Competition between superconductivity and topological Kondo effect in Majorana devices

Francesco Buccheri
Heinrich-Heine-Universität Düsseldorf

Topological States of matter
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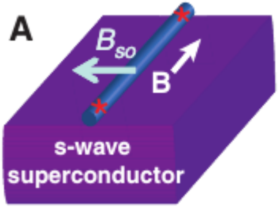
Work in collaboration with:
R. Egger, P. Sodano, A. Zazunov

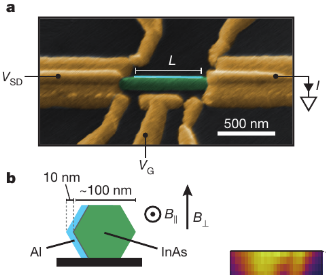
Outline

Ingredients:

Lutchyn et al. PRL 105, 077001 (2010); Oreg et al. PRL 105, 177002 (2010);

- nanowire with strong spin-orbit coupling (InAs, InSb)
- BCS superconductor (Nb, Al) $\Delta \sim 180\mu eV$
- magnetic field $B \sim 110mT$
- chemical potential $\mu(x)$
- $T \sim 50mK$

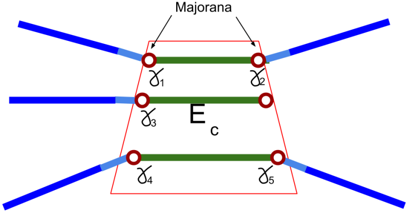




Mourik et al. Science 336, 1003 (2012); Rokhinson et al. Nat. Phys. 8, 795 (2012); Das et al. Nat. Phys. 8, 887 (2012); Deng et al. Nano Lett., 12, 6414 (2012); Churchill et al. PRB 87, 241401(R) (2013); Finck et al. PRL 110, 126406 (2013); Deng et al. Sc. Rep. 4, 7261 (2014); Nadj-Perge et al. Science 346, 602 (2014); Albrecht et al. Nature 531, 206 (2016)

The Majorana-Coulomb box

- Majorana end modes γ_α: $\{\gamma_\alpha, \gamma_\beta\} = 2\delta_{\alpha,\beta}$



- Spinless fermions in semi-infinite M one-dimensional wires

$$H_w = -i \sum_{\alpha=1}^{M} \int dx \left(\psi^\dagger_{\alpha,R}(x)\partial_x\psi_{\alpha,R}(x) - \psi^\dagger_{\alpha,L}(x)\partial_x\psi_{\alpha,L}(x) \right)$$

open boundary condition at one end: $\psi_{\alpha,R}(0) = \psi_{\alpha,L}(0)$

- Floating superconductor, large charging energy $E_c \gg \Delta, T$
- $2^{\lceil M/2 \rceil - 1}$ degeneracy $\rightarrow$ localized degree of freedom

The topological Kondo model

- Coupling between M Majorana modes and M external wires

Fu - PRL 104, 056402 (2010) ; Law et al. PRL 103, 237001 (2009); Zazunov et al. PRB 84 165440 (2011)

Effective model at $T \ll \Delta, E_c$:

$$H = -i \sum_{\alpha=1}^{M} \int dx \psi_\alpha^\dagger(x) \partial_x \psi_\alpha(x) + \sum_{\alpha \neq \beta} \lambda_{\alpha,\beta} \gamma_\alpha \gamma_\beta \psi_\alpha^\dagger(0) \psi_\beta(0)$$

$$\lambda_{\alpha,\beta} \sim \lambda_\alpha \lambda_\beta / E_c$$

Béri and Cooper PRL 109:156803 (2012); Altland and Egger PRL 110, 196401; Béri PRL 110, 216803 (2013)

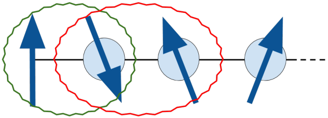
RG analysis:

- Unstable fixed point at $\lambda = 0$
- Strong coupling isotropic point $\rightarrow SO(M)_2$
  - Conductance $G_{j,k} = \frac{2e^2}{h} \frac{1}{M}$
  - $SO(3)_2 \sim SU(2)_4$
  - Stable against anisotropy in $\lambda_{\alpha,\beta}$
- Crossover temperature:

$$T_K \sim E_c e^{-\frac{\pi}{2\bar{\lambda}(M-2)}}$$

Kondo vs. BCS

Magnetic impurity in a BCS superconductor



- Transition temperature, density of states

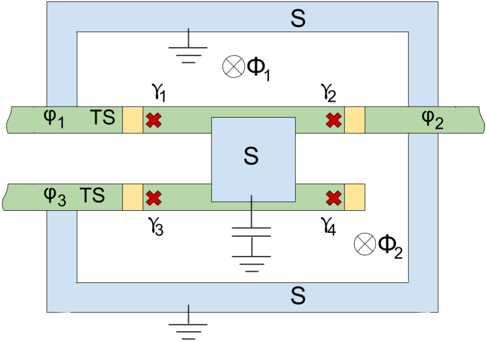
Abrikosov and Gorkov JETP 12 (1961), 1243; Zittartz and Müller-Hartmann Z. Physik 232, 11 (1970)

- Competition between pairing Δ and T_K in the Josephson effect.

Glazman et al. JETP 49, 570 (1989); Siano et al. PRL 93, 047002 (2004); Karrasch et al. PRB 77, 024517 (2008); van Dam et al., Nat. Lett. 442, 667 (2006); Eichler et al. PRB 79, 161407(R) (2009)

Question: what happens to the "topological" Kondo effect when we add pairing interactions in the leads?

The device



- M 1D topological superconductors:

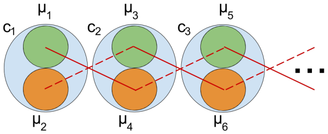
$$H_\alpha = \int_0^\infty dx \Psi_\alpha^\dagger(x) \left[-i\partial_x \sigma_z + \Delta_\alpha e^{-i\varphi_\alpha \sigma_z} \sigma_y \right] \Psi_\alpha(x) \qquad \Psi_\alpha = \begin{pmatrix} \psi_{\alpha,R} \\ \psi_{\alpha,L}^\dagger \end{pmatrix}$$

- Majorana island with large E_c
- Effective tunneling $H_K = \sum_{\alpha\neq\beta} \lambda_{\alpha,\beta}\gamma_\alpha\gamma_\beta\psi_\alpha^\dagger(0)\psi_\beta(0)$

Lattice description

Kitaev chain:

$$H_\alpha = -\sum_{j=1}^{L-1}\left[\frac{t}{2}\left(c_{j,\alpha}^\dagger c_{j+1,\alpha} + c_{j+1,\alpha}^\dagger c_{j,\alpha}\right) + \frac{\Delta}{2}\left(e^{i\phi_\alpha} c_{j,\alpha} c_{j+1,\alpha} + e^{-i\phi_\alpha} c_{j+1,\alpha}^\dagger c_{j,\alpha}^\dagger\right)\right]$$



Majorana representation $2c_{j,\alpha} = \tilde{\mu}_{2j-1,\alpha} + i\tilde{\mu}_{2j,\alpha}$

$$H_\alpha = -i\sum_{j=1}^{L-1}\Big((t+\Delta\cos\phi_\alpha)\,\tilde{\mu}_{2j-1,\alpha}\tilde{\mu}_{2j+2,\alpha} - (t-\Delta\cos\phi_\alpha)\,\tilde{\mu}_{2j,\alpha}\tilde{\mu}_{2j+1,\alpha}$$
$$+\Delta\sin\phi_\alpha\left(\tilde{\mu}_{2j-1,\alpha}\tilde{\mu}_{2j+1,\alpha} - \tilde{\mu}_{2j,\alpha}\tilde{\mu}_{2j+2,\alpha}\right)\Big)$$

Localized Majorana modes

Kitaev Usp. Fiz. 44, 131 (2001)

Perturbative RG

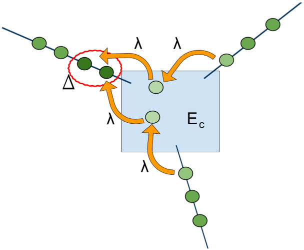
- Field theory description ($T \to 0$, $L \to \infty$). Boundary Green's function:

$$iG_\alpha(\omega) = \begin{pmatrix} \frac{\sqrt{\omega^2+\Delta_\alpha^2}}{\omega} & \frac{\Delta_\alpha}{\omega} e^{i\phi_\alpha} \\ \frac{\Delta_\alpha}{\omega} e^{-i\phi_\alpha} & \frac{\sqrt{\omega^2+\Delta_\alpha^2}}{\omega} \end{pmatrix}$$

Zazunov et al. PRB 94 014502 (2016)

- "Crossed Andreev reflection" generated by the RG

$$\mu_{\alpha,\beta}\gamma_\beta\gamma_\alpha\psi_\alpha\psi_\beta + \mu^*_{\alpha,\beta}\gamma_\beta\gamma_\alpha\psi^\dagger_\alpha\psi^\dagger_\beta$$



RG equations

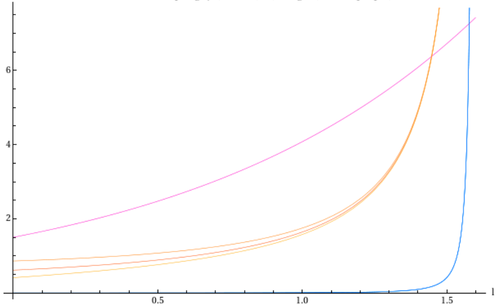
Running couplings: $\mu_{\alpha,\beta}(l)$, $\lambda_{\alpha,\beta}(l)$, $\delta(l) = \frac{\Delta}{\omega_c}$, $l = \log \frac{E_c}{\omega_c}$

$$\frac{d\lambda_{jk}}{dl} = \sum_{m\neq(j,k)}^{M} \left[\rho_{1,1}\left(\lambda_{jm}\lambda_{mk} + \mu_{jm}\mu^*_{mk}\right) + \rho_{1,2}\left(\lambda_{jm}\mu^*_{mk} + \mu_{jm}\lambda_{mk}\right)\right]$$

$$\frac{d\mu_{jk}}{dl} = \sum_{m\neq(j,k)}^{M} \left[\rho_{1,2}\left(\lambda_{jm}\lambda^*_{mk} + \mu_{jm}\mu_{mk}\right) + \rho_{1,1}\left(\lambda_{jm}\mu_{mk} + \mu_{jm}\lambda^*_{mk}\right)\right]$$

$$\rho_{1,1} = \frac{2}{\pi}\sqrt{1+\delta^2} \qquad \rho_{1,2} = \frac{2}{\pi}\delta$$

Flow of the couplings μ (blueish) λ (orangish) and Δ (purple)



The Kondo temperature and the gap

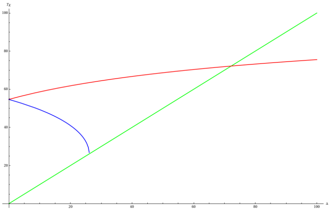
Define $\hat{\Lambda}_{\pm} = \hat{\lambda} \pm \hat{\mu}$

$$\frac{d}{dl}\hat{\Lambda}_{\pm} = \hat{\Lambda}_{\pm}\left(\hat{\rho}_{1,1} \pm \hat{\rho}_{2,1}\right)\hat{\Lambda}_{\pm}$$

Kondo temperature $T_K^{\pm}$: divergence of the running coupling $\Lambda_{\pm}$

$$\hat{\Lambda}_{\pm}(l) = \frac{\bar{\lambda}_0}{1 - \frac{2(M-2)}{\pi}\bar{\lambda}_0 \mathcal{F}_{\pm}(l)}$$

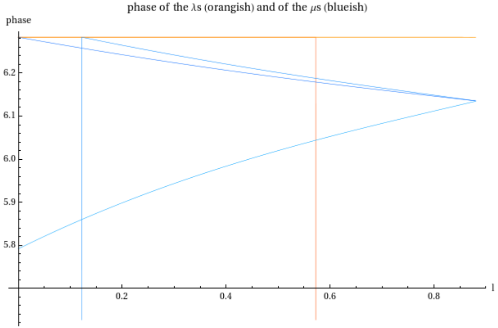
- $\Delta \ll E_c, T_K$: $\quad \frac{T_K^{\pm}}{E_C} \simeq e^{-\frac{\pi}{2\lambda_0(M-2)}} \pm \frac{\Delta}{E_C}\left(1 - e^{-\frac{\pi}{2\lambda_0(M-2)}}\right)$



- $T_K \ll E_c, \Delta$: no solution

Phase flow

Parametrization: $\lambda_{\alpha,\beta} = \tilde{\lambda}_{\alpha,\beta} e^{i\eta_{\alpha,\beta}}$ $\mu_{\alpha,\beta} = \tilde{\mu}_{\alpha,\beta} e^{i\theta_{\alpha,\beta}}$



Flow of the phases ($\tilde{\lambda}_{j,k} \simeq \tilde{\mu}_{j,k} \simeq \bar{\lambda}$):

$$\frac{1}{2\bar{\lambda}} \frac{d\eta_{\alpha,\beta}}{dl} = \sum_{\chi \neq \alpha,\beta} \sin\left(\frac{\eta_{\alpha,\chi} + \eta_{\chi,\beta} + \theta_{\alpha,\chi} - \theta_{\chi,\beta}}{2} - \eta_{\alpha,\beta}\right)$$
$$\left(\rho_{1,1} \cos \frac{\eta_{\alpha,\chi} + \eta_{\chi,\beta} - \theta_{\alpha,\chi} + \theta_{\chi,\beta}}{2} + |\rho_{1,2}| \cos\left(\frac{\eta_{\alpha,\chi} - \theta_{\chi,\beta} - \theta_{\alpha,\chi} - \eta_{\chi,\beta}}{2} + \phi_\chi\right)\right)$$
$$\frac{1}{2\bar{\lambda}} \frac{d\theta_{\alpha,\beta}}{dl} = \sum_{\chi \neq \alpha,\beta} \sin\left(\frac{\eta_{\alpha,\chi} + \theta_{\chi,\beta} + \theta_{\alpha,\chi} + \eta_{\beta,\chi}}{2} - \theta_{\alpha,\beta}\right)$$
$$\left(\rho_{1,1} \cos \frac{\eta_{\alpha,\chi} + \theta_{\chi,\beta} - \theta_{\alpha,\chi} - \eta_{\beta,\chi}}{2} + |\rho_{1,2}| \cos\left(\frac{\eta_{\alpha,\chi} + \eta_{\beta,\chi} - \theta_{\alpha,\chi} - \theta_{\chi,\beta}}{2} + \phi_\chi\right)\right)$$

Phases at strong coupling

Focus on $M = 3$ wires

- Observation $\frac{d\eta_{\alpha,\beta}}{dl} \to 0$ and $\frac{d\theta_{\alpha,\beta}}{dl} \to 0$

$$\begin{aligned} \eta_{1,2} + \eta_{2,3} + \eta_{3,1} &= 0 \\ \theta_{1,3} - \theta_{1,2} &= -\eta_{2,3} \\ \theta_{2,3} - \theta_{1,2} &= \eta_{3,1} \end{aligned}$$

satisfied by

$$\eta_{j,k} = 0 \qquad \theta_{j,k} = \theta_0$$

- Gauge invariance + symmetry $\to \theta_0 = \frac{\phi_1+\phi_2+\phi_3}{3}$
- Strong coupling Hamiltonian:

$$H_K^{scfp} = \frac{1}{2}\Lambda_+ \sum_{\alpha\neq\beta} \gamma_\beta\gamma_\alpha \tilde{\mu}_{1,\alpha}\tilde{\mu}_{1,\beta}$$

$$\tilde{\mu}_{1,\alpha} = \frac{1}{\sqrt{2}}\left(e^{-i\frac{\theta_0}{2}} c_{1,\alpha}^\dagger + e^{i\frac{\theta_0}{2}} c_{1,\alpha}\right)$$

$\to$ 2 channel Kondo model

Coleman et al: PRB 52, 6611 (1995); Tsvelik PRL 110, 147202, (2013)

The strong coupling fixed point

Coleman et al: PRB 52, 6611 (1995); Tsvelik PRL 110, 147202, (2013); Giuliano et al. NPB 909, 135 (2016)

- Add a fourth Kitaev chain $H_0(\phi_0, \Delta)$, which stays decoupled from the system
- Define $d_{j,\uparrow} = \tilde{\mu}_{j,1} + i\tilde{\mu}_{j,2}$, $\qquad d_{j,\downarrow} = \tilde{\mu}_{j,3} + i\tilde{\mu}_{j,0}$

$$H = -t \sum_{j=1}^{L-1} \sum_{\sigma=\uparrow,\downarrow} \left(d^\dagger_{2j-1,\sigma} d_{2j+2,\sigma} + d^\dagger_{2j,\sigma} d_{2j+1,\sigma} \right) - \Delta \ldots$$

- Kondo term

$$H_K = \Lambda_+ \vec{S} \cdot \mathcal{D}^\dagger_1 \left(\vec{\sigma} + \vec{\tau} \right) \mathcal{D}_1$$

$$S^a = \frac{1}{2i} \varepsilon^{\alpha,\beta,\eta} \gamma_\beta \gamma_\eta$$

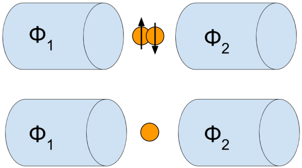
$$\mathcal{D}^\dagger_j = \left(d^\dagger_{j,\uparrow}, d^\dagger_{j,\downarrow}, d_{j,\downarrow}, -d_{j,\uparrow} \right) \qquad \sigma^a = \sigma^a_{Pauli} \otimes \mathbb{I} \qquad \tau^a = \mathbb{I} \otimes \sigma^a_{Pauli}$$

$$d^\dagger_{j,\updownarrow} |0\rangle \rightarrow \{\sigma = \pm 1, \tau = 0\} \qquad d^\dagger_{j,\uparrow} d^\dagger_{j,\downarrow} |0\rangle \rightarrow \{0, 1\} \qquad |0\rangle \rightarrow \{0, -1\}$$

Strong coupling analysis

Open questions:

- Is the 2CK the low-T fixed point?
- How does the Majorana interact with the rest of the system?
- What is the Josephson current?
  - Fractional Josephson effect is a signature of Majorana modes



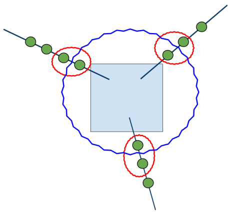
Kitaev Usp. Fiz. 44, 131 (2001); Kwon et al. Low T Phys. 30, 613 (2004); Fu and Kane PRB 79, 161408(R) (2009)

  - Detection

Rokhinson et al. Nat. Phys. 8, 795 (2012); Bocquillon et al. Nnano 159 (2016)

The Majorana fermion $\tilde{\mu}_1$ and the island "spin" $\vec{S}$ are strongly entangled. What else is competing?

Competing interactions around the SCFP



- Subleading Kondo interaction:

$$H_K \quad = \quad \Lambda_+ \vec{S} \cdot \mathcal{D}_1^\dagger \left(\vec{\sigma} + \vec{\tau}\right) \mathcal{D}_1 + \Lambda_- \vec{S} \cdot \mathcal{D}_2^\dagger \left(\vec{\sigma} + \vec{\tau}\right) \mathcal{D}_2$$

- Interaction of the boundary with the bulk

$$H_r = -i\left(t + \Delta \cos \tilde{\phi}_\alpha\right) \tilde{\mu}_{1,\alpha} \tilde{\mu}_{4,\alpha} - i \Delta \sin \tilde{\phi}_\alpha \tilde{\mu}_{1,\alpha} \tilde{\mu}_{3,\alpha}$$

$$\tilde{\phi}_\alpha \equiv \phi_\alpha - \frac{\phi_1 + \phi_2 + \phi_3}{3}$$

Around the fixed point

- Ground state manifold

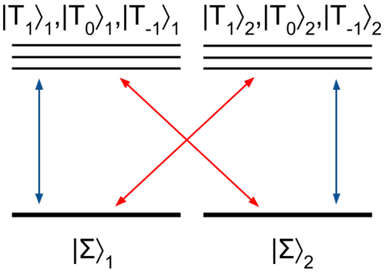
$$|\Sigma_1\rangle = \frac{1}{\sqrt{2}}\left(d^\dagger_{1,\uparrow}|\Downarrow\rangle - d^\dagger_{1,\downarrow}|\Uparrow\rangle\right) \qquad |\Sigma_2\rangle = \frac{1}{\sqrt{2}}\left(d^\dagger_{1,\uparrow}d^\dagger_{1,\downarrow}|\Downarrow\rangle - |\Uparrow\rangle\right)$$

- Excited state manifold

$$|T_1\rangle_1 = d^\dagger_{1,\uparrow}|\Uparrow\rangle \qquad |T_0\rangle_1 = \frac{1}{\sqrt{2}}\left(d^\dagger_{1,\uparrow}|\Downarrow\rangle + d^\dagger_{1,\downarrow}|\Uparrow\rangle\right) \qquad |T_{-1}\rangle_1 = d^\dagger_{1,\downarrow}|\Downarrow\rangle$$

$$|T_1\rangle_2 = d^\dagger_{1,\uparrow}d^\dagger_{1,\downarrow}|\Uparrow\rangle \qquad |T_0\rangle_1 = \frac{1}{\sqrt{2}}\left(d^\dagger_{1,\uparrow}d^\dagger_{1,\downarrow}|\Downarrow\rangle + |\Uparrow\rangle\right) \qquad |T_{-1}\rangle_1 = |\Downarrow\rangle$$

- Transitions



The strong coupling Hamiltonian

We write an effective Hamiltonian that takes into account the transitions to excited states:

- To second order $\vec{S} \cdot (\vec{\sigma}_2 + \vec{\tau}_2)$, H_r
- To third order H_r^3

$$
\begin{aligned}
H_{sc} \quad \sim \quad & \left(-\frac{3\Lambda_-^2}{4\Lambda_+} - \frac{3\left(\Delta^2 + t^2\right)}{4\Lambda_+} - \frac{\Delta t}{2\Lambda_+} \sum_\alpha \cos \tilde{\phi}_\alpha \right) \mathbb{I} \\
& + \frac{\Lambda_-}{\Lambda_+} \gamma_0 \left[t \tilde{\mu}_{4,\alpha} + \Delta \left(\sin \tilde{\phi}_\alpha \tilde{\mu}_{3,\alpha} + \cos \tilde{\phi}_\alpha \tilde{\mu}_{4,\alpha} \right) \right] \tilde{\mu}_{2,\beta} \tilde{\mu}_{2,\gamma} + cycl. \\
& + \frac{1}{\Lambda_+^2} \gamma_0 \prod_{\alpha=1}^{3} \left[\left(t + \Delta \cos \tilde{\phi}_\alpha \right) \tilde{\mu}_{4,\alpha} + \Delta \sin \tilde{\phi}_\alpha \tilde{\mu}_{3,\alpha} \right]
\end{aligned}
$$

$\gamma_0 = i \tilde{\mu}_{1,1} \tilde{\mu}_{2,1} \tilde{\mu}_{3,1}$

- The perturbing operator is irrelevant

Giuliano et al. EPJ B 89, 251 (2016)

Josephson current

- Field theory, $T \ll T_K$, $t \gg \Delta$

$$c_{\alpha,j} \to \eta_\alpha(x) + i\xi_\alpha(x)$$

- Boundary conditions $\eta(-x) = -\eta(x)$, $\xi(-x) = \xi(x)$

$$\langle T_\tau \eta_\alpha(\tau)\eta_\beta(0)\rangle = \langle T_\tau \xi_\alpha(\tau)\xi_\beta(0)\rangle = -\delta_{\alpha,\beta}\partial_\tau f(\tau)$$
$$\langle T_\tau \eta_\alpha(\tau)\xi_\beta(0)\rangle = -\langle T_\tau \xi_\alpha(\tau)\eta_\beta(0)\rangle = -i\delta_{\alpha,\beta}\cos\tilde{\phi}_\alpha f(\tau)$$

where $\quad f(\tau) = \int_0^{T_K} \frac{d\omega}{2\pi} \frac{Q_\omega \cos(\omega\tau)}{\sqrt{\omega^2+\Delta^2}} \qquad Q_\omega = 1 - e^{-\sqrt{\omega^2+\Delta^2}/T_K}$

Free energy variation

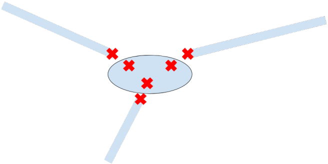
$$\begin{aligned} I_\alpha &\propto \frac{2e}{\hbar}\frac{\partial}{\partial\phi_\alpha}\lim_{T\to 0} F(T) \\ &= \frac{I_0}{3}\sum_{\beta\neq\eta(\neq\alpha)}^{3}\left(3\sin(\phi_\alpha-\phi_\beta) + \sin\frac{\phi_\alpha+\phi_\beta-2\phi_\eta}{3} - \sin\frac{\phi_\beta+\phi_\eta-2\phi_\alpha}{3}\right) \end{aligned}$$

$$I_0 = \frac{e\Delta^2\zeta}{T_K\hbar} \qquad \zeta = \frac{1}{3}\Lambda_-^2 f(0)^3$$

6π-periodicity.

The strong pairing limit

Pairing is very large: $\Delta \gg T_K$.



$$H_K = \sum_{\alpha<\beta} \lambda_{\alpha,\beta} \sqrt{\Delta_\alpha \Delta_\beta} \gamma_\beta \gamma_\alpha \left(\xi_\alpha \xi_\beta e^{i \frac{\phi_\alpha - \phi_\beta}{2}} - \xi_\beta \xi_\alpha e^{-i \frac{\phi_\alpha - \phi_\beta}{2}} \right)$$

Dirac fermion $d_\alpha = \frac{\xi_\alpha + i\gamma_\alpha}{2}$ and $\sigma_\alpha = 2d_\alpha^\dagger d_\alpha - 1$

Josephson current

$$I_\alpha = \frac{e}{\hbar} \sum_{\beta \neq \alpha} \lambda_{\alpha,\beta} \sqrt{\Delta_\alpha \Delta_\beta} \sigma_\alpha \sigma_\beta \sin \frac{\phi_\alpha - \phi_\beta}{2}$$

4π-periodicity

Kitaev Usp. Fiz. 44, 131 (2001)

Conclusions

- Model involving Majorana modes in solid state devices, realistically realizable in the lab
- For $M = 3$, the pairing interactions drive the system to a different (NFL) fixed point
- Josephson current periodicity is a multiple of 2π
- Competition between characteristic energy scales creates two distinct regimes

To-do list:

- Characterization of the transition from one regime to the other
- More wires $M > 3$
- Experiment

A. Zazunov, F. B., P. Sodano and R. Egger, Phys. Rev. Lett. 118, 057001 (2017), arXiv:1611.07307