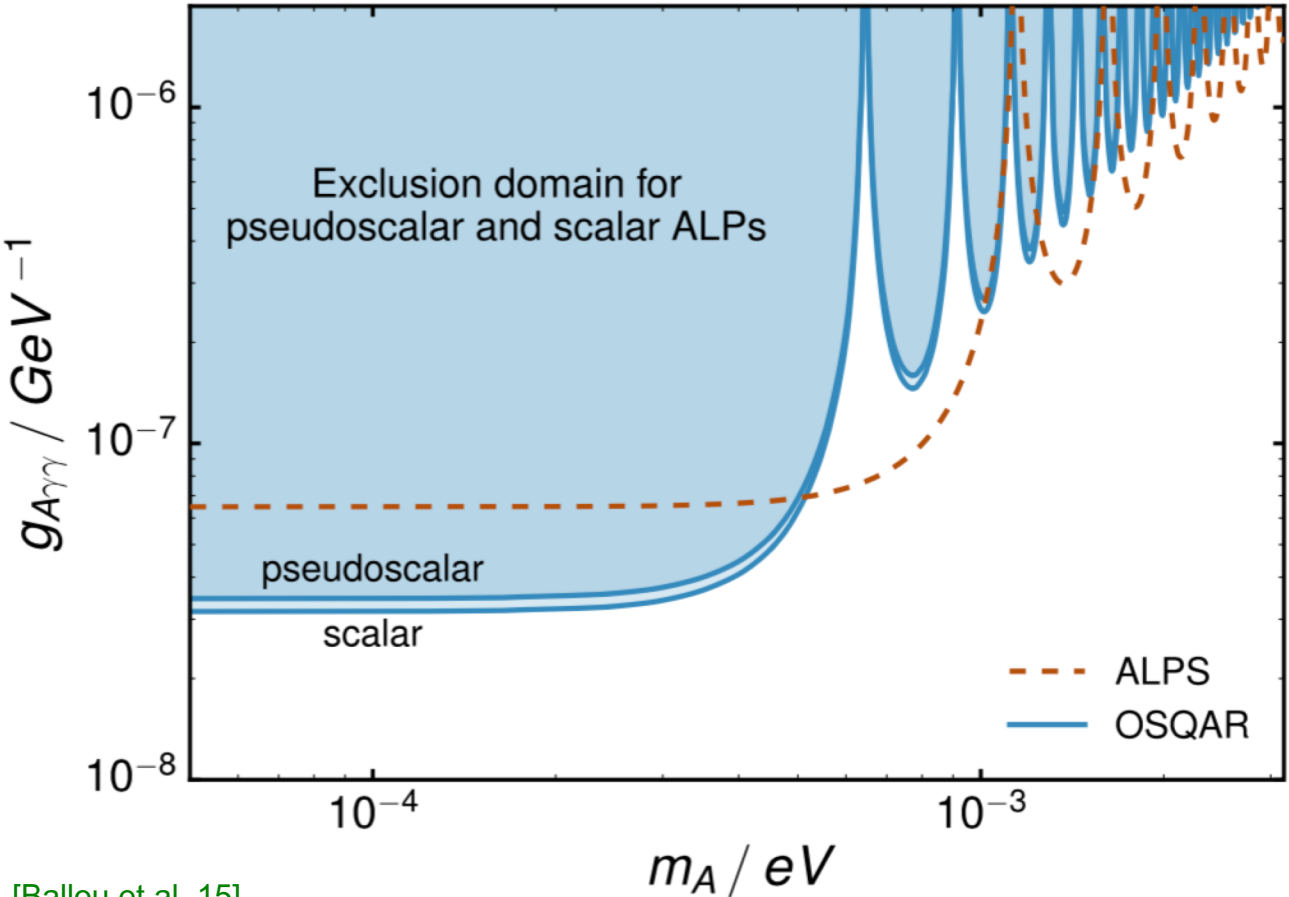
- ALPS I @ DESY (in collaboration with AEI Hannover and U Hamburg)

[Ehret et al. 10]



Light-Shining-through-a-Wall Searches

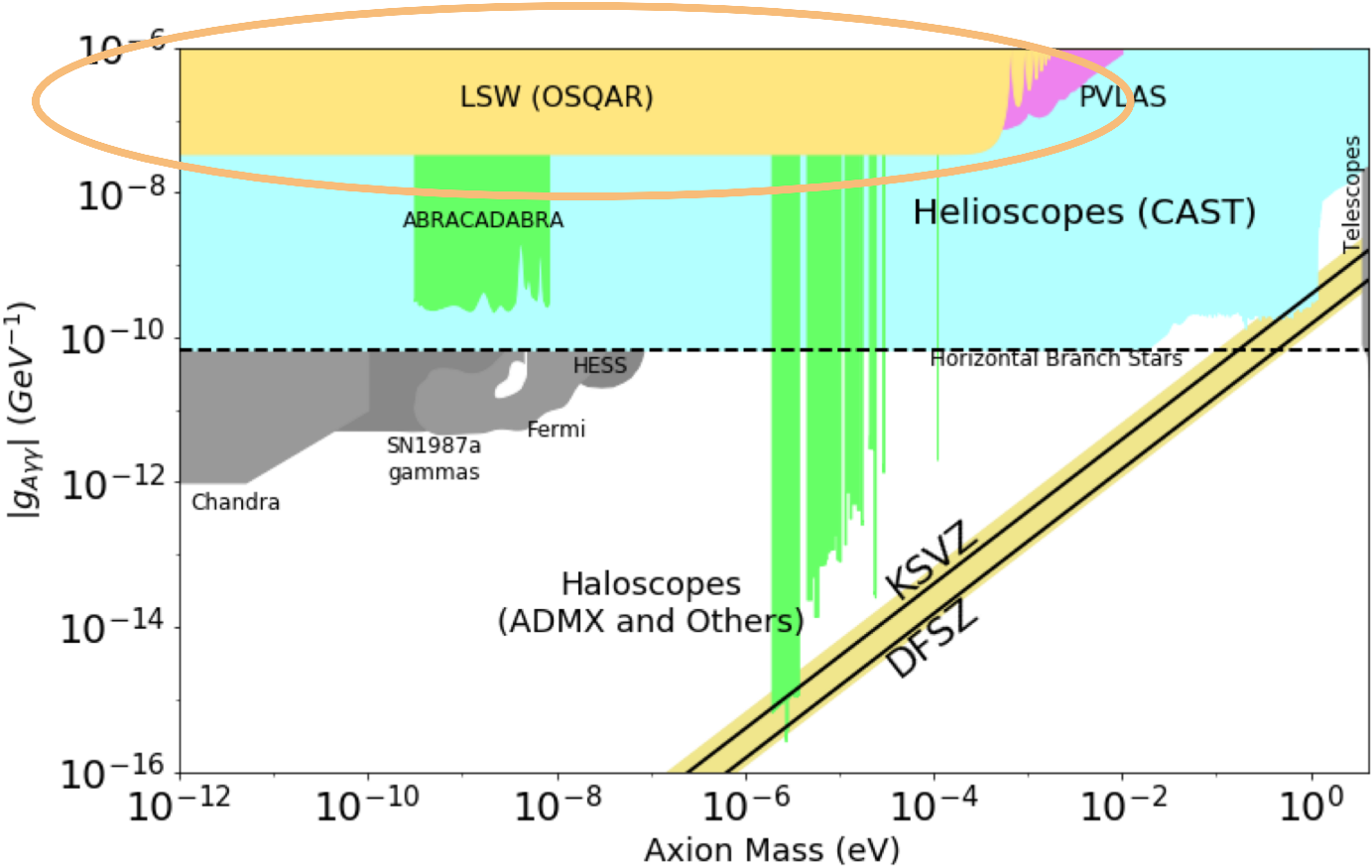
- **ALPS I** @ DESY (in collaboration with AEI Hannover and U Hamburg) [Ehret et al. 10]
- LSW experiments **ALPS I** and **OSQAR** @ CERN give currently the best purely laboratory limit on low mass axions:



[Ballou et al. 15]

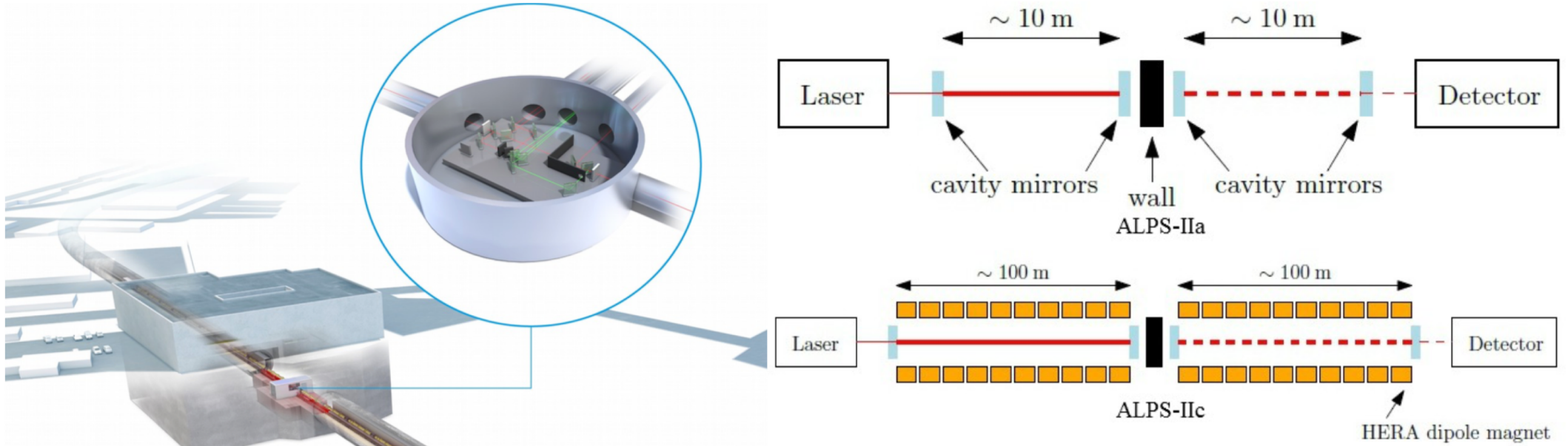
Light-Shining-through-a-Wall Searches

- **ALPS I** @ DESY (in collaboration with AEI Hannover and U Hamburg) [Ehret et al. 10]
- LSW experiments **ALPS I** and **OSQAR** @ CERN give currently the best purely laboratory limit on low mass axions:



Light-Shining-through-a-Wall Searches

- ALPS II @ DESY (in collaboration with AEI Hannover, U Cardiff, U Florida, U Mainz) [Bähre et al (ALPS II TDR) 13]



Parameter	Scaling	ALPS I	ALPS IIc	Sens. gain
Effective laser power P_{laser}	$g_{a\gamma} \propto P_{\text{laser}}^{-1/4}$	1 kW	150 kW	3.5
Rel. photon number flux n_γ	$g_{a\gamma} \propto n_\gamma^{-1/4}$	1 (532 nm)	2 (1064 nm)	1.2
Power built up in RC P_{RC}	$g_{a\gamma} \propto P_{\text{reg}}^{-1/4}$	1	40,000	14
BL (before& after the wall)	$g_{a\gamma} \propto (BL)^{-1}$	22 Tm	468 Tm	21
Detector efficiency QE	$g_{a\gamma} \propto QE^{-1/4}$	0.9	0.75	0.96
Detector noise DC	$g_{a\gamma} \propto DC^{1/8}$	0.0018 s^{-1}	0.000001 s^{-1}	2.6
Combined improvements				3082

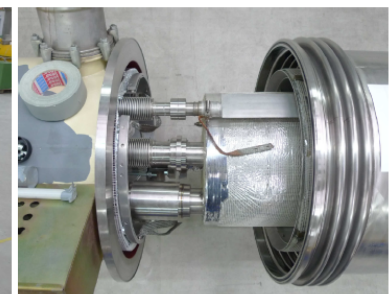
Light-Shining-through-a-Wall Searches

ALPS II at DESY

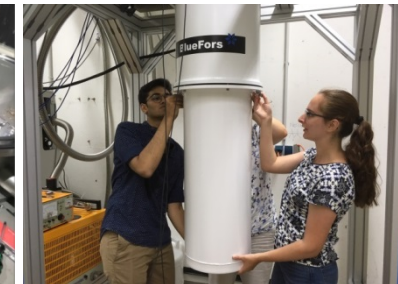
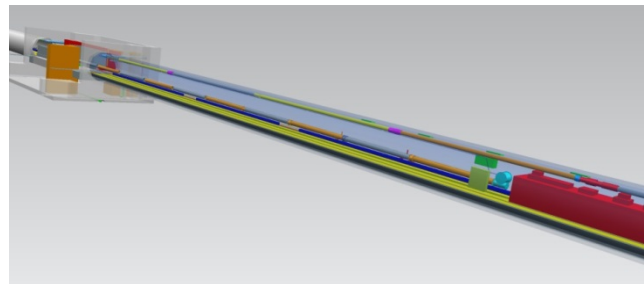
- HERA tunnel on about 300 m cleared



- 22 magnets straightened and tested (no failure)



- Construction will be finished end of 2020
- Data taking 2021 and 2022

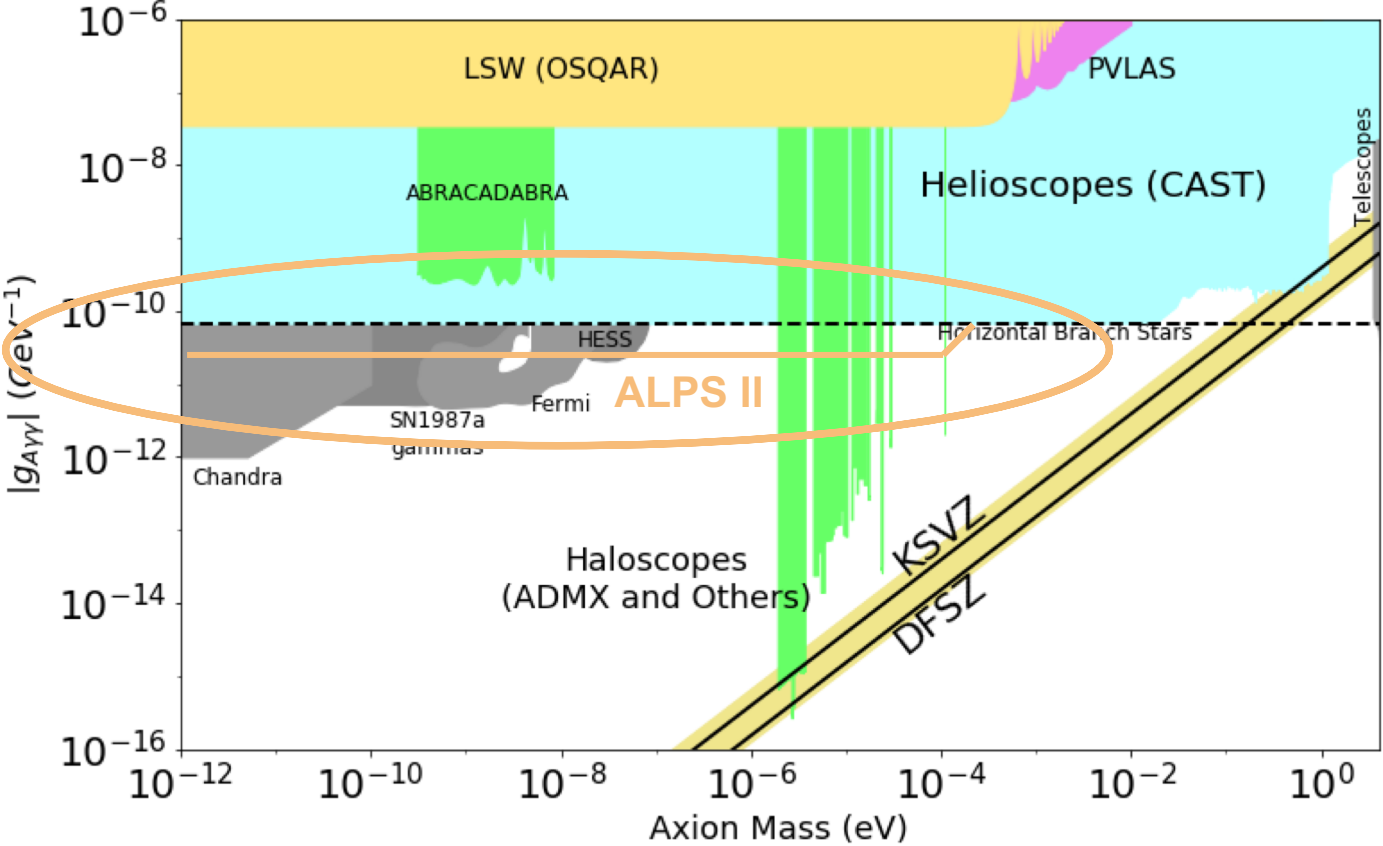


Albert Einstein Institute
Hannover



Light-Shining-through-a-Wall Searches

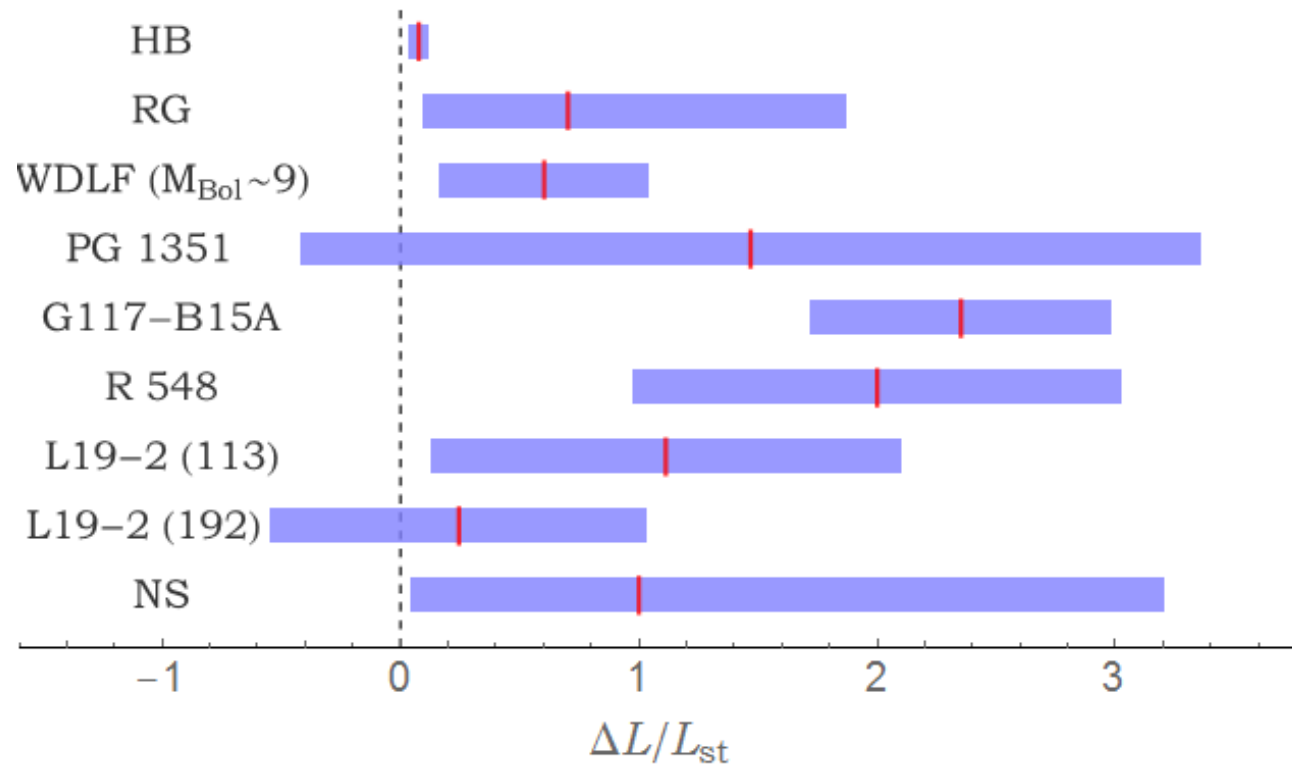
- **ALPS II** will probe previously uncharted territory, in particular part of parameter space relevant for dark matter and astro hints (excessive energy losses of Horizontal Branch stars in globular clusters)



Intermezzo

Stellar hints for axions and ALPs

- Practically every stellar systems seems to be cooling a bit faster than predicted by models based on SM:



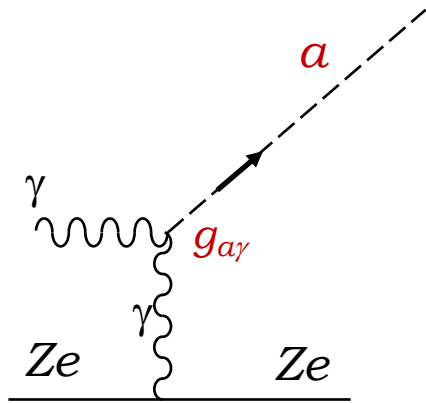
[Giannotti, Irastorza, Redondo, AR (2015); Giannotti, Irastorza, Redondo, AR (in preparation)]

Intermezzo

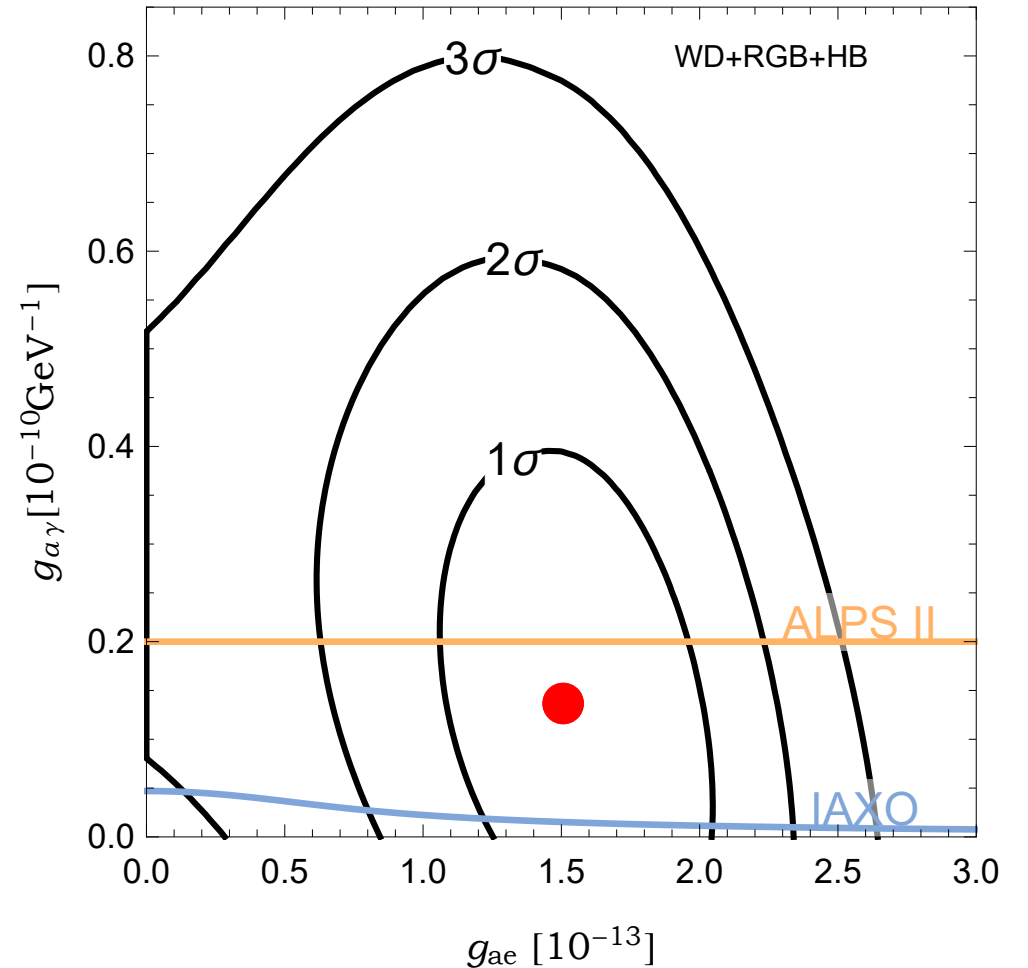
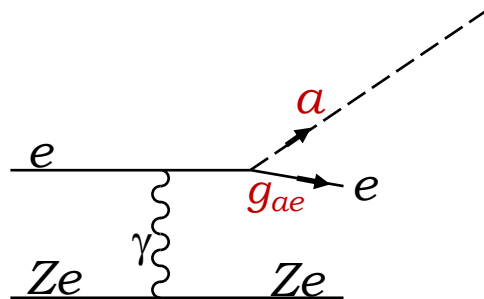
Stellar hints for axions and ALPs

- Excessive energy losses of stars in various stages of their evolution may be explained by axion production

$$\mathcal{L} \supset -\frac{g_{a\gamma}}{4} a F_{\mu\nu} \tilde{F}^{\mu\nu}$$

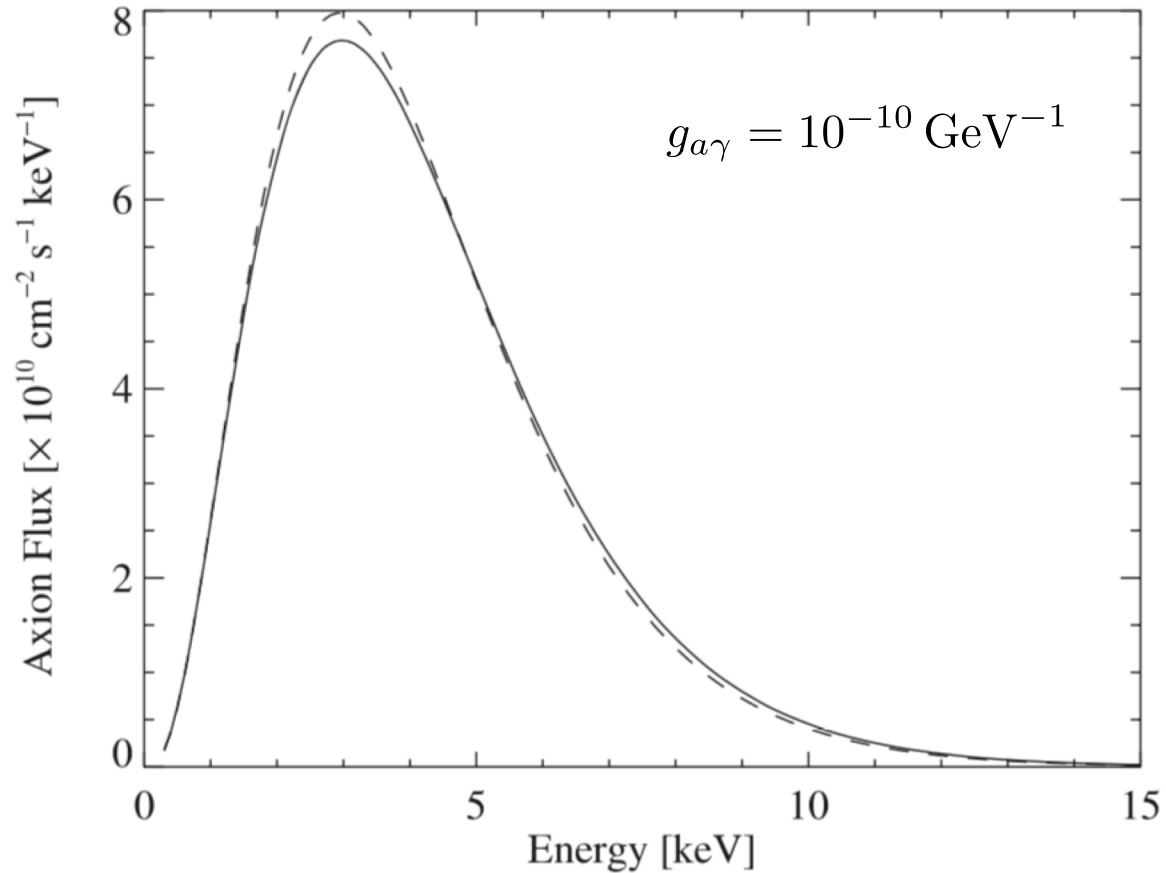


$$\mathcal{L} \supset -ig_{ae} a \bar{\psi}_e \gamma_5 \psi_e$$

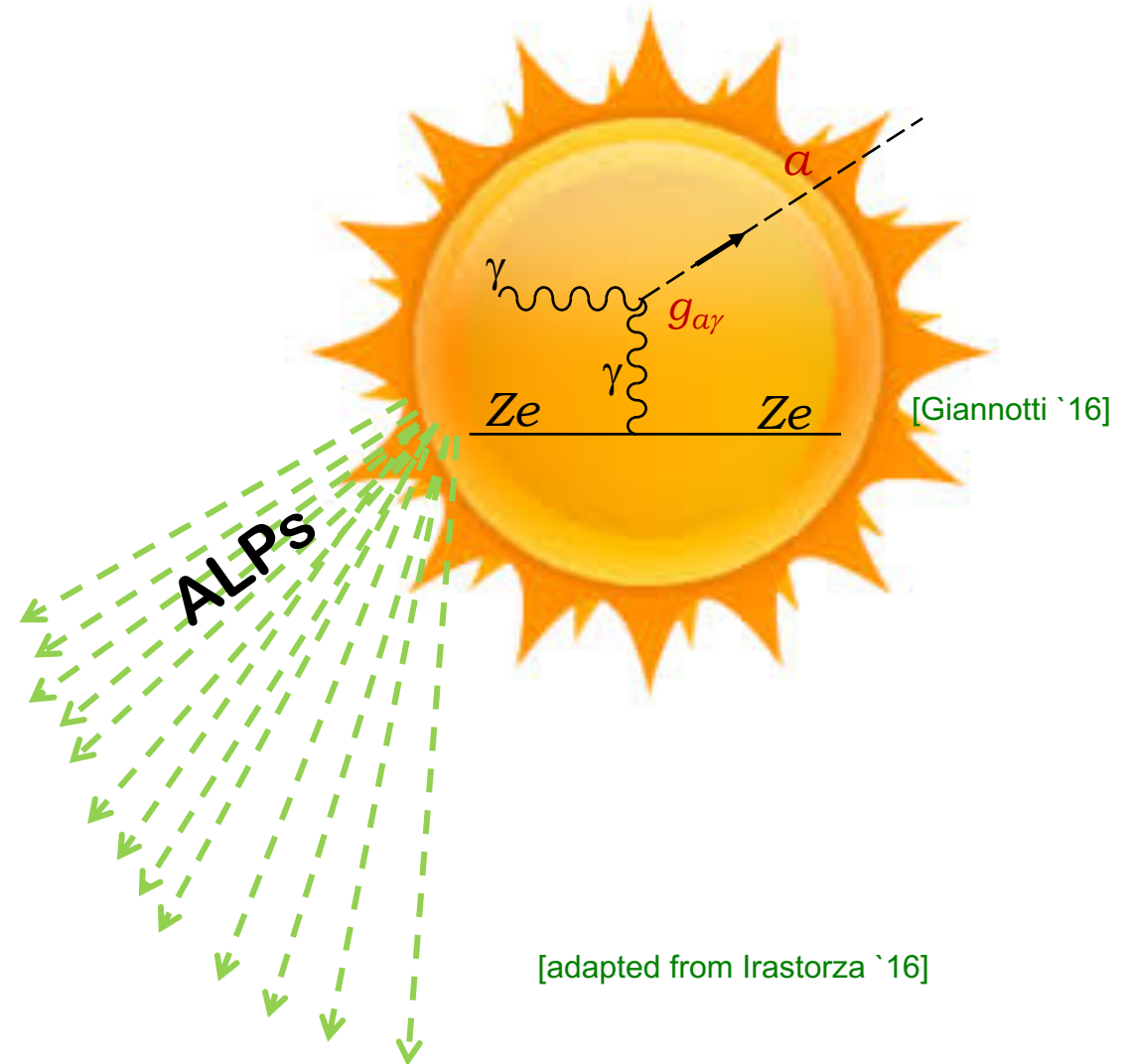


Searches for Solar Axions

- Flux of solar axions/ALPs produced by two photon process in core:

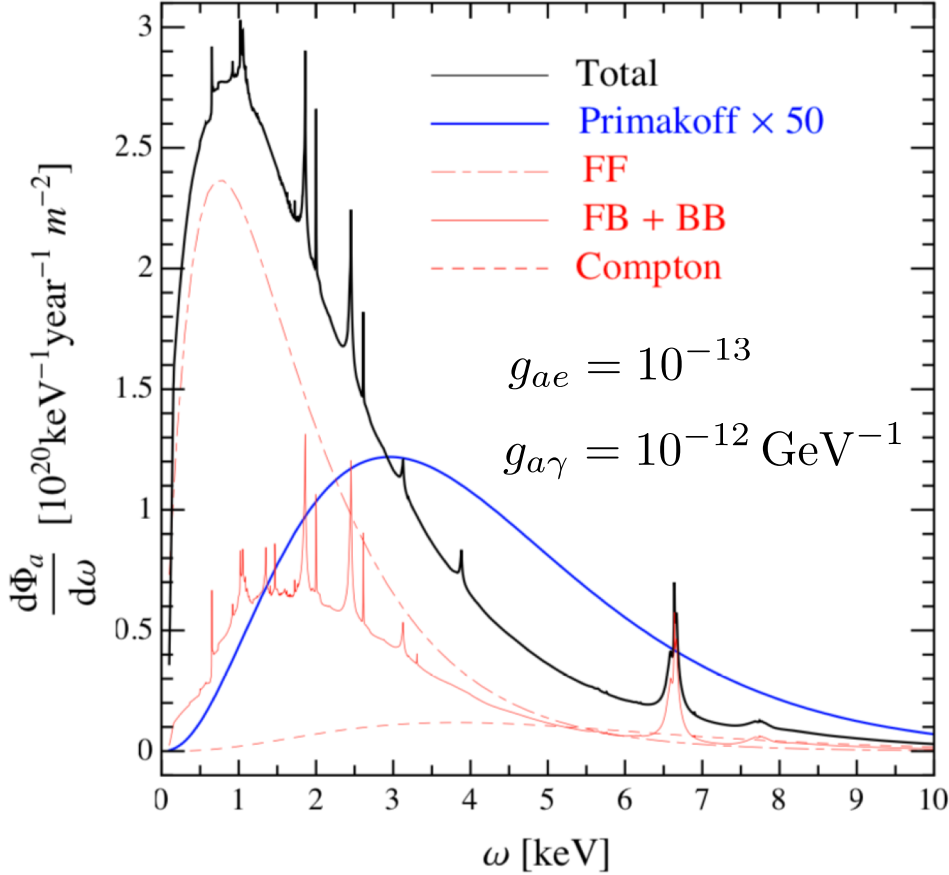


[Adriamonje et al. '07]

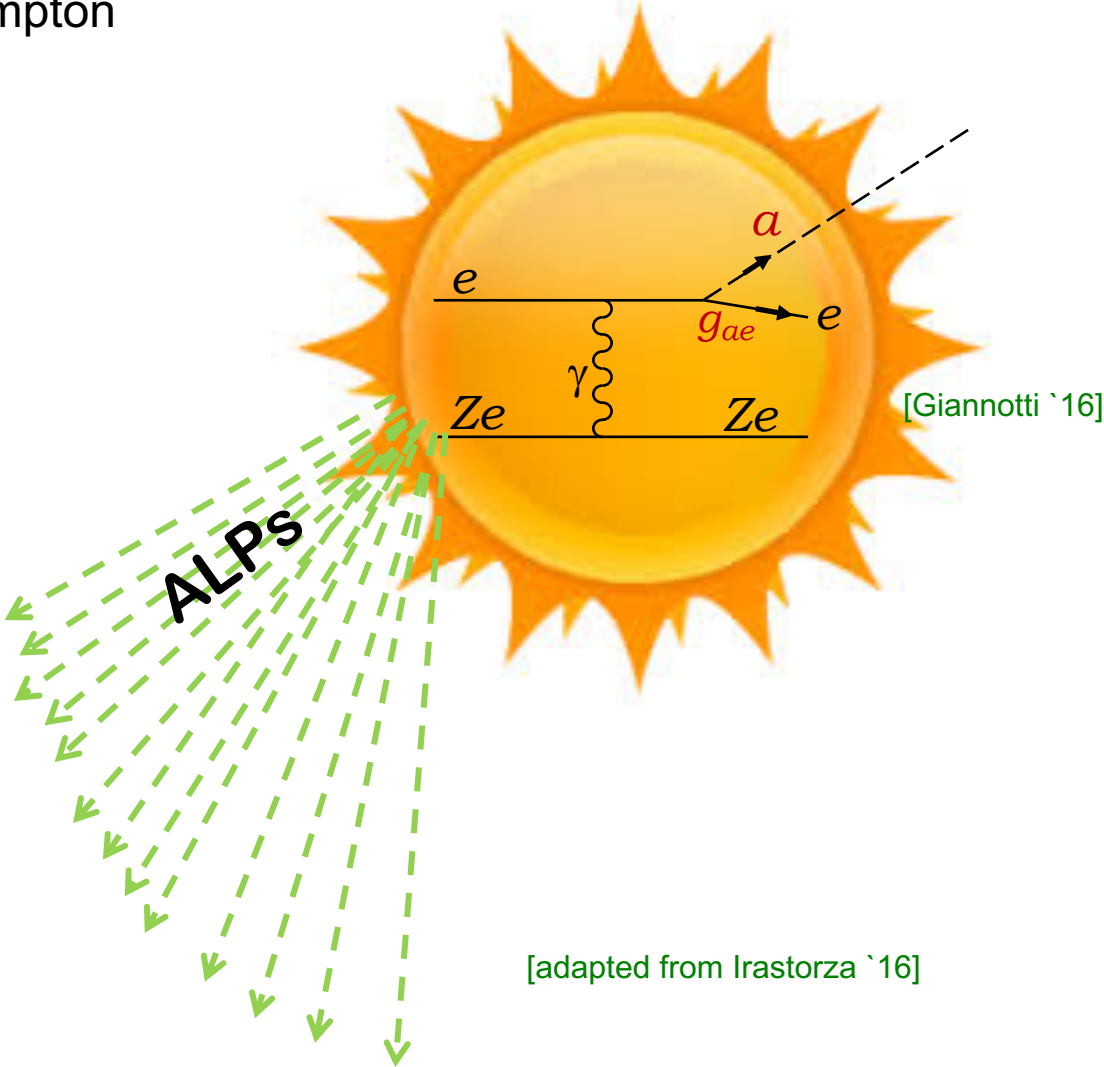


Searches for Solar Axions

- If axion/ALP couples to electron, even higher flux of solar axion/ALPs produced by atomic recombination and deexcitation (FB+BB), Bremsstrahlung (FF) and Compton



[Redondo `13]

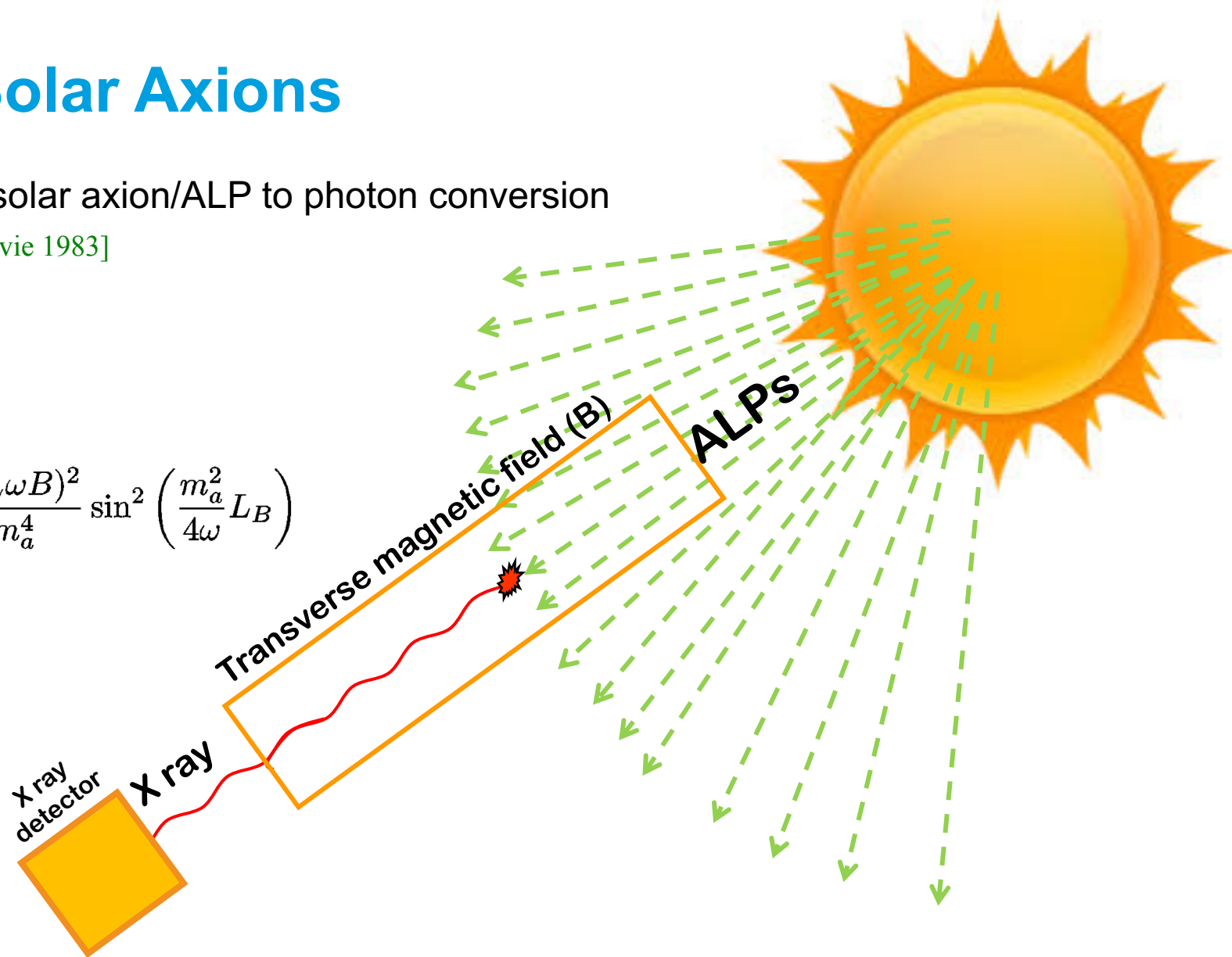


[adapted from Irastorza `16]

Searches for Solar Axions

- Helioscope concept: solar axion/ALP to photon conversion in magnetic field [Sikivie 1983]

$$P(a \leftrightarrow \gamma) = 4 \frac{(g_{a\gamma} \omega B)^2}{m_a^4} \sin^2 \left(\frac{m_a^2}{4\omega} L_B \right)$$



[adapted from Irastorza `16]

Searches for Solar Axions

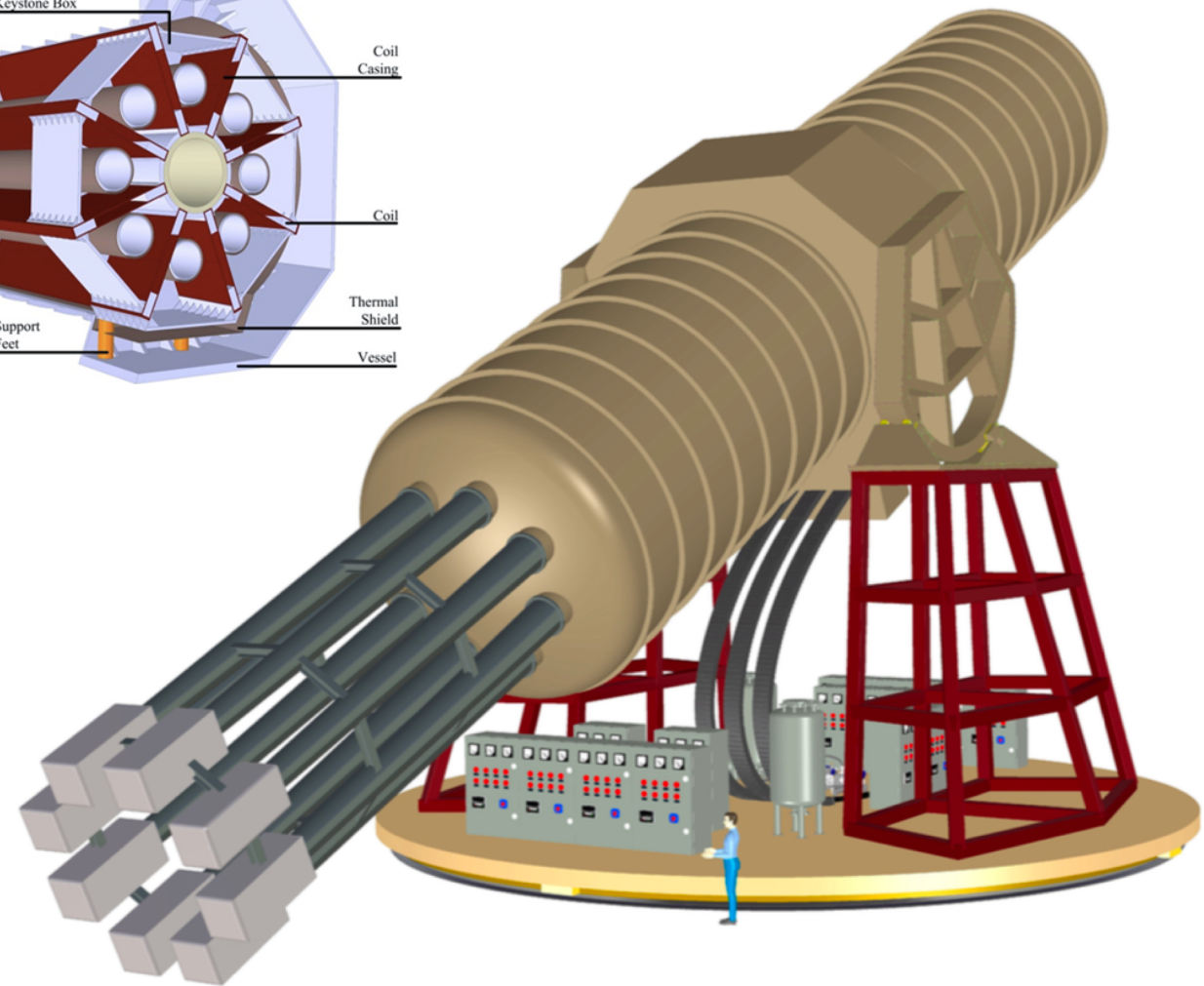
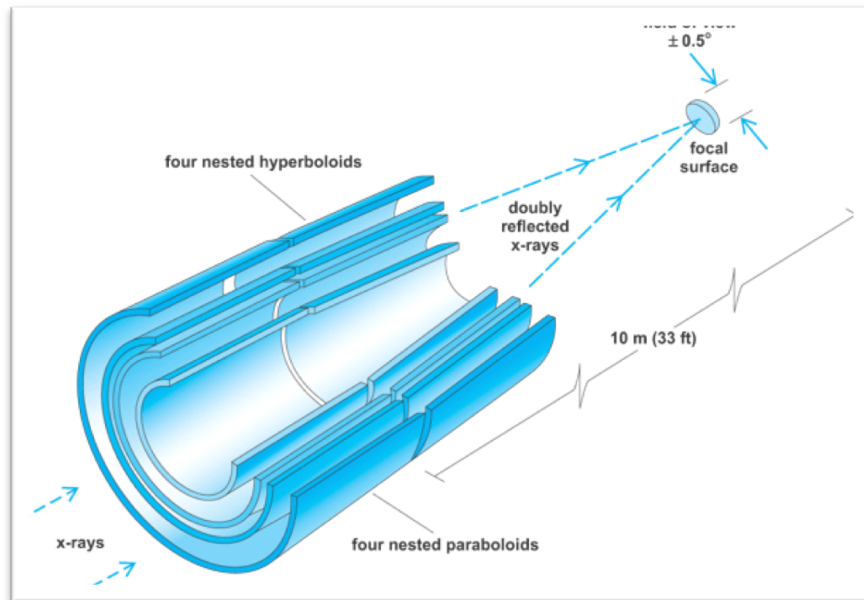
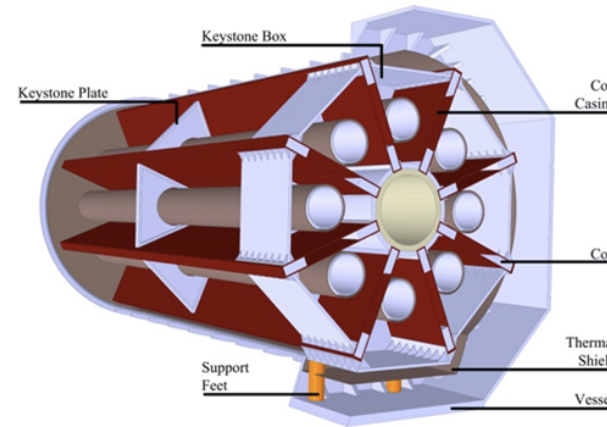
- Most sensitive until now: [CERN Axion Solar Telescope \(CAST\)](#)
 - Superconducting LHC dipole magnet
 - X-ray detectors
 - Use of buffer gas to extend sensitivity to higher masses (axion band)



Searches for Solar Axions

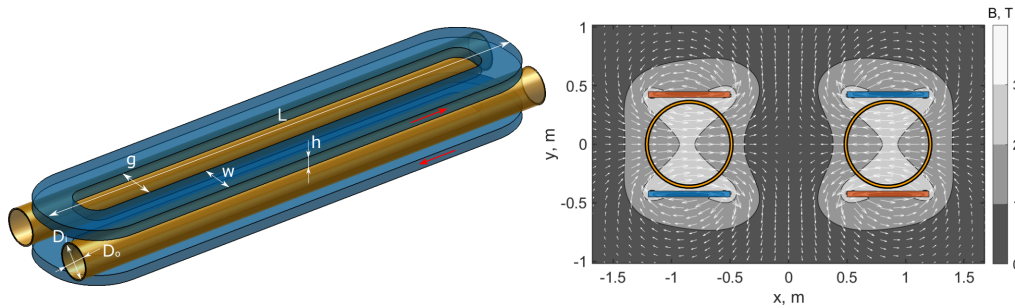
- International Axion Observatory (IAXO)
 - Large toroidal 8-coil magnet $L = \sim 20$ m
 - 8 bores: 600 mm diameter each
 - 8 X-ray telescopes + 8 detection systems
 - Rotating platform with services
- Proposed site: [DESY](#)

[IAXO CDR: JINST 9 (2014) T05002 (arXiv:1401.3233)]

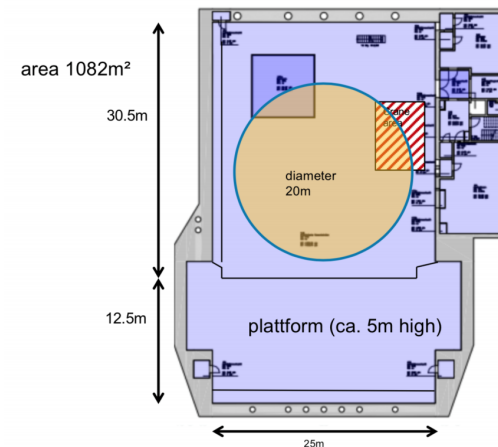
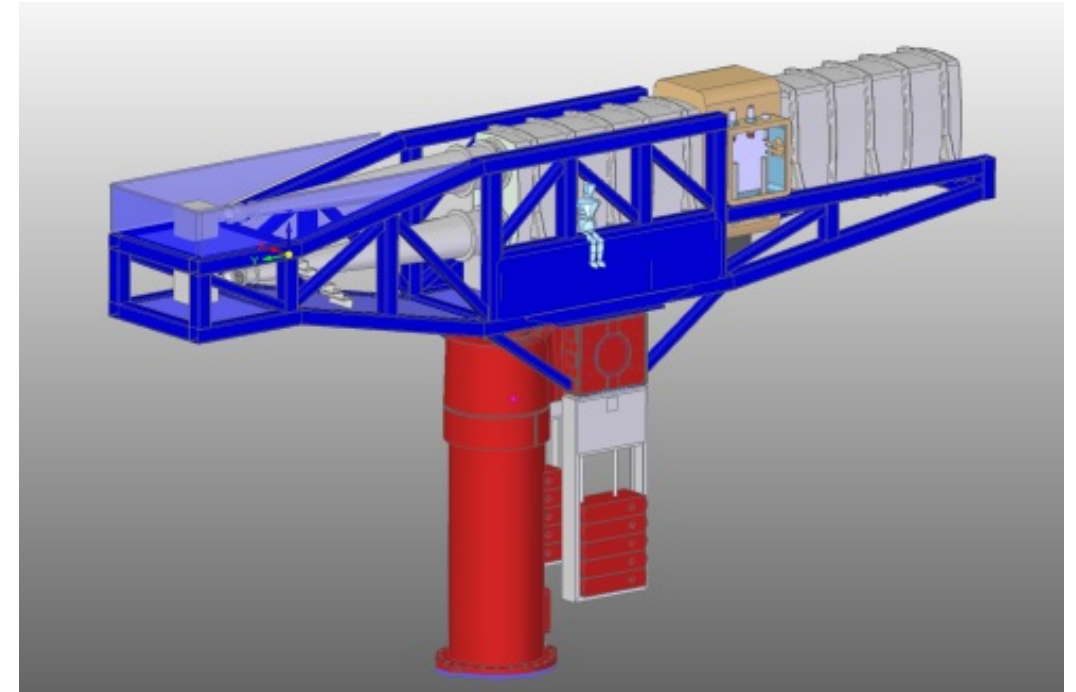


Searches for Solar Axions

- Prototype for IAXO: [BabyIAXO](#)
 - Two bores of dimensions similar to final IAXO bores
 - Detection lines representative of final ones
 - Test & improve all systems
- Magnet technical design ongoing at CERN

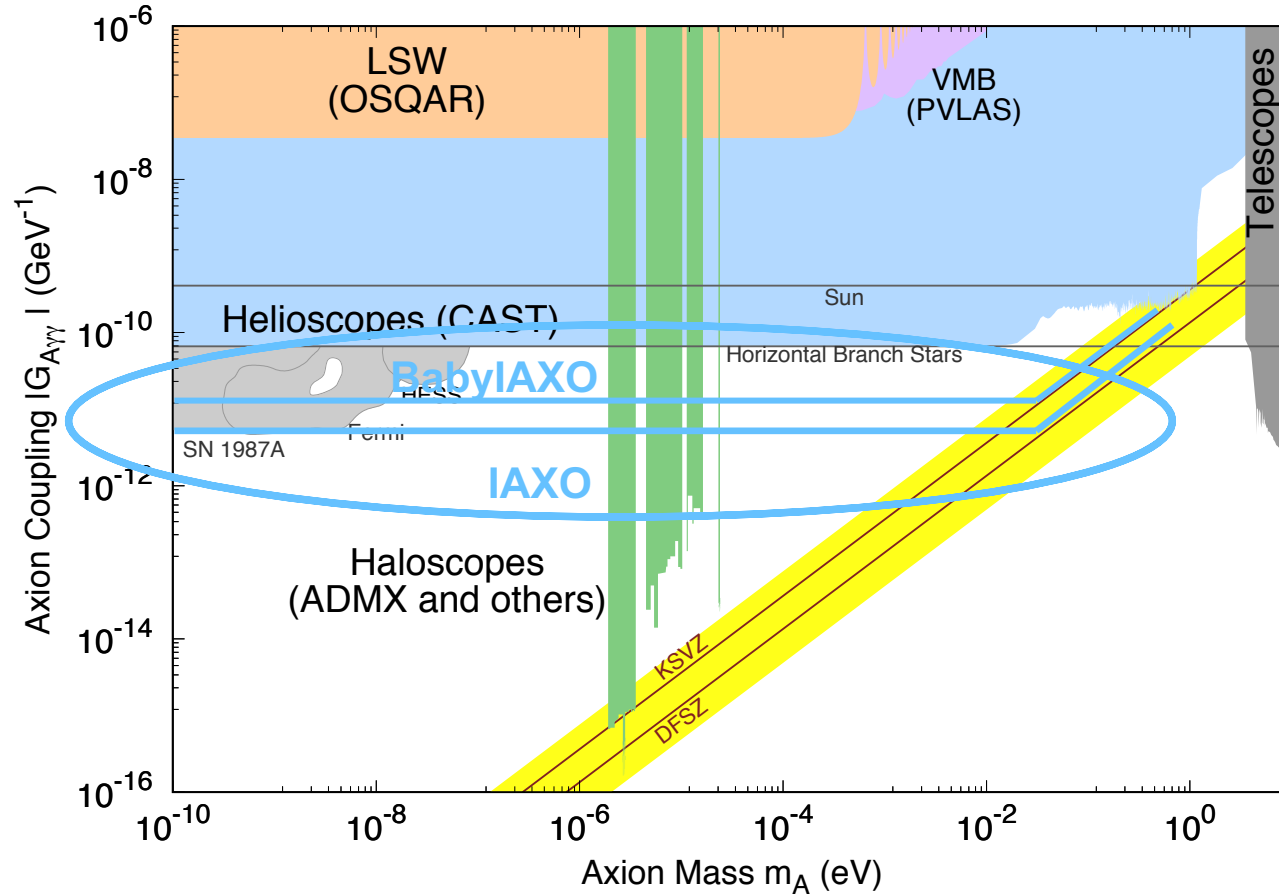


- Funded mainly via [Irastorza: ERC-AvG 2017 IAXO+](#)
- Preferred site: HERA South Hall at [DESY](#)
- Construction may start in 2020
- Data taking may start in 2024



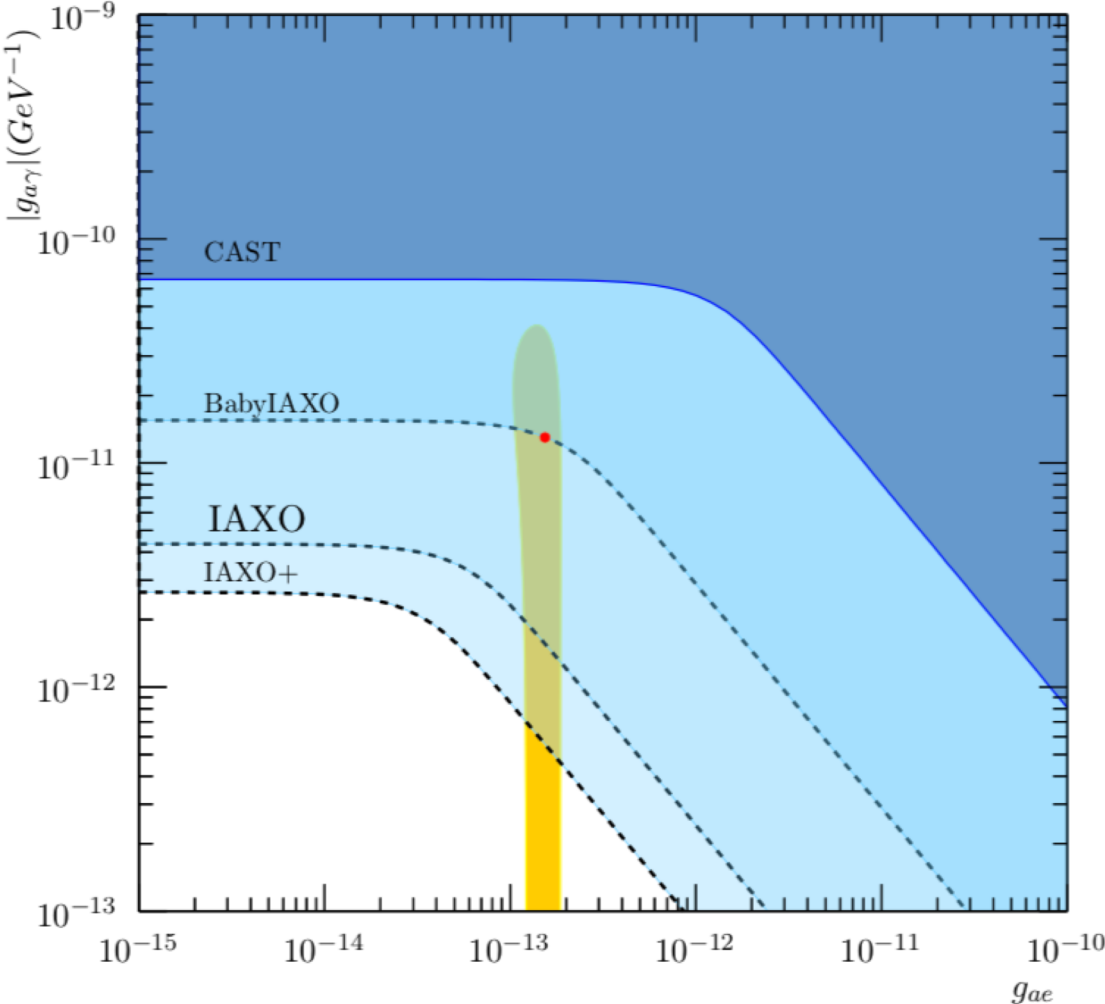
Searches for Solar Axions

- (Baby)IAXO probes meV mass QCD axion and covers most of parameter space relevant for astro hints

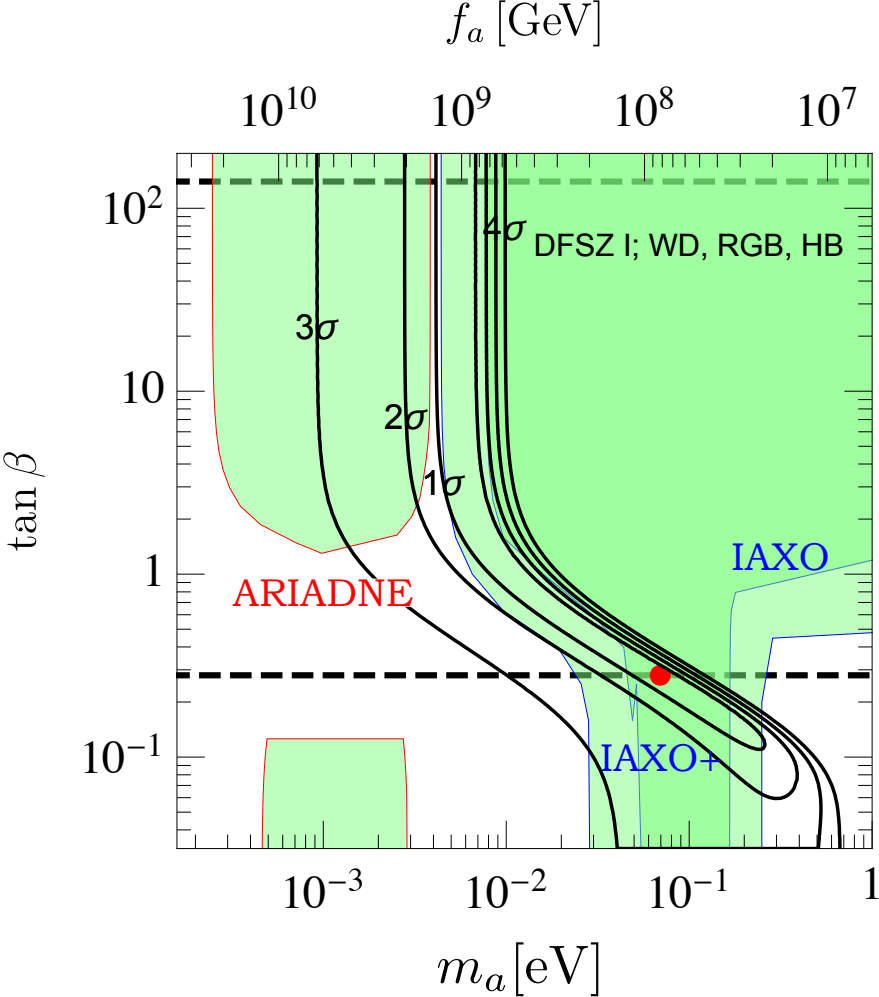


Searches for Solar Axions

- (Baby)IAXO also sensitive to electron coupling hinted at by stellar energy losses



[Armengaud et al. 19]



[Giannotti,Irastorza,Redondo,AR,Saikawa 17]

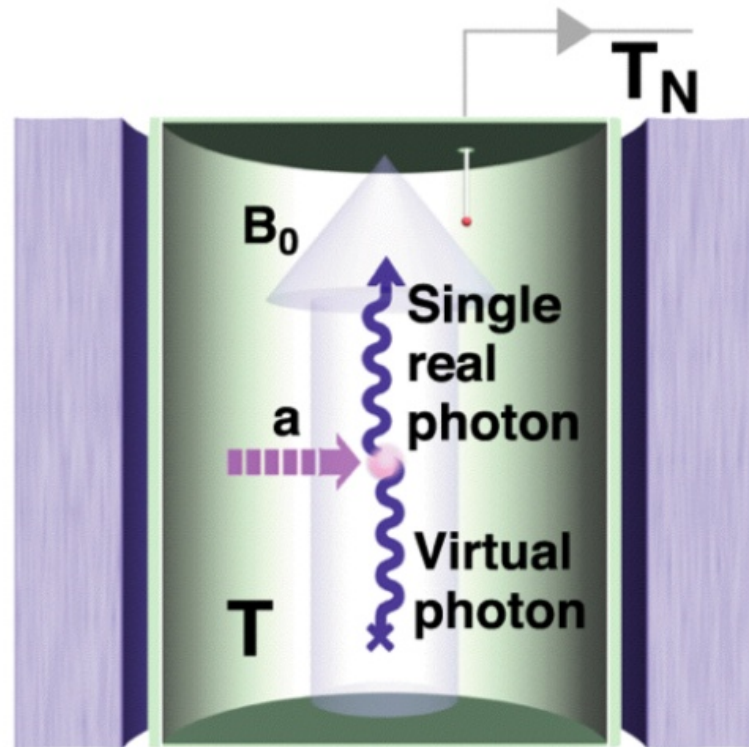
Searches for Dark Matter Axions

Microwave cavities

- Axion or ALP DM – photon conversion in microwave cavity placed in magnetic field

[Sikivie 83]

- Best sensitivity: mass = resonance frequency $m_a = 2\pi\nu \sim 4 \mu\text{eV} \left(\frac{\nu}{\text{GHz}} \right)$

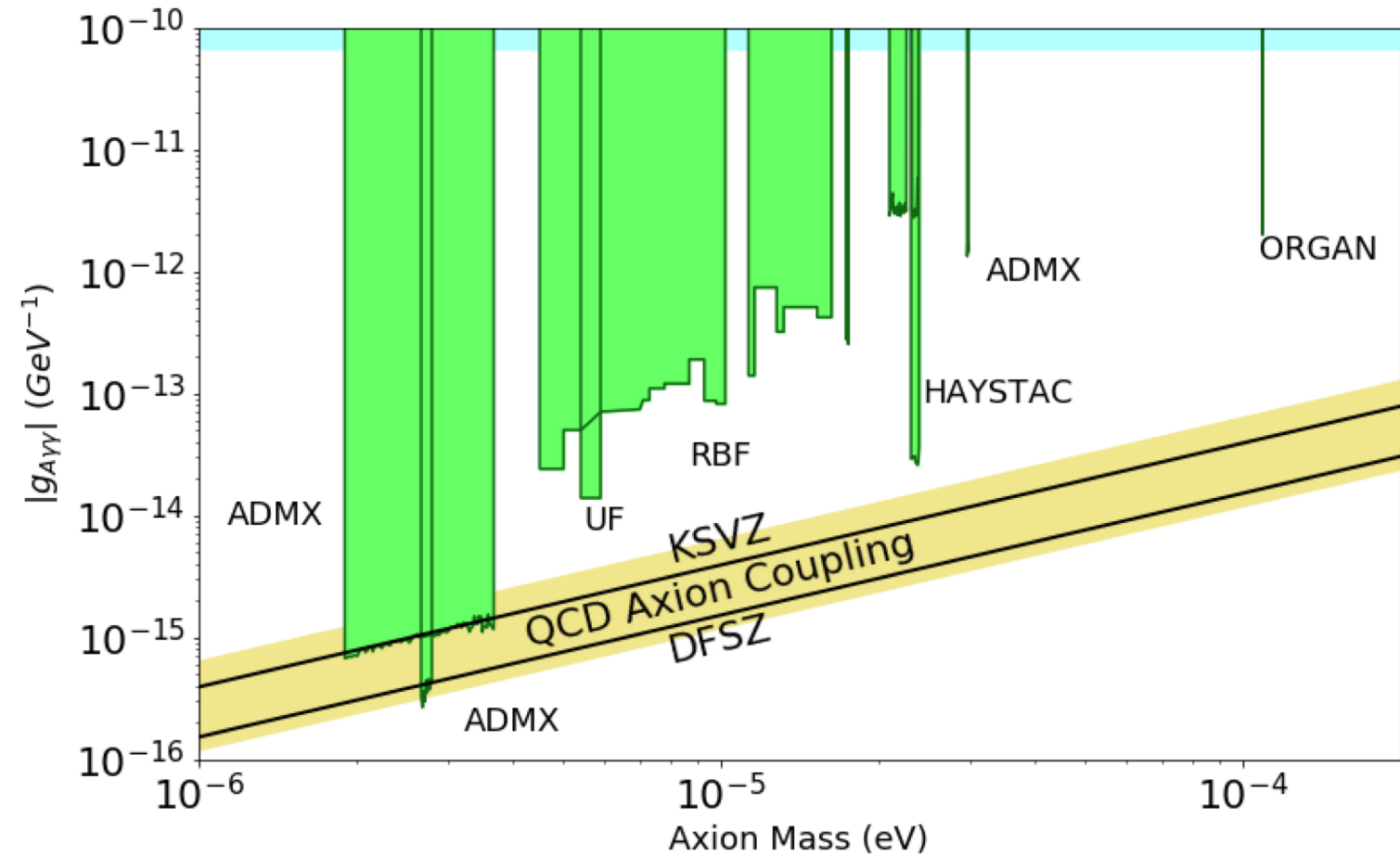


$$P_{\text{out}} \sim g^2 | \mathbf{B}_0 |^2 \rho_{\text{DM}} V Q / m_a$$

Searches for Dark Matter Axions

Microwave cavities

- Currently running:
 - ADMX
 - HAYSTAC
 - ORGAN
 - CAST-CAPP
 - RADES

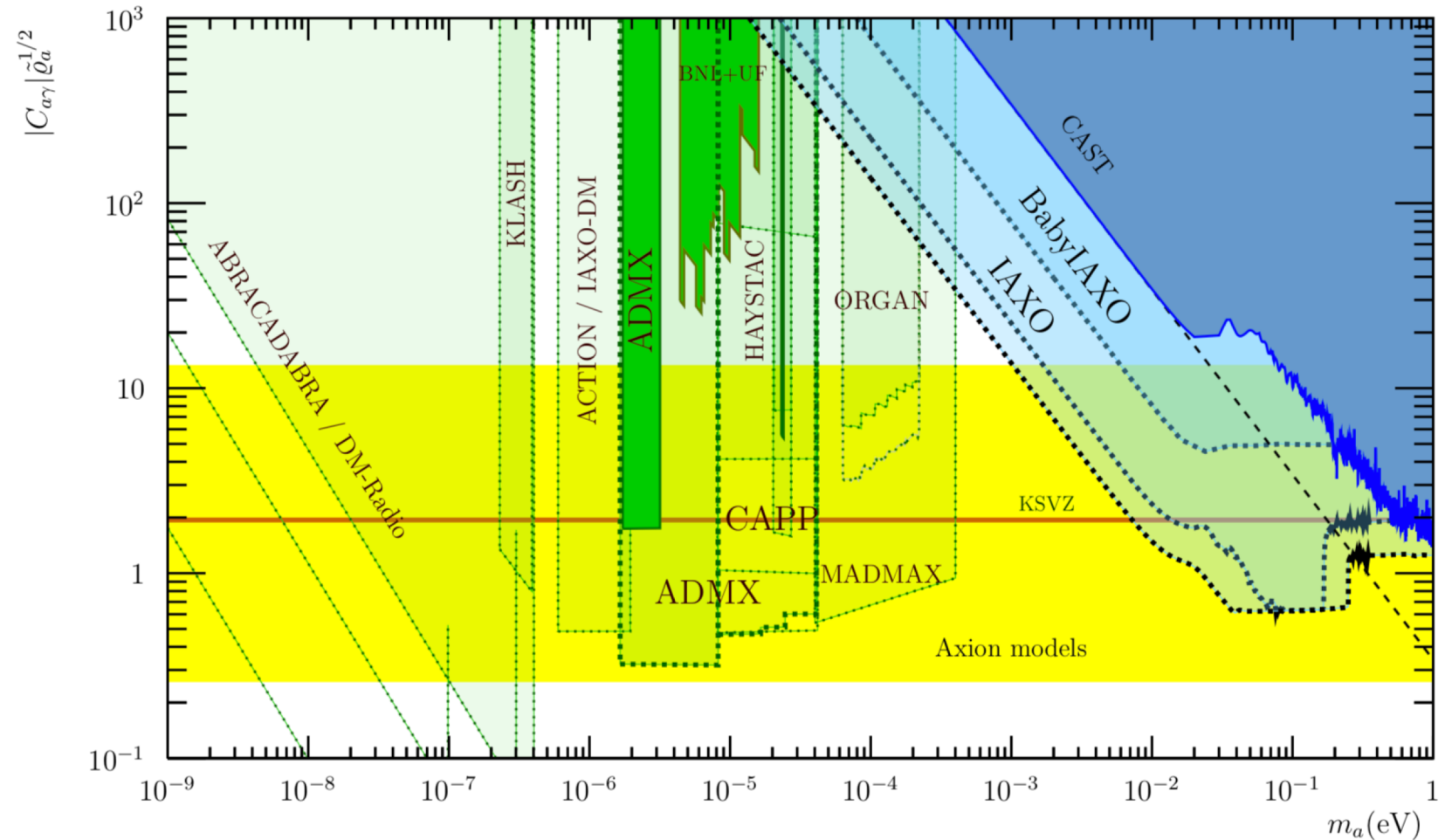


[AR,Rybka,Rosenberg in update draft PDG RPP 2019]

Searches for Dark Matter Axions

Microwave cavities

- Currently running:
 - ADMX
 - HAYSTAC
 - ORGAN
 - CAST-CAPP
 - RADES
- Currently in construction:
 - CULTASK
- Proposed:
 - KLASH
 - ACTION
 - IAXO-DM

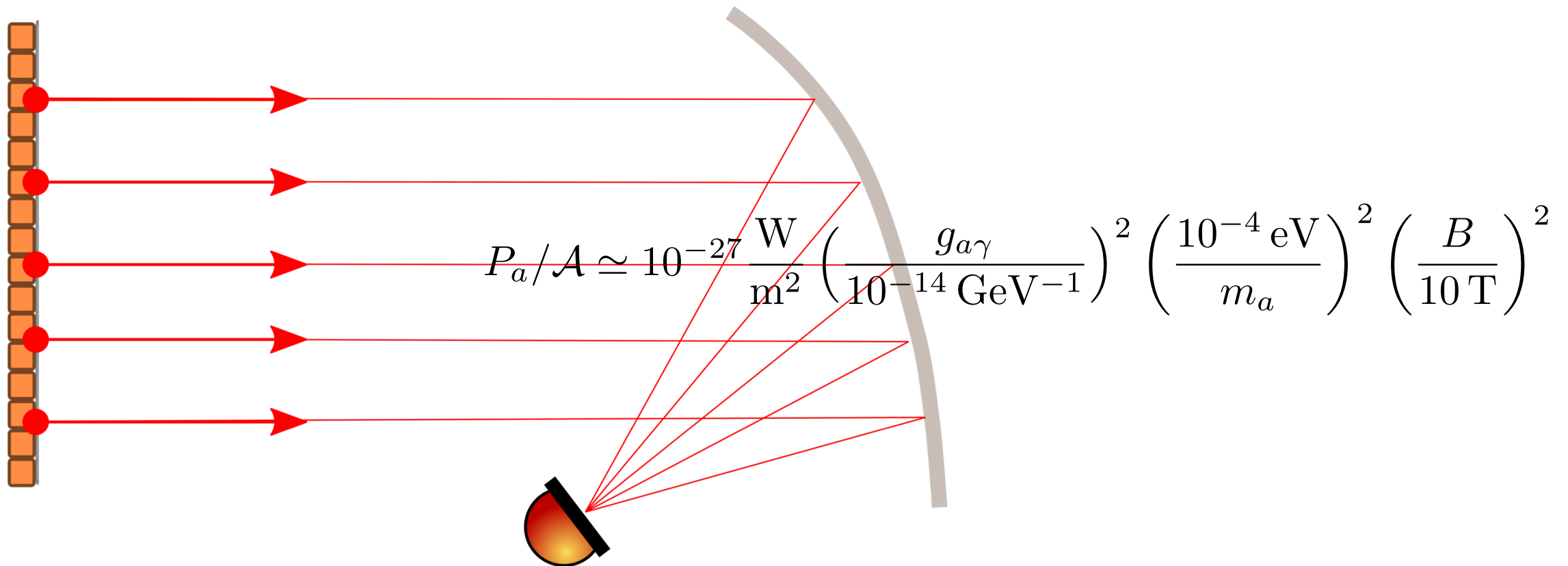


[Irastorza, Redondo 18]

Searches for Dark Matter Axions

Dish antennas

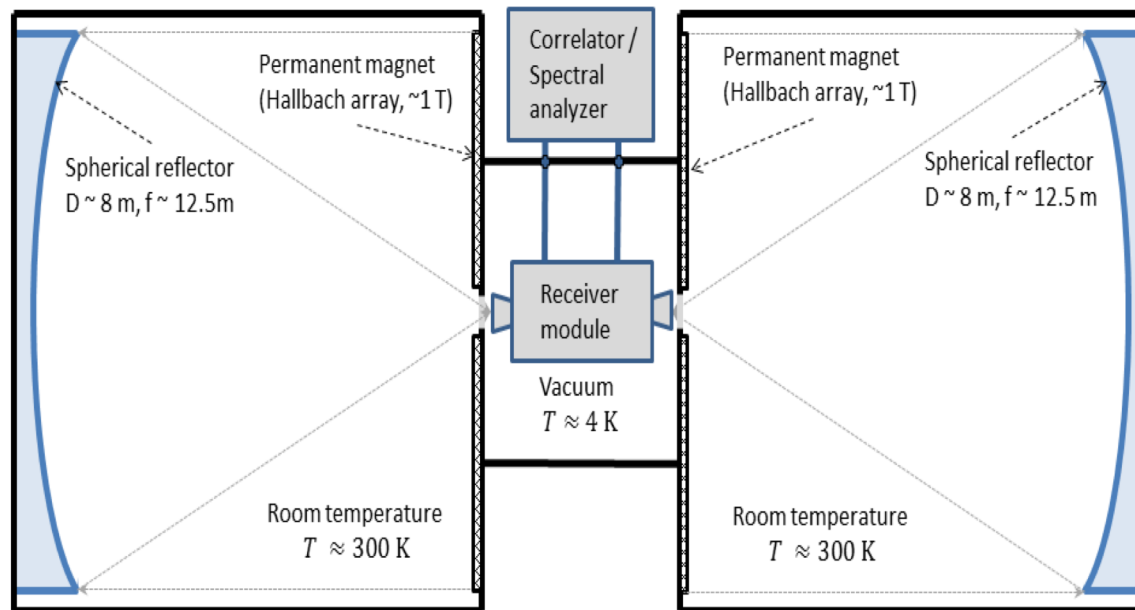
- Oscillating axion/ALP DM in a background magnetic field carries a small electric field component
- A magnetised mirror in axion/ALP DM background radiates photons [Horns,Jaeckel,Lindner,Lobanov,Redondo,AR 13]



Searches for Dark Matter Axions

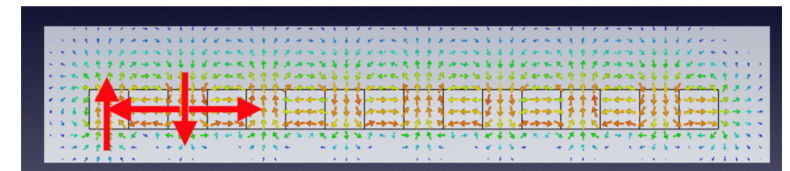
Dish antennas

- Oscillating axion/ALP DM in a background magnetic field carries a small electric field component
- A magnetised mirror in axion/ALP DM background radiates photons [Horns,Jaeckel,Lindner,Lobanov,Redondo,AR 13]
- Proposed axion/ALP DM dish antenna experiment: **BRASS** (U Hamburg)



[Horns et al. (unpublished)]

- Permanently magnetized surface for axion/ALP photon conversion (Hallbach array)

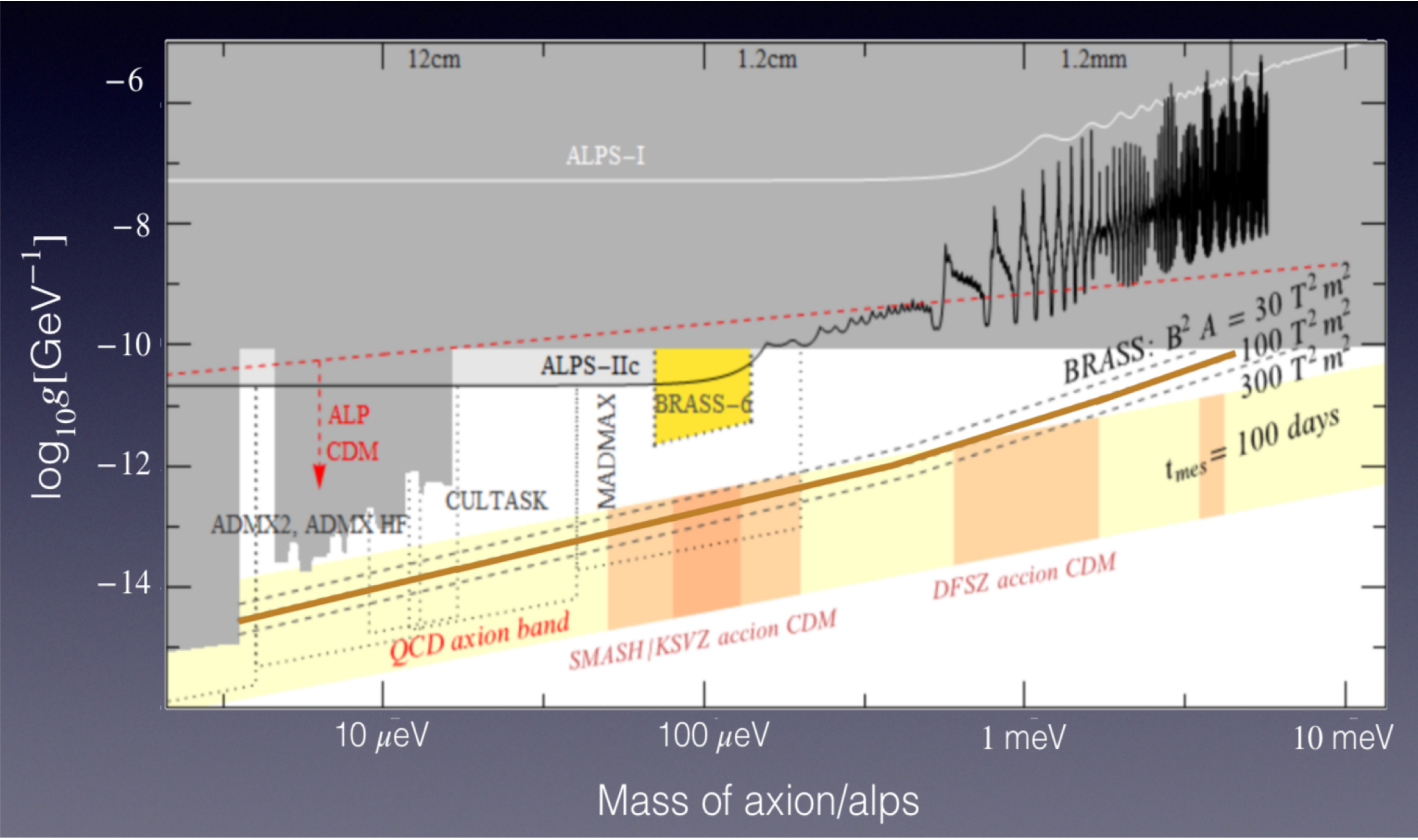
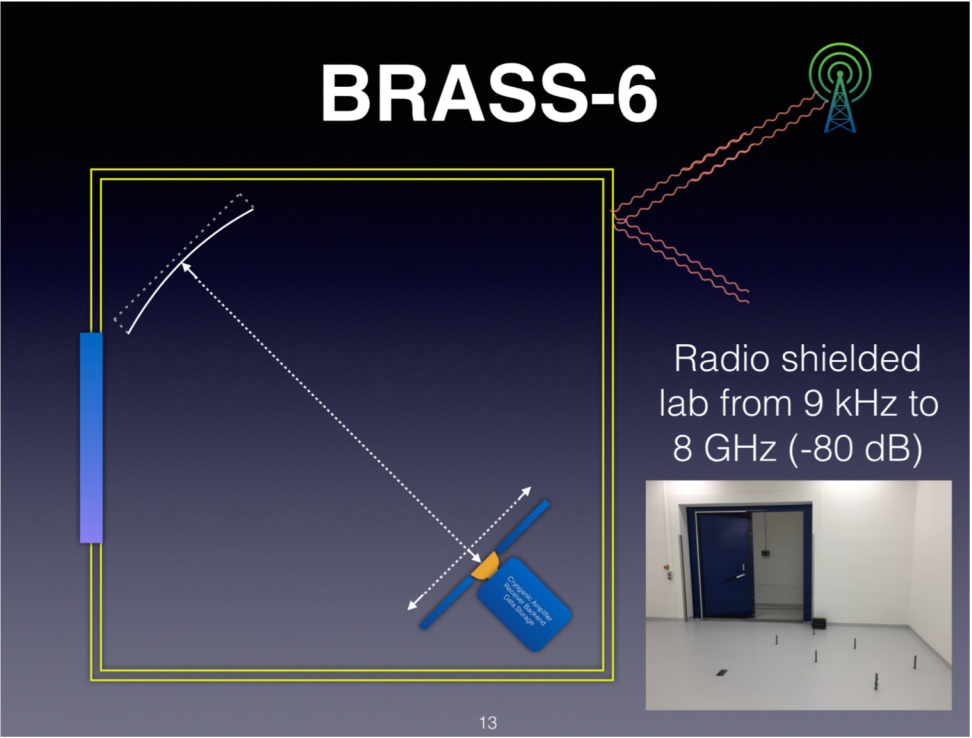


- Dish antenna for photon signal concentration
- Broadband acquisition (16 GHz bandwidth, 10^7 channels)

Searches for Dark Matter Axions

Dish antennas

- BRASS-6 in construction, data taking starting 2020



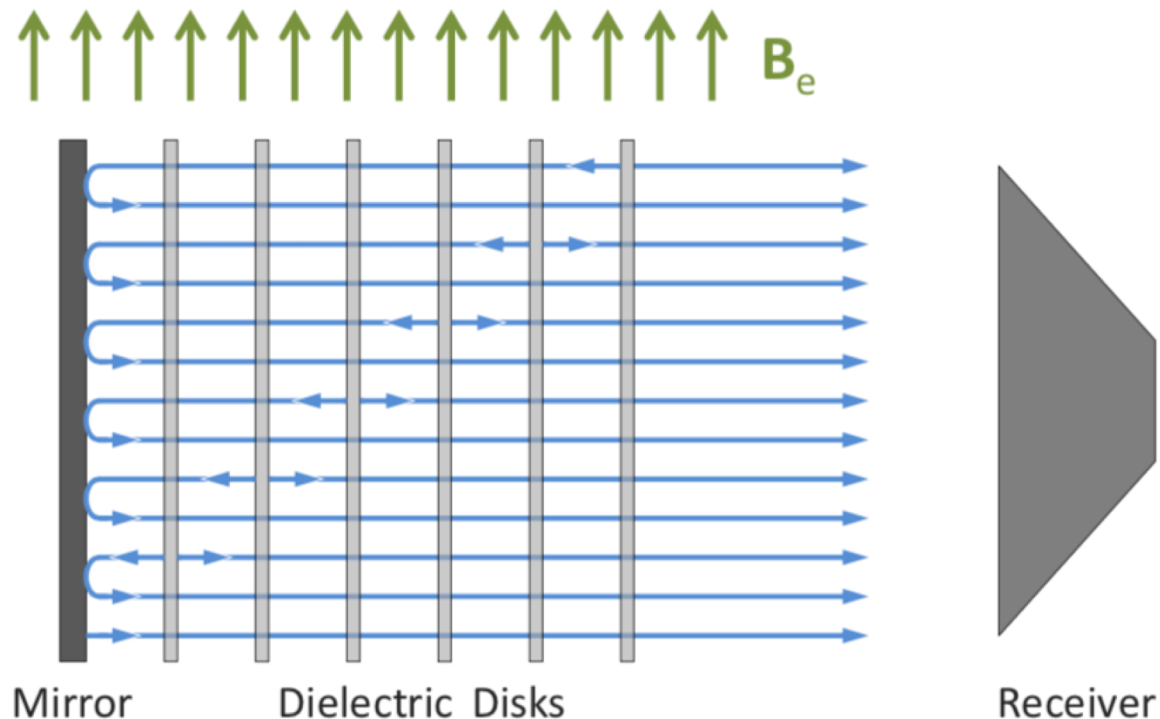
[Le Hoang Nguyen, Patras Workshop 2019]

Searches for Dark Matter Axions

Dish antennas

- Boosted dish antenna: Open dielectric resonator
 - Add stack of dielectric disks with $\sim \lambda/2$ spacing in front of mirror (all immersed in magnetic field) [Jaeckel,Redondo 13]
 - Constructive interference of photon part of wave function [Millar,Raffelt,Redondo,Steffen 16]

[Baryakhtar,Huang,Lasenby18]



Searches for Dark Matter Axions

Dish Antennas

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 - $|\beta|^2 > 10^4$ achievable with 80 discs of LaAlO_3 ($\epsilon = 24$) [Baryakhtar, Huang, Lasenby 18]

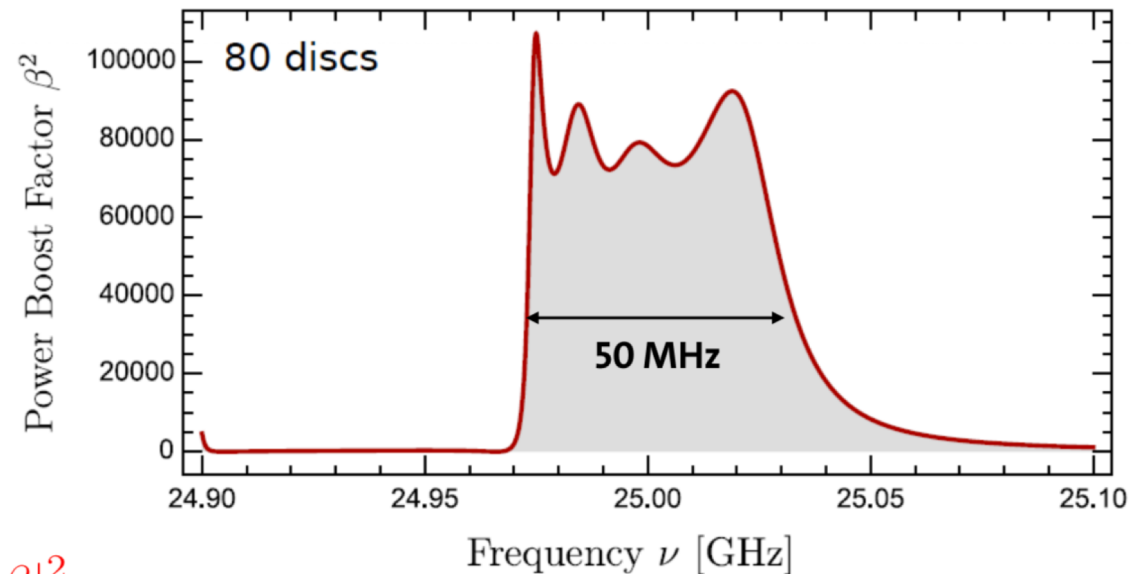
- “Quasi-broadband” achieved by:
- positioning the discs with relative spacing of $\sim \lambda/2$ according to simulation prediction
 - with precision better than $10 \mu\text{m}$

Photon power :

$$\frac{P}{A} = 2 \cdot 10^{-27} \frac{\text{W}}{\text{m}^2} C_{a\gamma\gamma}^2 \left(\frac{B}{10\text{T}} \right)^2 |\beta|^2$$

$$\text{FoM} = B^2 \text{m}^2 = 100 \text{T}^2 \text{m}^2$$

$$\beta^2 = \frac{P_{\text{Diel.Halosc.}}}{P_{\text{Mirror}}}$$



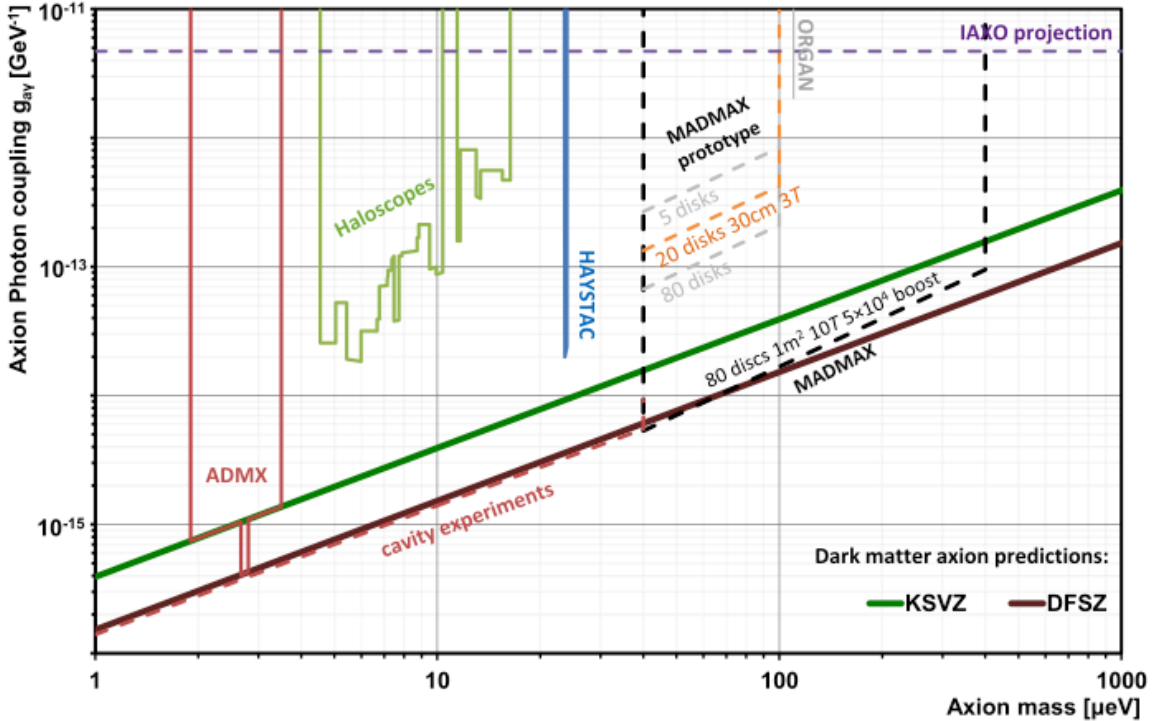
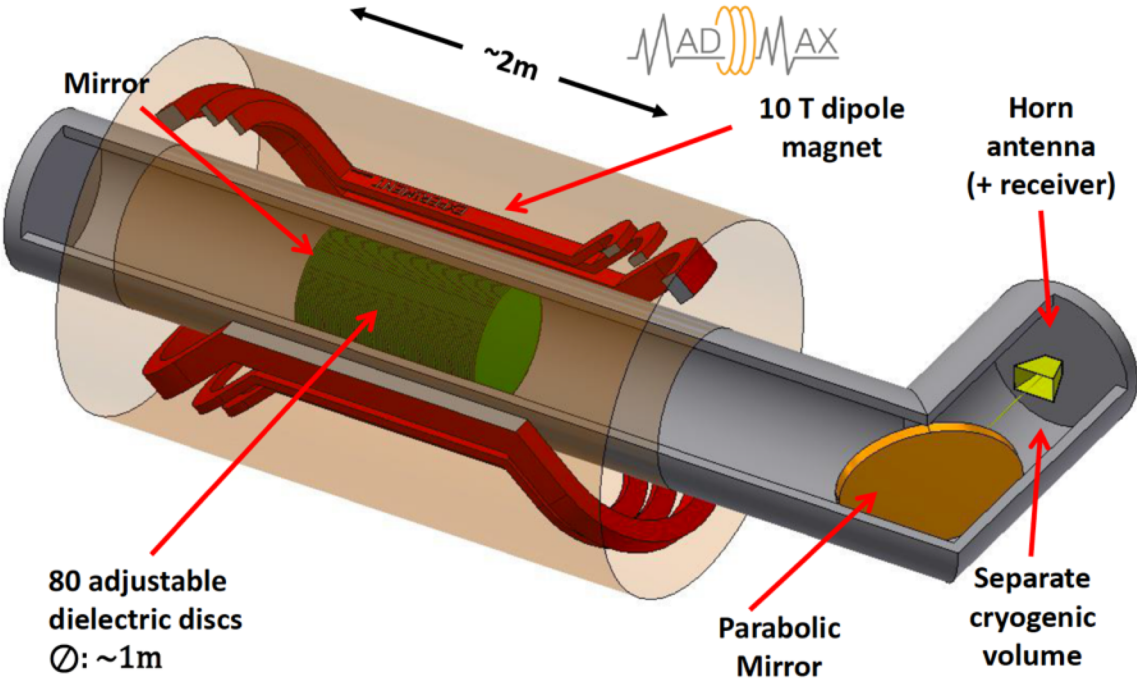
[Garutti, Patras Workshop 19]

White paper: MADMAX Collaboration,
Eur. Phys. J. C 79, 186 (2019), [arXiv:1901.07401]

Searches for Dark Matter Axions

Dish antennas

- Booster dish antenna: Proposed **MADMAX** experiment [Caldwell et al. '16]



[The MADMAX collaboration: Brun et al. 1901.07401]

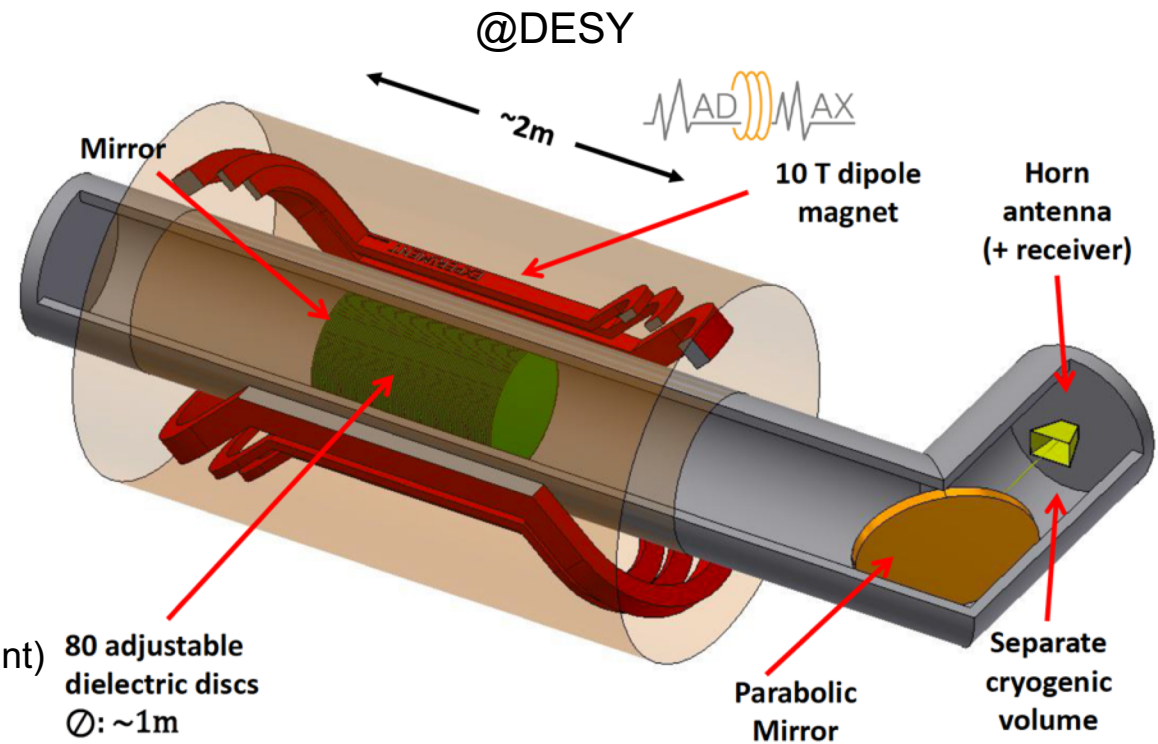
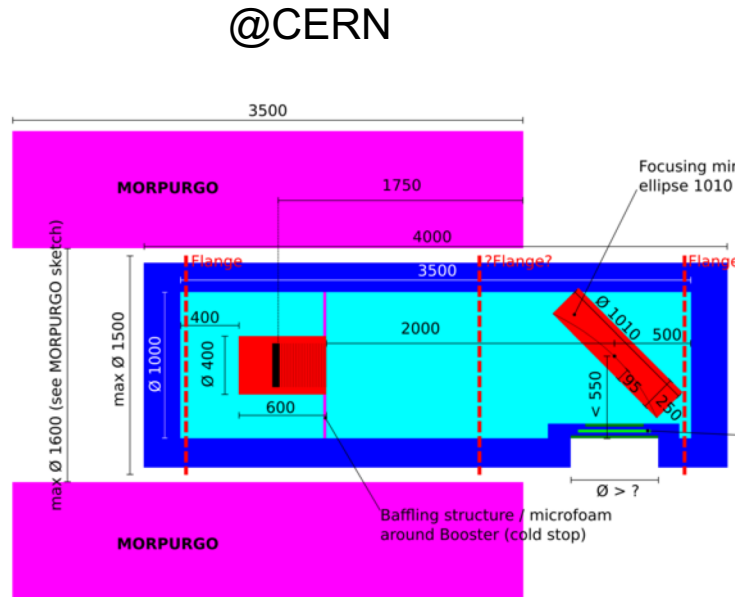
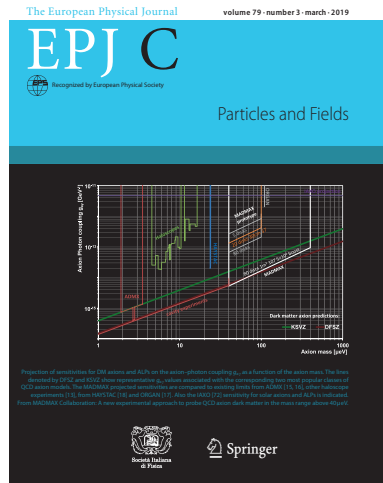
Searches for Dark Matter Axions

Dish antennas

2017 -2019
Design

2020 -2025
Prototype

2025 -2035
Experiment



Scaling: Area 1/10 (of final experiment)
 # discs 1/4
 B [T] 1/5

80 adjustable dielectric discs
 Ø: ~1m

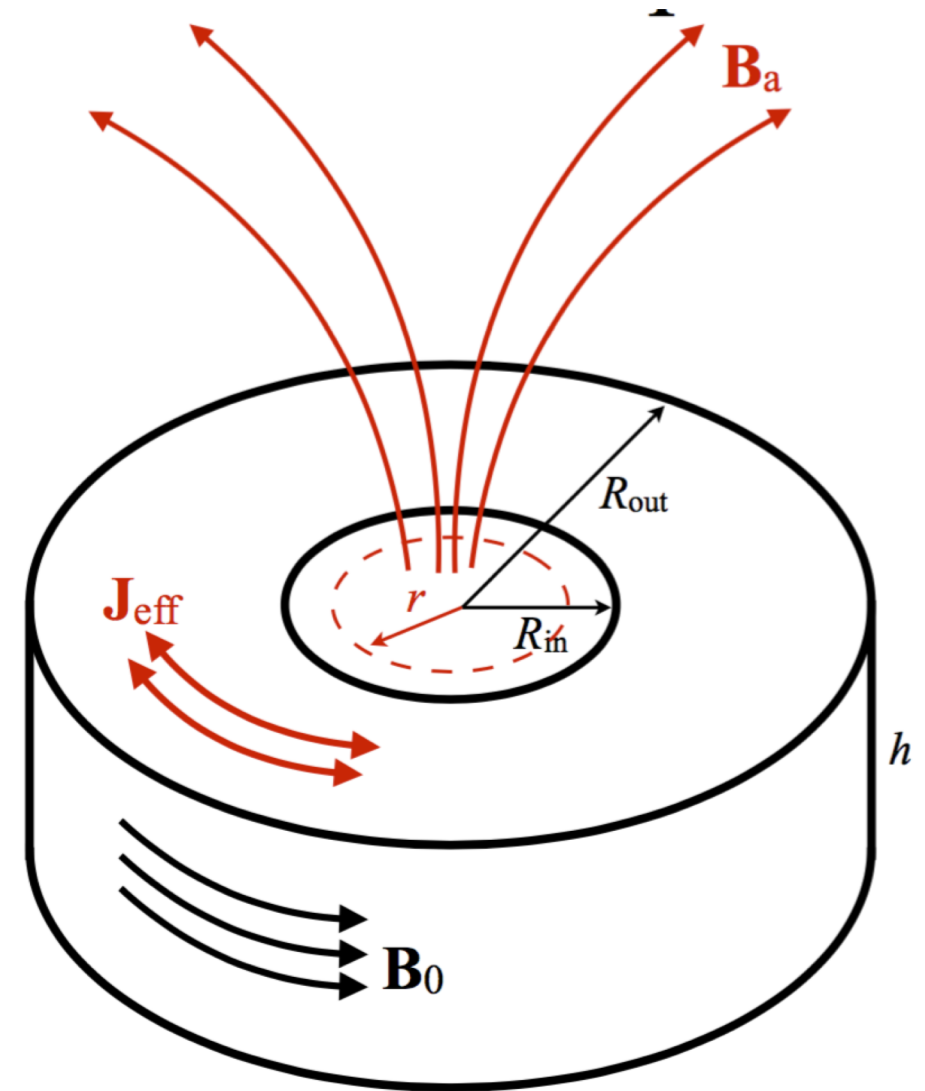
Searches for Dark Matter Axions

Searching for axion-induced magnetic fields

[Sikivie, Sullivan, Tanner 14; Kahn, Safdi, Thaler '16]

- **ABRACADABRA** (MIT) currently being set-up
 - Exploit toroidal magnet with fixed magnetic field:
 - Axion DM generates oscillating effective current around ring
 - ... this generates oscillating magnetic field through center
 - ... this can be detected by pickup loop
- **DM-Radio** (Stanford): similar experiment in path-finder status

[Silva-Feaver et al. 16]



[Ouellet '16; adapted from Kahn, Safdi, Thaler '16]

Searches for Dark Matter Axions

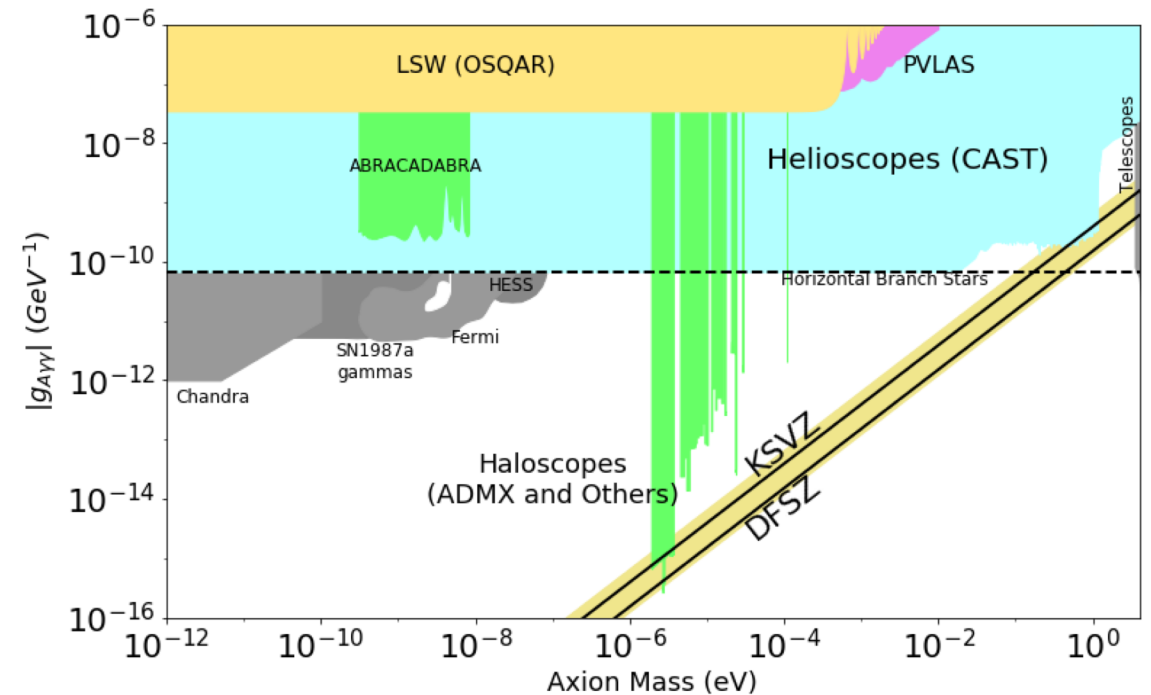
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[Silva-Feaver et al. 16]

ABRACADABRA-10 cm Run 1:



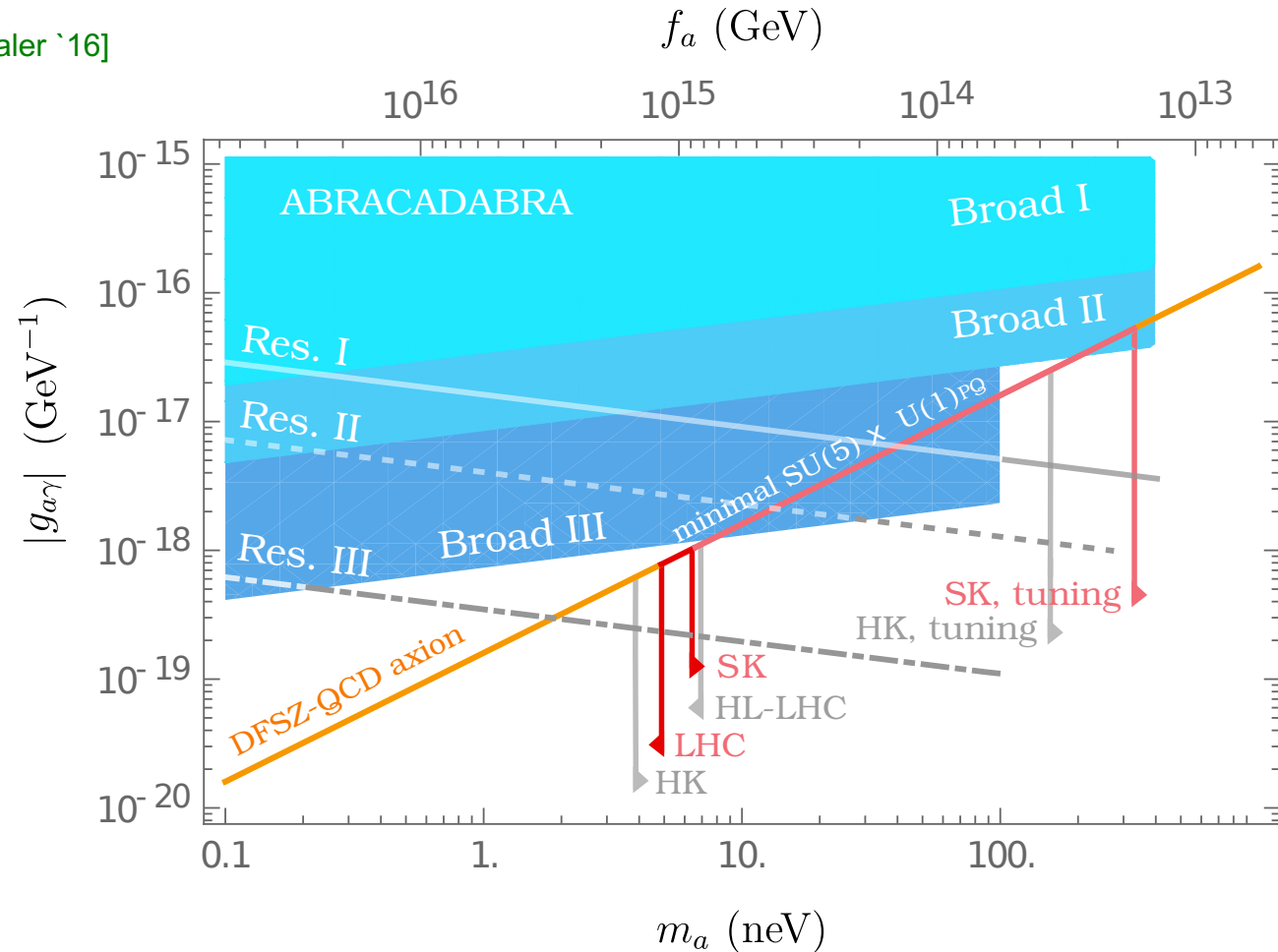
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- Probe QCD axion dark matter in mass range predicted by Grand Unified Theories (GUTs)

[Ernst, AR, Tamarit 18; Di Luzio, AR, Tamarit 18]



[Di Luzio, AR, Tamarit, arXiv:1807.09769]

Searches for Dark Matter Axions

Magnetic resonance searches

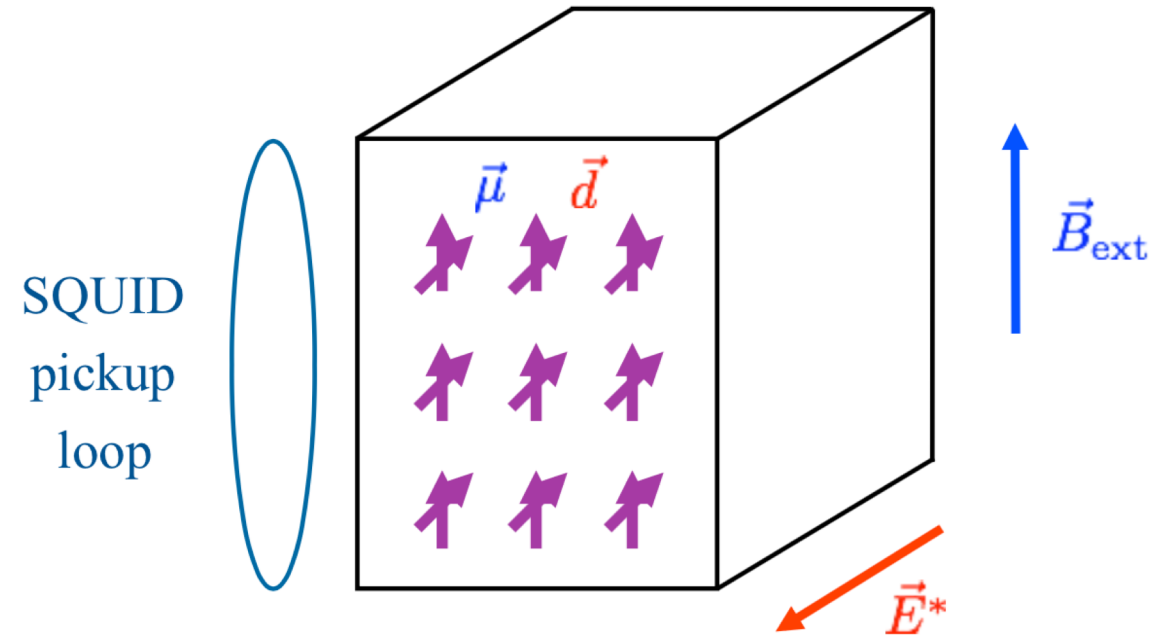
- Axion DM field induces oscillating NEDMs:

$$d_N(t) = g_d \sqrt{2\rho_{\text{DM}}} \cos(m_A t) / m_A$$

- Place a ferroelectric crystal (permanent electric polarisation fields \vec{E}^*) in external $\vec{B}_{\text{ext}} \perp \vec{E}^*$
- Nuclear spins are polarised along \vec{B}_{ext} and precess at Larmor frequency $\omega_L = 2\mu_N B_{\text{ext}}$
- Interaction $\epsilon_S \vec{d}_N(t) \cdot \vec{E}^*$ of DM induced NEDM with the \vec{E}^* -field leads to resonant increase of transverse magnetisation of sample when $\omega_L = m_A$

[Graham,Rajendran 13; Budker et al. 14]

- CASPER-Electric currently being set-up in Boston



[Budker et al. 14]

Searches for Dark Matter Axions

Magnetic resonance searches

- Axion DM field induces oscillating NEDMs:

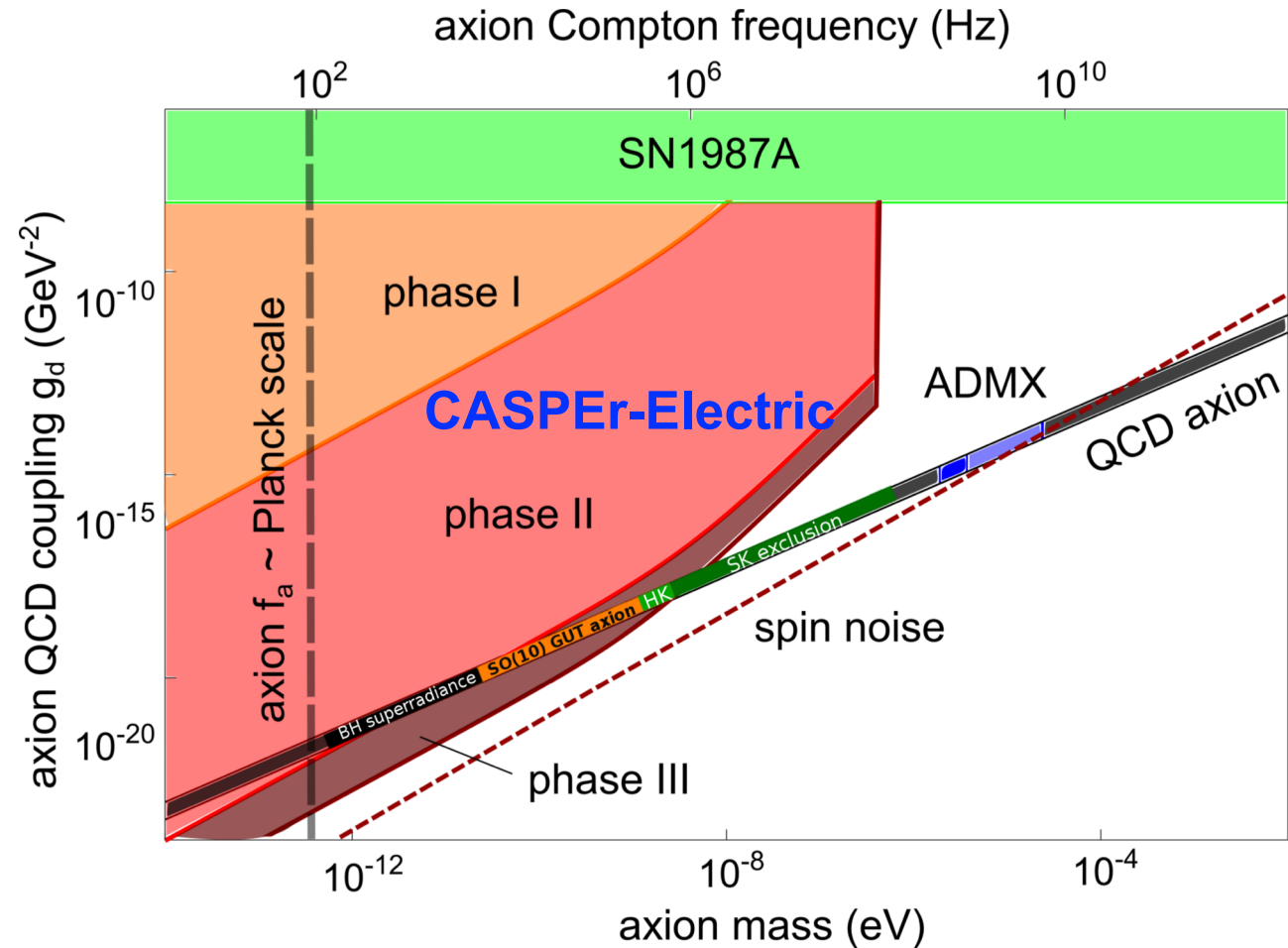
$$d_N(t) = g_d \sqrt{2\rho_{\text{DM}}} \cos(m_A t) / m_A$$

- Place a ferroelectric crystal (permanent electric polarisation fields \vec{E}^*) in external $\vec{B}_{\text{ext}} \perp \vec{E}^*$
- Nuclear spins are polarised along \vec{B}_{ext} and precess at Larmor frequency $\omega_L = 2\mu_N B_{\text{ext}}$
- Interaction $\epsilon_S \vec{d}_N(t) \cdot \vec{E}^*$ of DM induced NEDM with the \vec{E}^* -field leads to resonant increase of transverse magnetisation of sample when $\omega_L = m_A$

[Graham,Rajendran 13; Budker et al. 14]

- CASPER-Electric** currently being set-up in Boston

- Probe QCD axion dark matter in mass range predicted by GUTs [Ernst,AR,Tamarit 18; Di Luzio,AR,Tamarit 18]



[Ernst,Di Luzio,AR,Tamarit 18]

Searches for Dark Matter Axions

Magnetic resonance searches

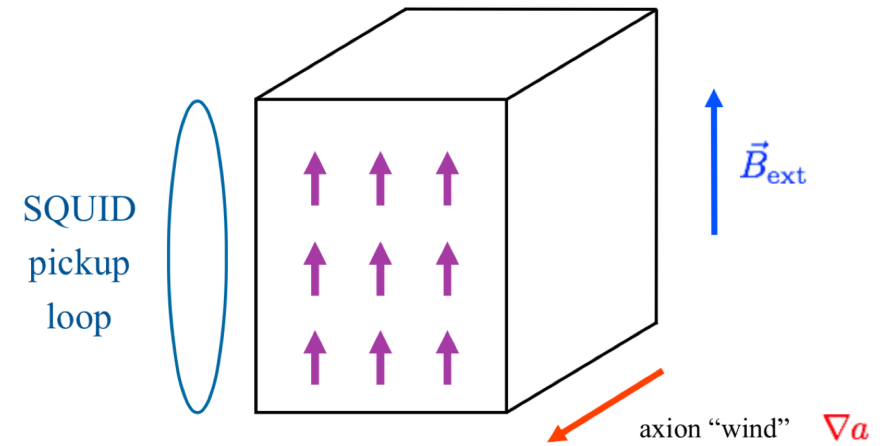
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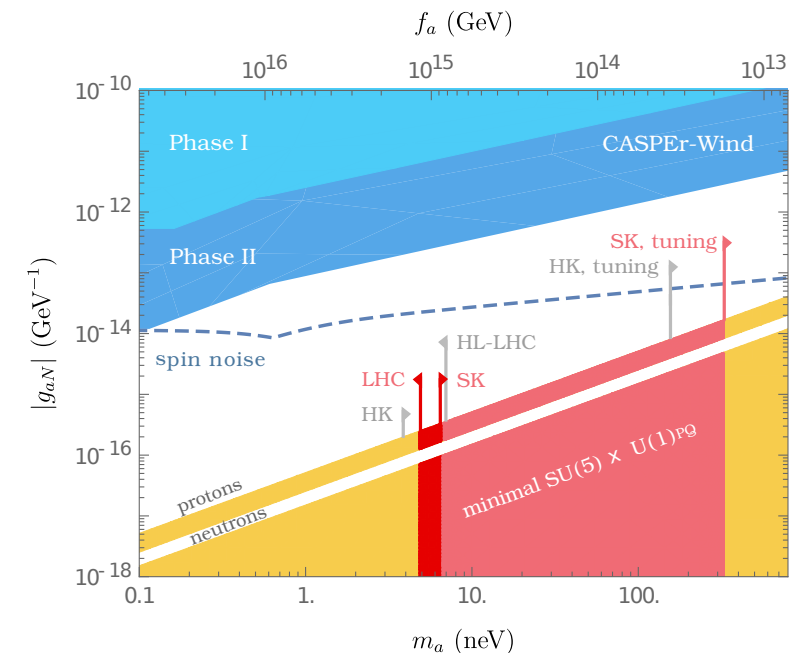
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[Budker et al. 14]



[Di Luzio,AR,Tamarit 18]

Searches for Axion Mediated Forces

Magnetic resonance searches

- Experiments searching for axion mediated forces particularly effective in meV mass range
- Monopole-dipole interaction between nucleon and fermion:

$$U_{\text{mon-dip}}(r) = \frac{g_{aN\bar{N}} g_{af\bar{f}}}{8\pi m_f} \left(\frac{m_a}{r} + \frac{1}{r^2} \right) e^{-m_a r} (\hat{\sigma} \cdot \hat{r})$$

$$\mathcal{L}_{\text{int}} = g_{aN\bar{N}} a \bar{N} N - i g_{af\bar{f}} a \bar{f} \gamma_5 f$$

- Proposed ARIADNE experiment searches for forces between a rotating cylinder, made of unpolarized material, and a vessel containing hyperpolarized ^3He gas
 - Since ^3He magnetic moment dominated by neutron contribution: sensitive to monopole-dipole interaction between nucleus and neutrons, $|g_{aN\bar{N}} g_{an\bar{n}}|$

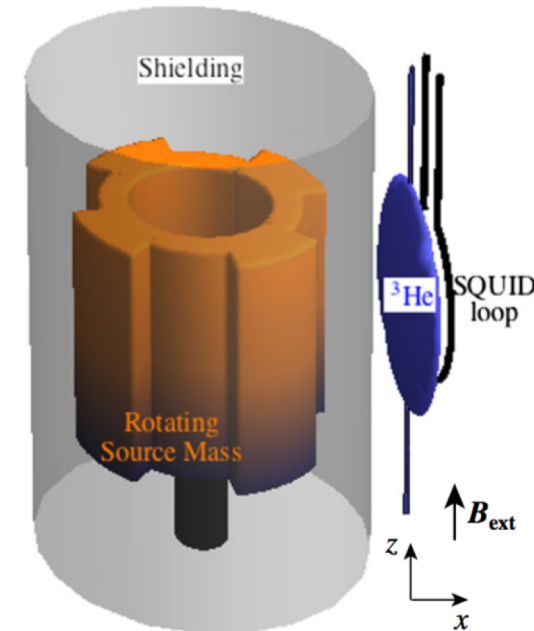


FIG. 1 (color online). A source mass consisting of a segmented cylinder with n sections is rotated around its axis of symmetry at frequency ω_{rot} , which results in a resonance between the frequency $\omega = n\omega_{\text{rot}}$ at which the segments pass near the sample and the resonant frequency $2\vec{\mu}_N \cdot \vec{B}_{\text{ext}}/\hbar$ of the NMR sample. Superconducting cylinders screen the NMR sample from the source mass and (not shown) the setup from the environment.

[Arvanitaki, Geraci 14]

Searches for Axion Mediated Forces

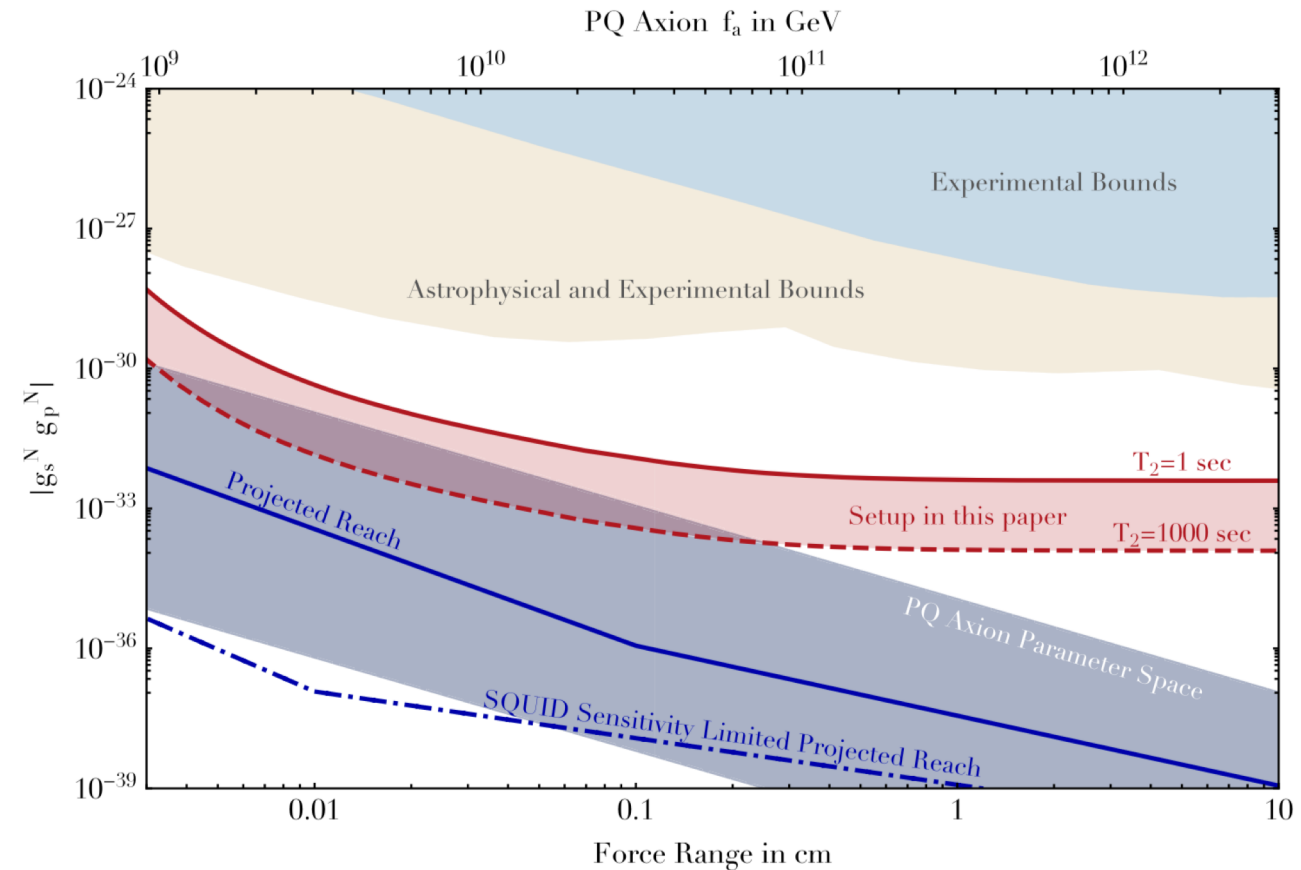
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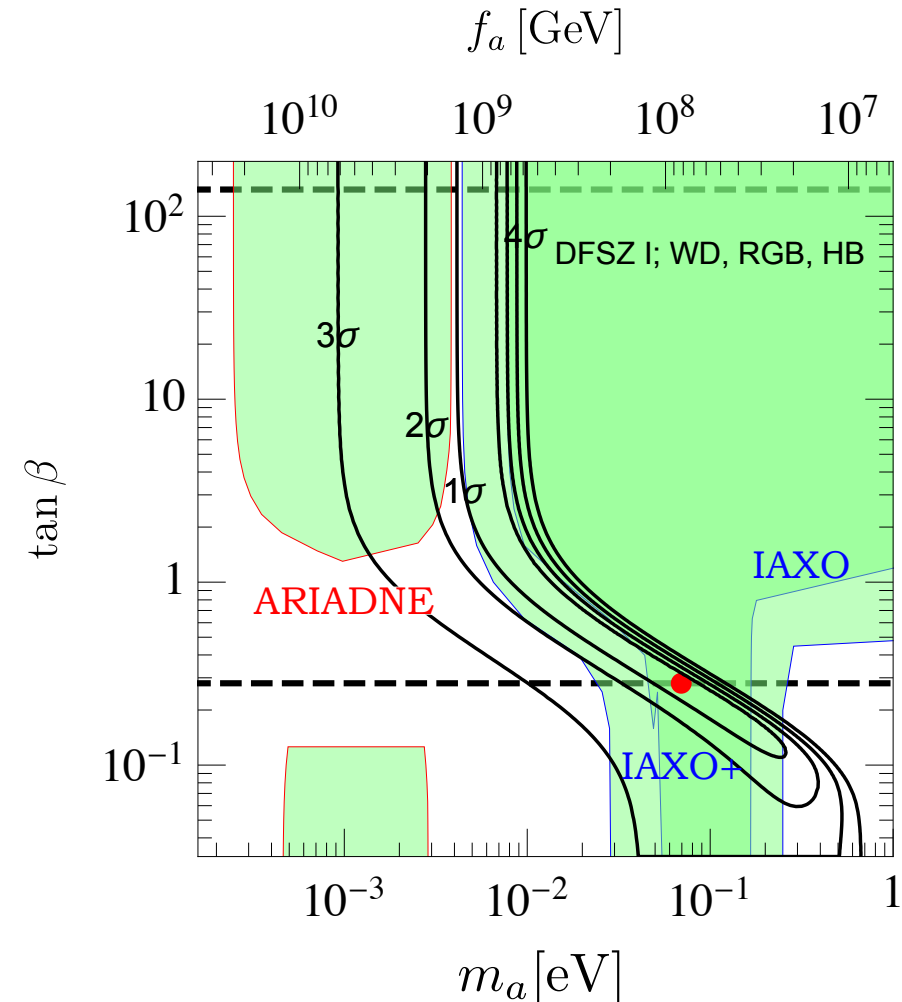
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[Giannotti, Irastorza, Redondo, AR, Saikawa 17]

Conclusions

- Boom in axion/ALP searches!
- Large parts in axion/ALP parameter space will be tackled in the upcoming decade by a number of terrestrial experiments:
 - Light-shining-through-a-wall experiments ([ALPS II](#), ...)
 - Solar axion searches ([IAXO](#), ...)
 - Axion Dark Matter searches ([ABRACADABRA](#), [BRASS](#), [ADMX](#), [CASPEr](#), [CULTASK](#), [HAYSTAC](#), [MADMAX](#), [ORGAN](#), [QUAX](#), ...)
 - Searches for axion-mediated forces ([ARIADNE](#), ...)

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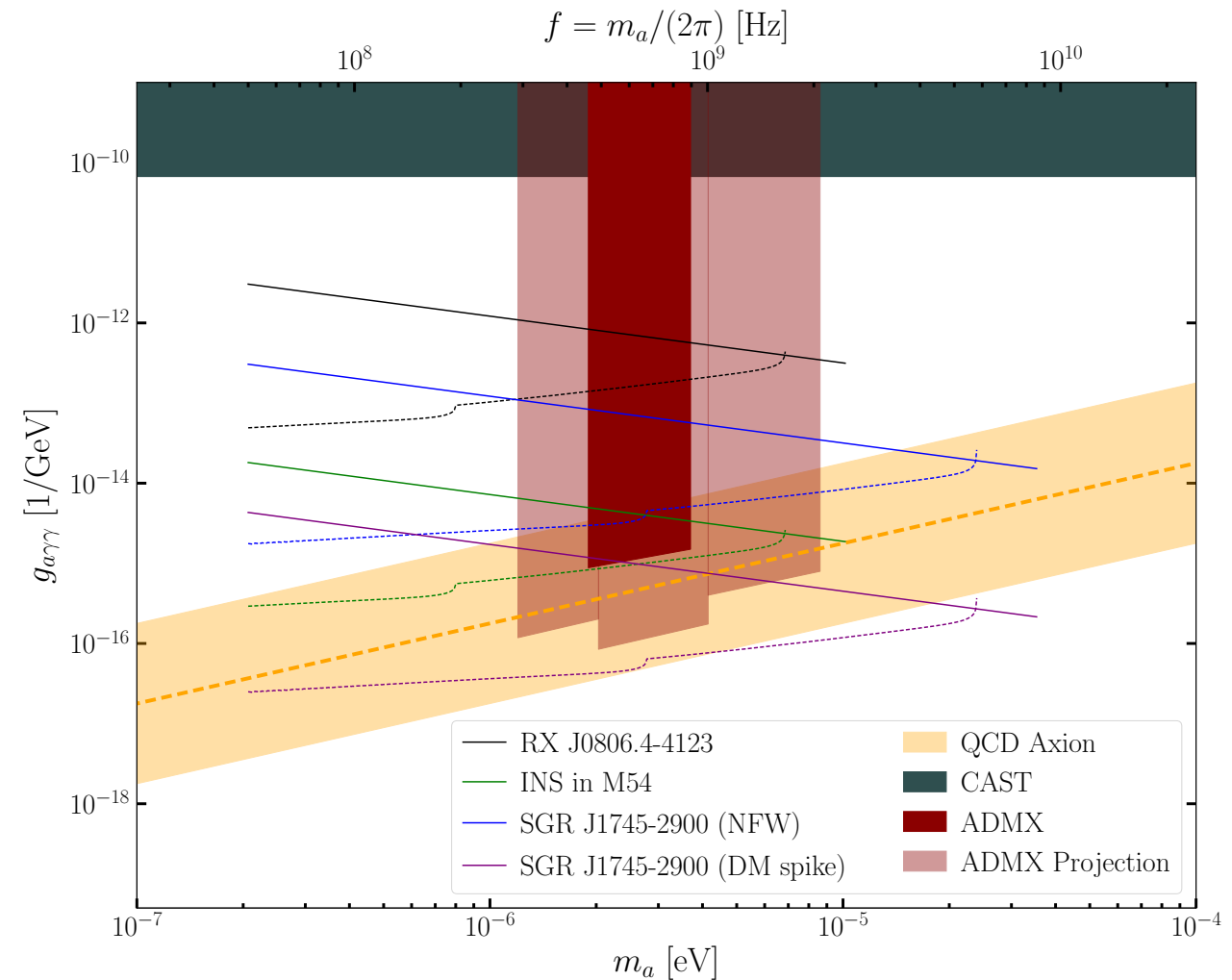
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 - Searches for axion-mediated forces ([ARIADNE](#), ...)
- If 100 % of DM consists of QCD axions, one of the ADM searches will see a signal in the upcoming decade!

STAY TUNED!

Indirect Detection of Axion Dark Matter

Telescope searches

- In presence of axion DM, monochromatic radio signals are emitted from neutron stars (NSs) due to axion-photon conversion within high magnetic field regions in NS magnetosphere
 - Forthcoming radiotelescopes such as SKA can search for those [Huang et al. `18; Hook et al. `18]
- In dwarf spheroidal magnetic fields expected to be small enough that monochromatic radio line due to axion DM decay dominates over signal due to axion-photon conversion in magnetic field
 - SKA sensitivity not enough to probe QCD axion DM, but sufficient for ALP DM [Caputo et al. `18]

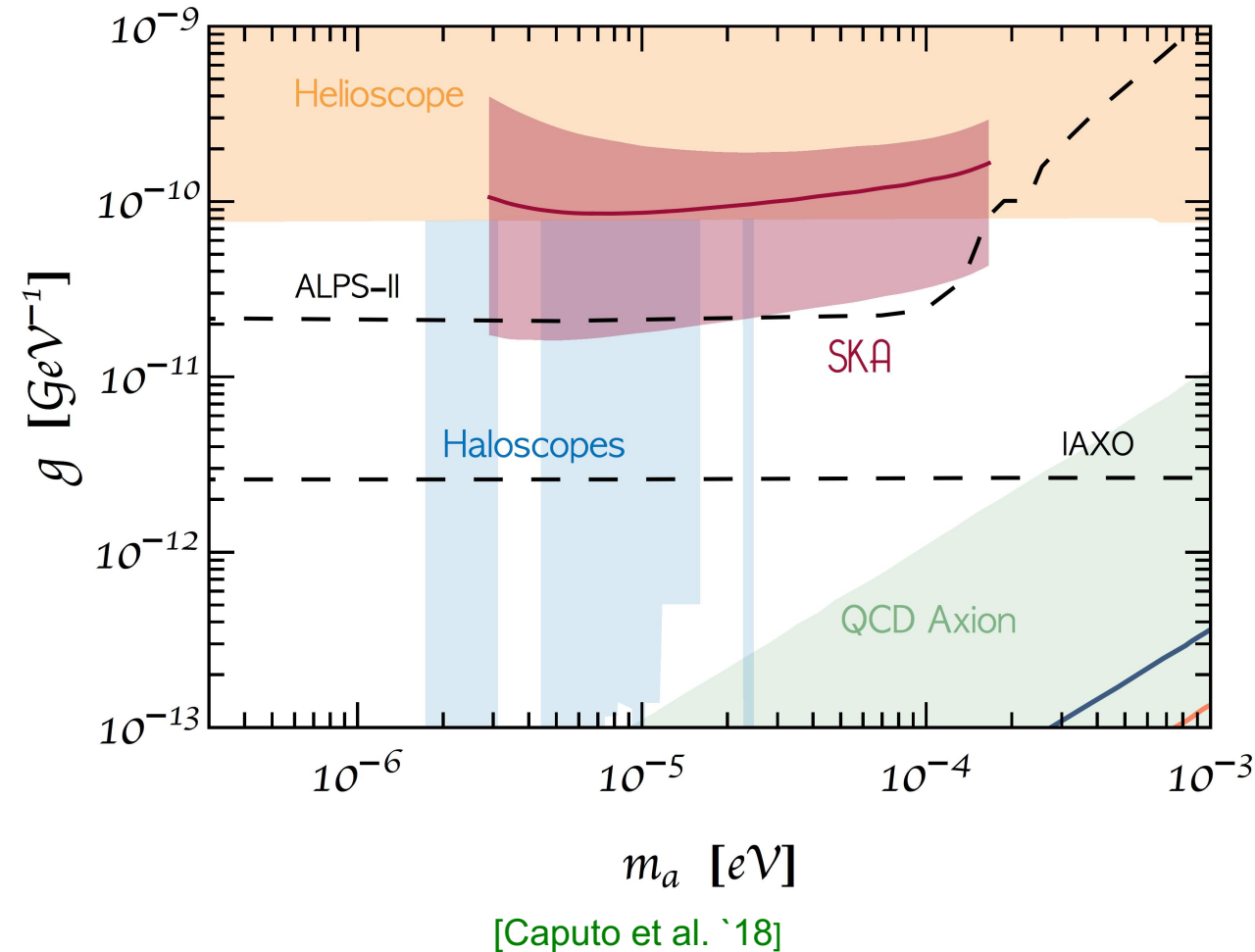


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Searches for FCNC Interactions

Minimal flavored SMASH

- Introduce right-handed singlet neutrinos N_α and complex scalar field σ with flavor-dependent spontaneously broken global chiral $U(1)_{\text{FN}}$ Froggatt-Nielsen (FN) symmetry

- Explains SM fermionic mass hierarchies and mixings by FN mechanism
- Explains neutrino masses and mixing by seesaw
- FN symmetry is at the same time a PQ symmetry
 - Nambu-Goldstone boson of FN breaking solves also strong CP problem (“axion”): “flaxion” or “axiflavor”
- Flaxion has flavor changing neutral current (FCNC) interactions

- Currently best bound from $\text{Br}(K^+ \rightarrow \pi^+ A) \lesssim 7.3 \times 10^{-11}$

$$\Rightarrow f_A \gtrsim 2 \times 10^{10} \text{ GeV} \left(\frac{26}{N_{\text{DW}}} \right) \left| \frac{(\kappa_{\text{ah}}^d)_{12}}{m_s} \right|$$

[Ema et al., 1612.05492;1802.07739; Calibbi et al., 1612.08040]

- Flaxion properties:

- Decay constant:

$$f_A = \frac{v_{\text{FN}}}{N_{\text{DW}}}, \quad N_{\text{DW}} = \text{Tr}(n^u + n^d)$$

- Typically large domain wall number

- Viable example:

$$\begin{pmatrix} q_{Q_1} & q_{Q_2} & q_{Q_3} \\ q_u & q_c & q_t \\ q_d & q_s & q_b \end{pmatrix} = \begin{pmatrix} 3 & 2 & 0 \\ -5 & -1 & 0 \\ -4 & -3 & -3 \end{pmatrix}$$

$$\begin{pmatrix} q_{L_1} & q_{L_2} & q_{L_3} \\ q_e & q_\mu & q_\tau \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ -8 & -5 & -3 \end{pmatrix}$$

$$\Rightarrow n_{ij}^u = \begin{pmatrix} 8 & 4 & 3 \\ 7 & 3 & 2 \\ 5 & 1 & 0 \end{pmatrix}, \quad n_{ij}^d = \begin{pmatrix} 7 & 6 & 6 \\ 6 & 5 & 5 \\ 4 & 3 & 3 \end{pmatrix}, \quad n_{ij}^l = \begin{pmatrix} 9 & 6 & 4 \\ 8 & 5 & 3 \\ 8 & 5 & 3 \end{pmatrix}$$

$$\Rightarrow N_{\text{DW}} = 26$$

- Coupling to photon:

$$C_{A\gamma} = \frac{2}{N_{\text{DW}}} \sum_{f=u,d,l} \left[N_f \text{Tr}(n^f) \left(q_f^{(\text{em})} \right)^2 \right] - 1.92(2)$$

- For example above: $C_{A\gamma} = 113/39 - 1.92 \simeq 0.97$

- **NA62:** reach in decay constant increased by order of magnitude!