The International Institute of Physics, Natal, 2017

Majorana States in the Coulomb Blockade Regime – Signatures and Applications

Eran Sela (Tel Aviv)

With: **Aviad Landau** (TAU), Alex Altland (Köln), Reinhold Egger, Stephan Plugge (Düsseldorf), Karsten Flensberg (Copenhagen)

Karen Michaeli (Weizmann) and Liang Fu (MIT)

Majorana zero-bias peaks

SCIENCE VOL 336 25 MAY 2012

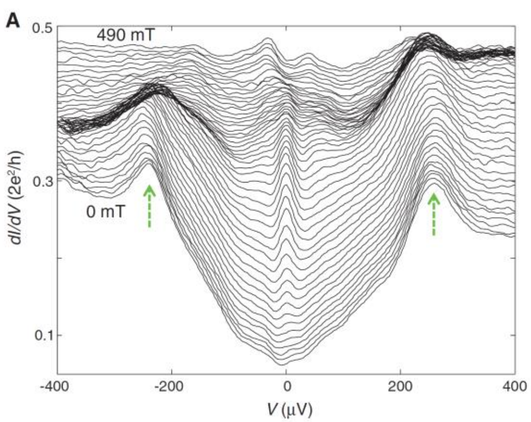
Signatures of Majorana Fermions in Hybrid Superconductor-Semiconductor Nanowire Devices

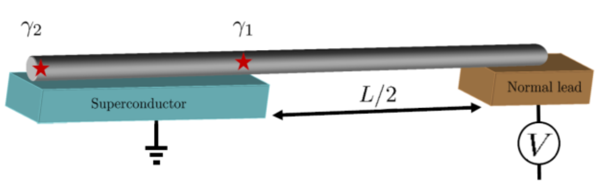
V. Mourik,[1*] K. Zuo,[1*] S. M. Frolov,[1] S. R. Plissard,[2] E. P. A. M. Bakkers,[1,2] L. P. Kouwenhoven[1]†

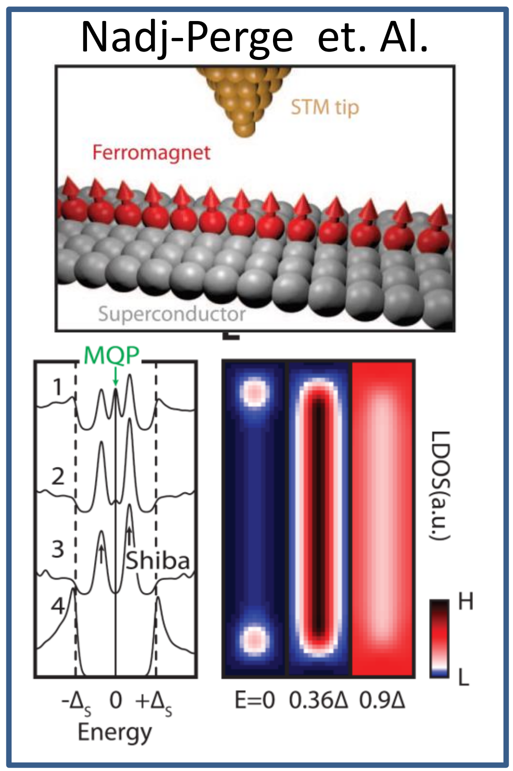
nature physics ARTICLES
PUBLISHED ONLINE: 11 NOVEMBER 2012 | DOI: 10.1038/NPHYS2479

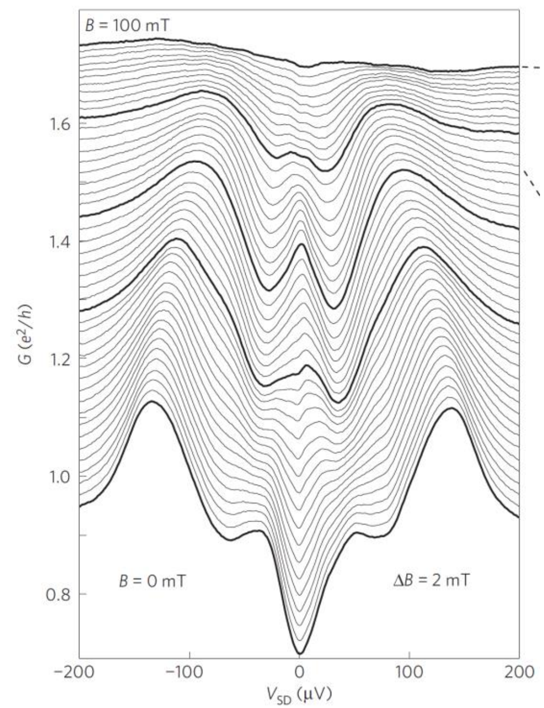
Zero-bias peaks and splitting in an Al-InAs nanowire topological superconductor as a signature of Majorana fermions

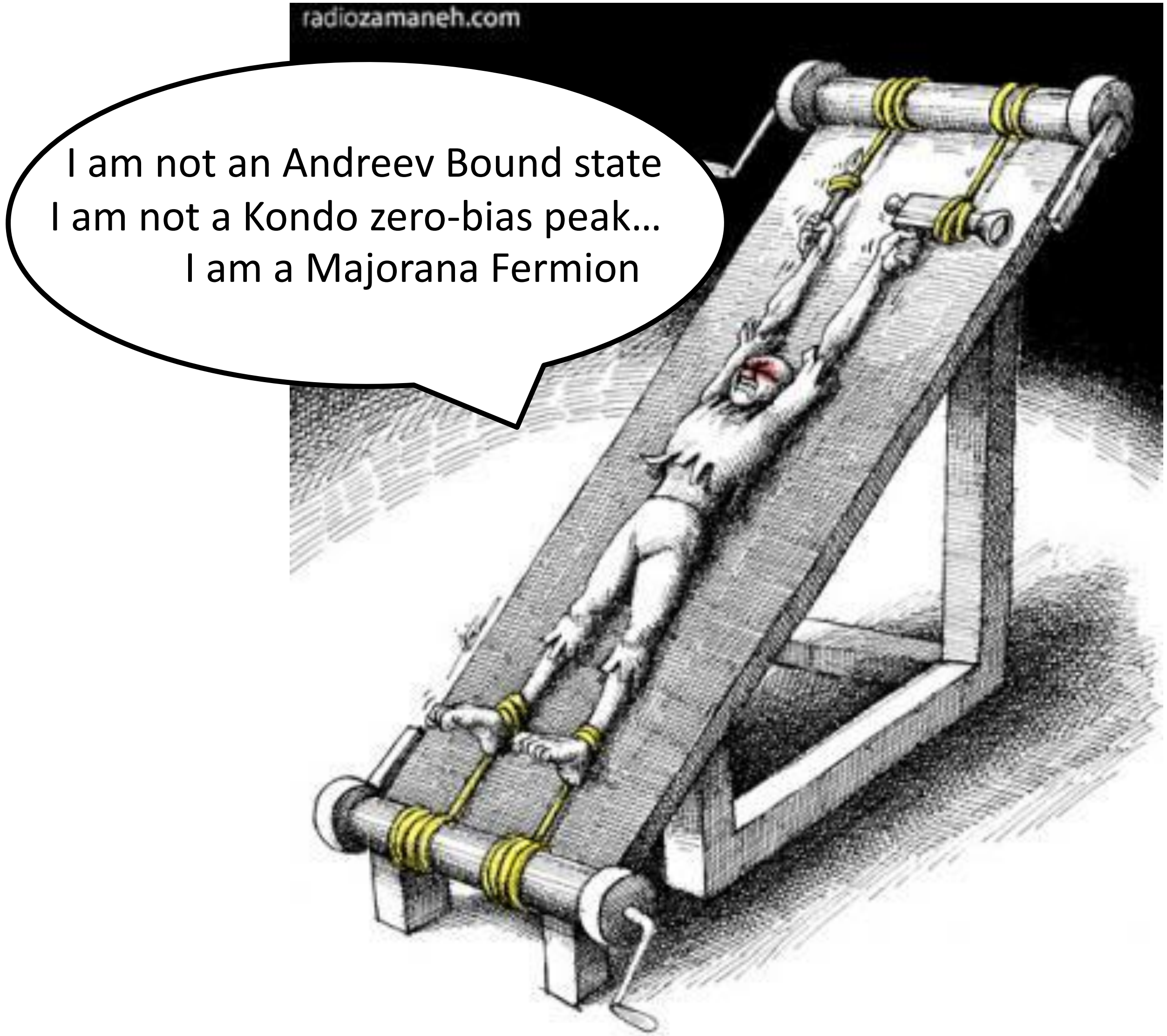
Anindya Das†, Yuval Ronen†, Yonatan Most, Yuval Oreg, Moty Heiblum* and Hadas Shtrikman





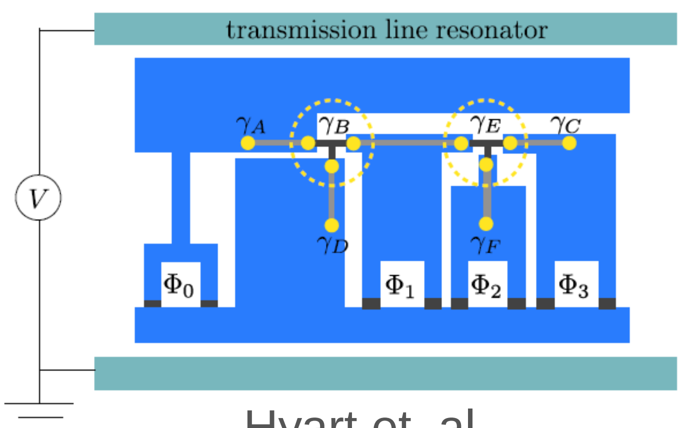




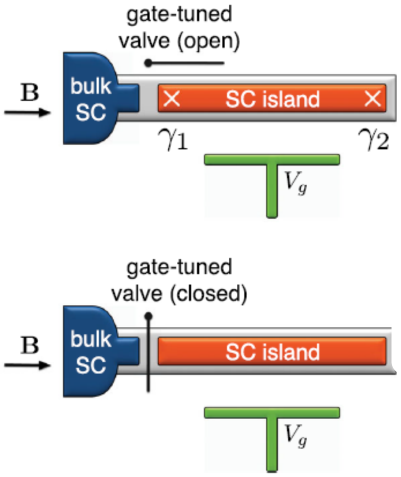

radiozamaneh.com
I am not an Andreev Bound state
I am not a Kondo zero-bias peak…
I am a Majorana Fermion

Majoranas and Coulomb Blockade

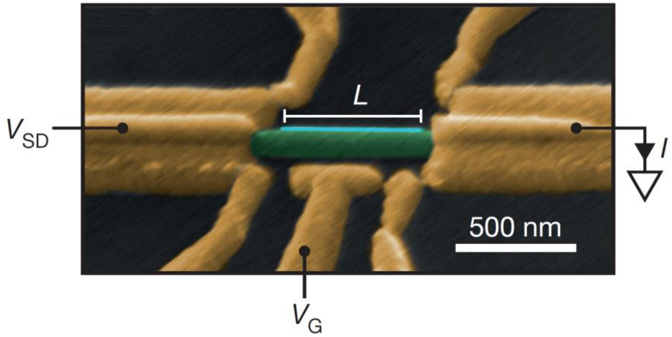
- Challenge: use and control them
- Need non-local control like Coulomb interaction
- Majorana box qubits
- Create exotic strongly correlated states



Hyart et. al.
PRB 2013



Aasen et. al.
PRX 2016



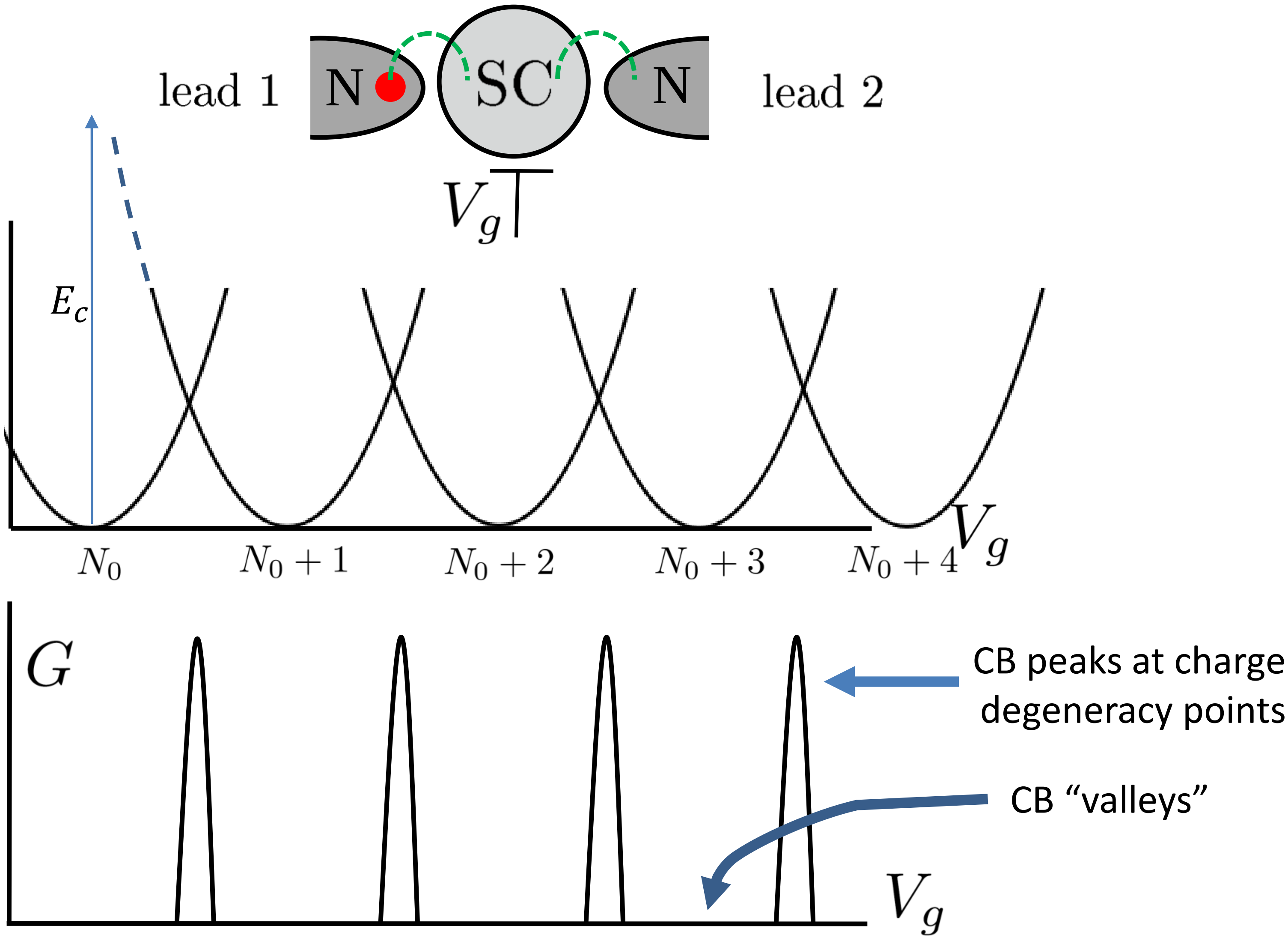
Albrecht et. al.
Nature 2016

Outline

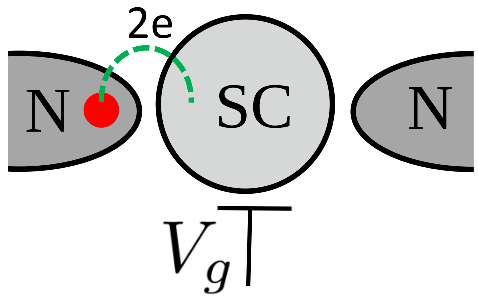
- Introduction to the “Copenhagen experiment”
- Topological Kondo effect
- Statistical transmutation
- Topological Kondo effect at charge degeneracy points
- Competition with Josephson interaction
- Outlook and Summary

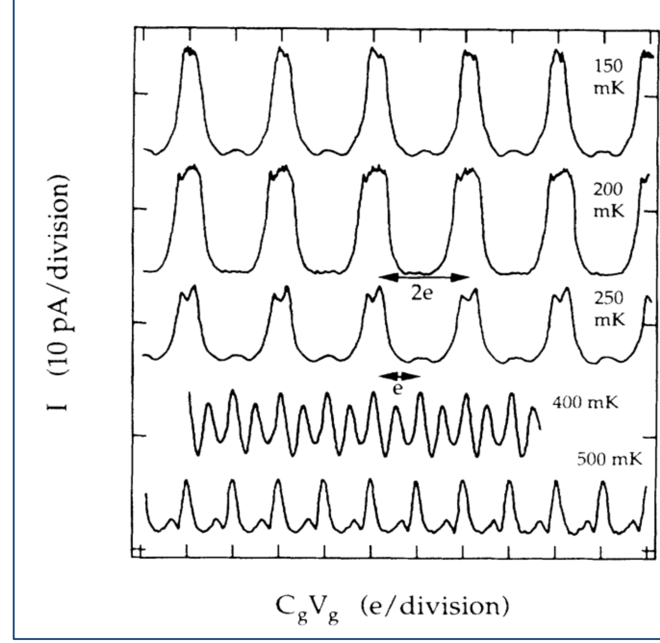
Introduction to the Copenhagen experiment

Coulomb Blockade in Quantum dots

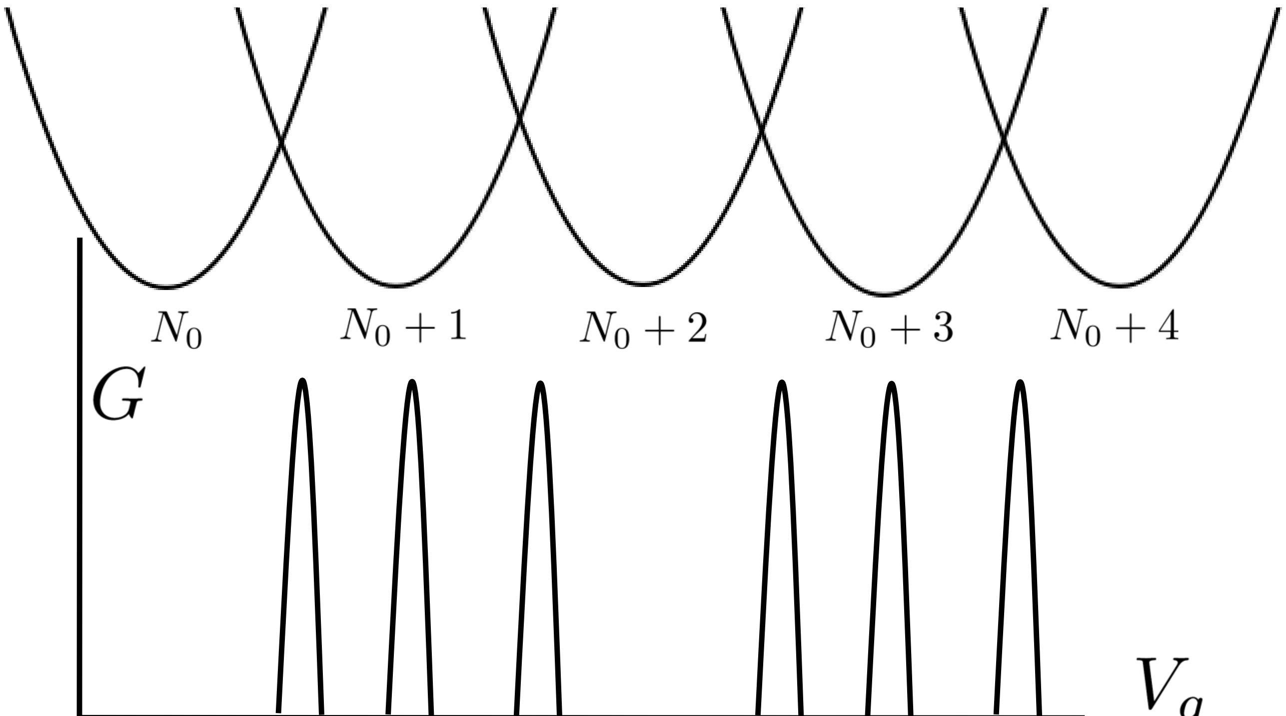


Conventional superconductor – Cooper pair tunneling



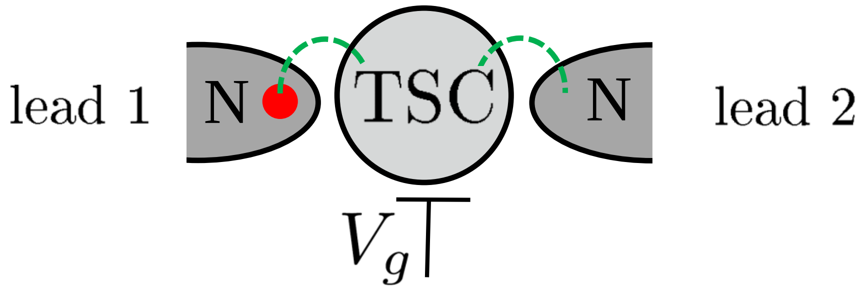


Tinkham 92



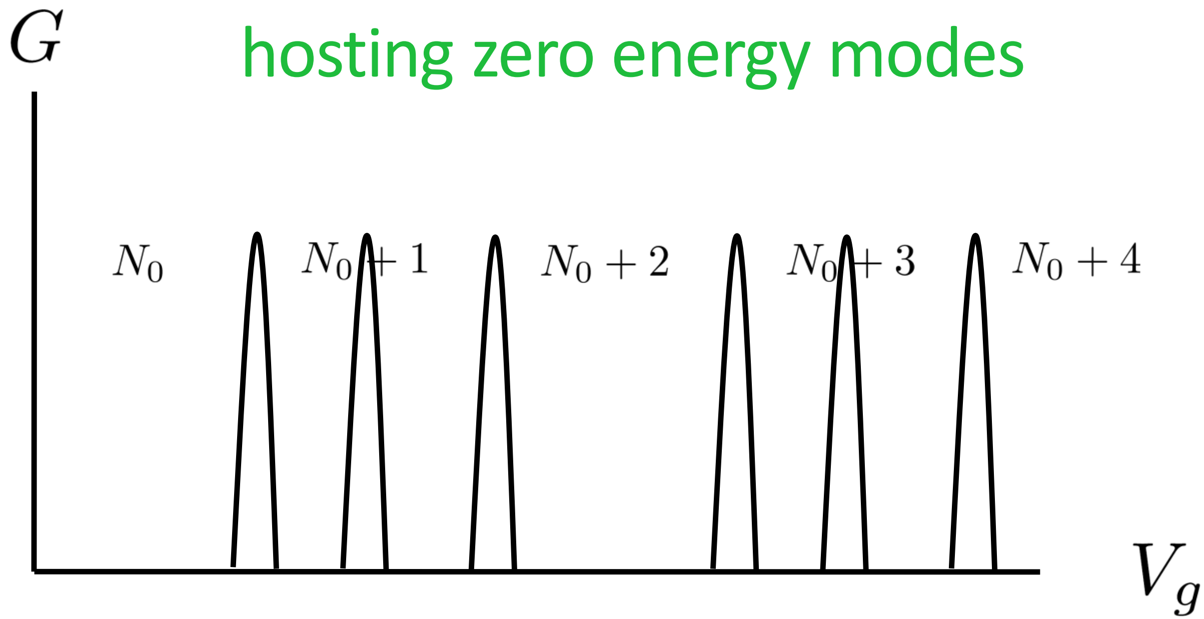
Energy gap Δ

Topological Superconductor – single electron tunneling ?



Energy gap Δ

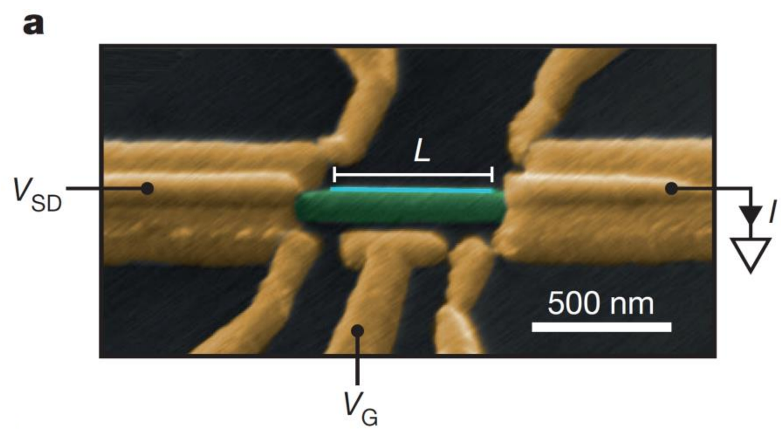
hosting zero energy modes

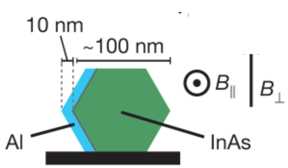


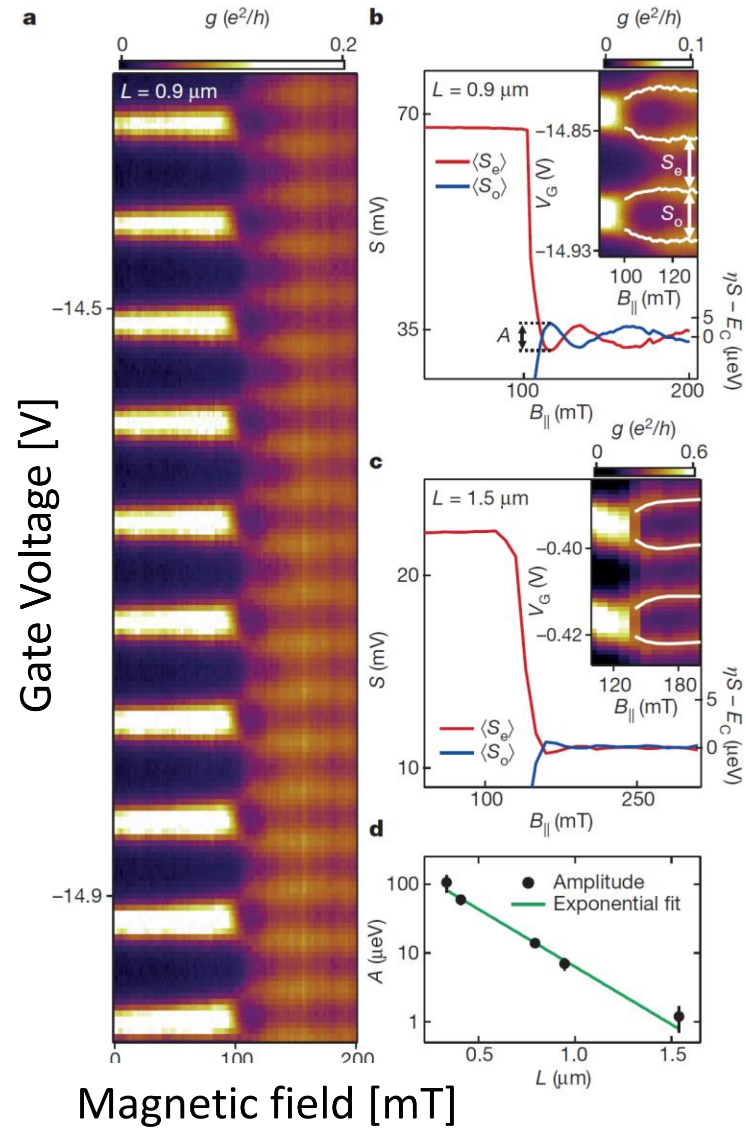
Exponential protection of zero modes in Majorana islands

Nature 2016

S. M. Albrecht[1]*, A. P. Higginbotham[1,2]*, M. Madsen[1], F. Kuemmeth[1], T. S. Jespersen[1], J. Nygård[1], P. Krogstrup[1] & C. M. Marcus[1]







Questions / remarks:

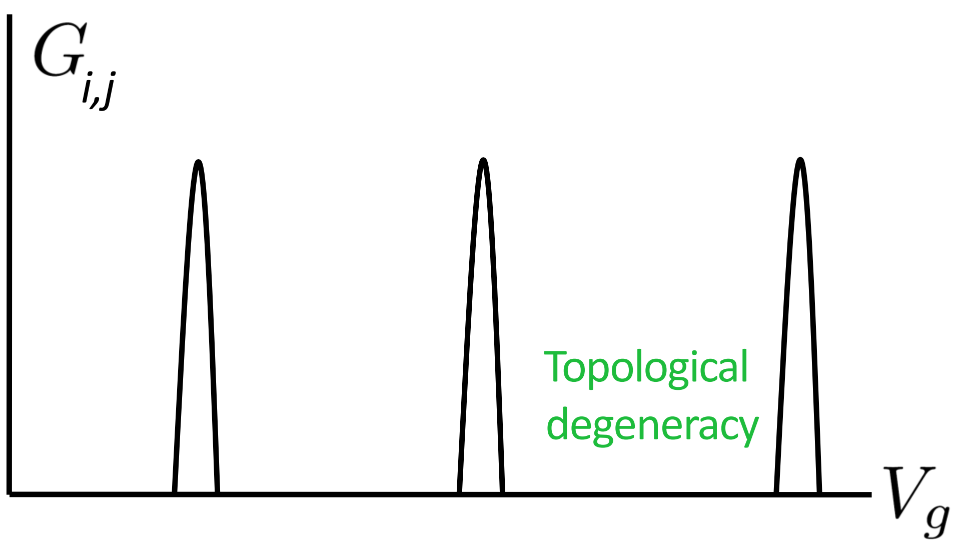
- Not direct evidence of Majorana physics
- Theoretical value of e^2/h peak not reached Fu PRL 2010 , B. van Heck PRB 2016
- Topological degeneracy lifted by Charging energy
- ...

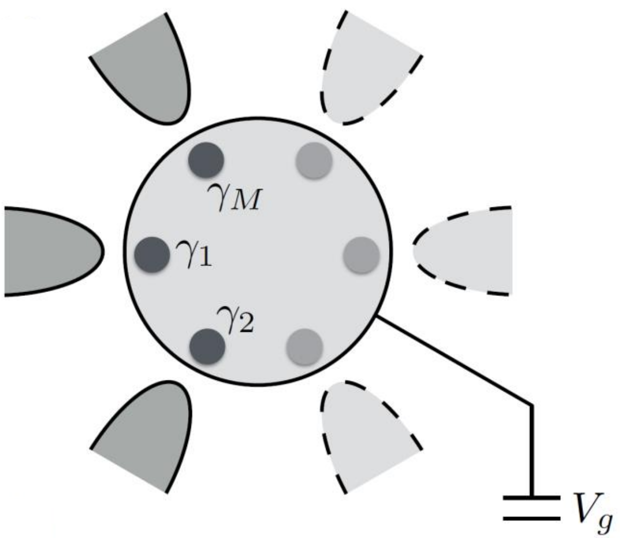
Béri and Cooper(PRL 2012)
Altland and Egger (PRL 2013)
Beri (PRL 2013)

Topological Kondo Effect

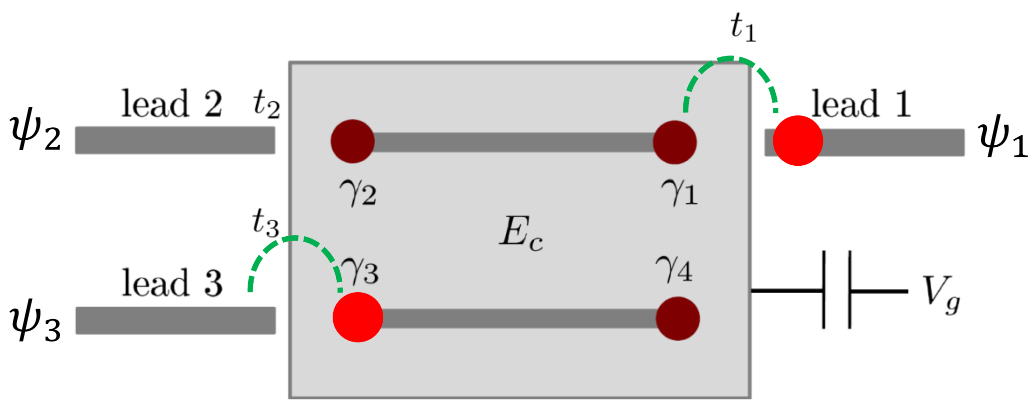
Multi (M)-terminal multi-majorana Devices

- Topological Degeneracy $2^{M/2}/2$ of Coulomb valley
- Kondo type interaction with the leads





Model Hamiltonian



transition between degenerate states $\rightarrow \gamma_3\gamma_1|\Psi\rangle$

effective Kondo Hamiltonian: $H_{\text{eff}} = \sum_{i\neq j}^{M} \lambda_{ij}\psi_i^\dagger\psi_j\gamma_i\gamma_j \qquad \lambda_{ij} \sim \frac{t_i t_j}{E_c}$

Tunnel width $\Gamma = \sum_j \Gamma_j = \sum_j \rho t_j^2$

Kondo temperature $T_K \sim e^{-E_c/\Gamma}$

Results

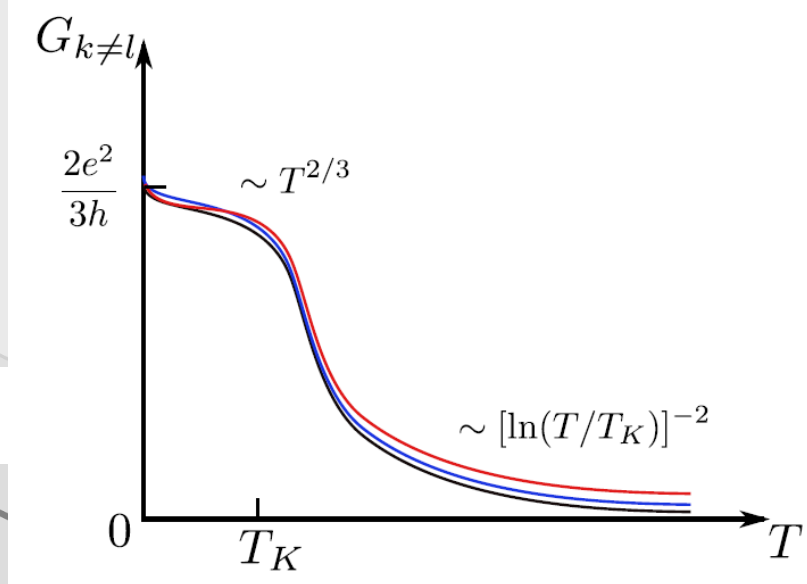
Béri and Cooper(PRL 2012)
Altland and Egger (PRL 2013)
Beri (PRL 2013)

T=0 conductance

$$G_{i\neq j} = \frac{2e^2}{h}\frac{1}{M}$$

Low T conductance fractional power law

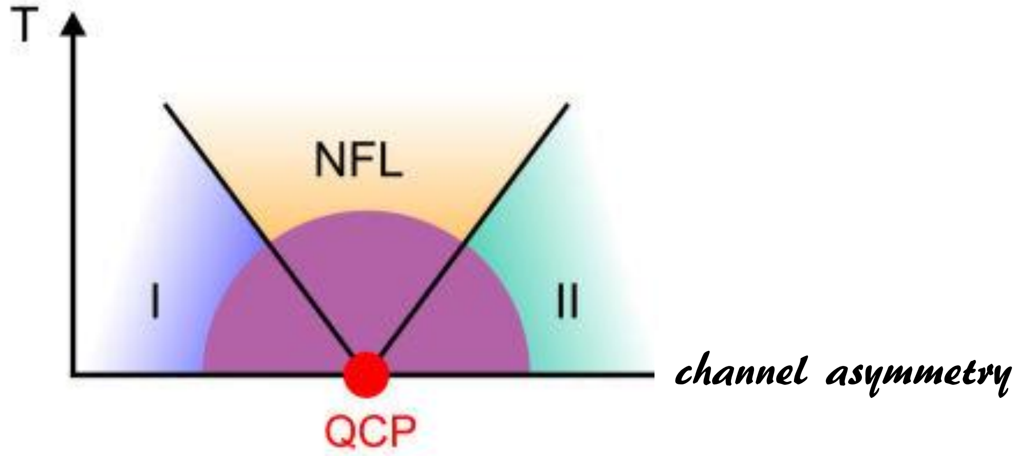
$$G_{ij} = \frac{e^2}{h}\frac{2}{M}\left[1 + CT^{2\left(1-\frac{1}{M}\right)}\right]$$

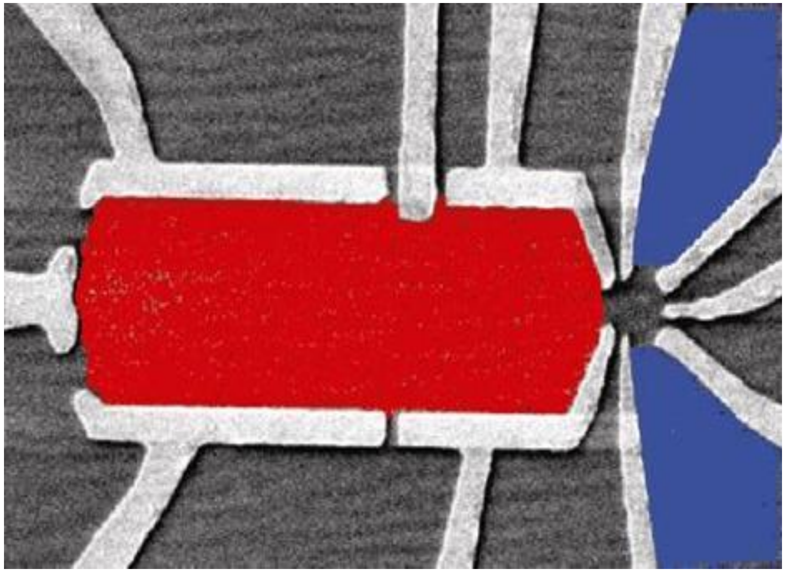


Non-Fermi liquid with interesting degrees of freedom

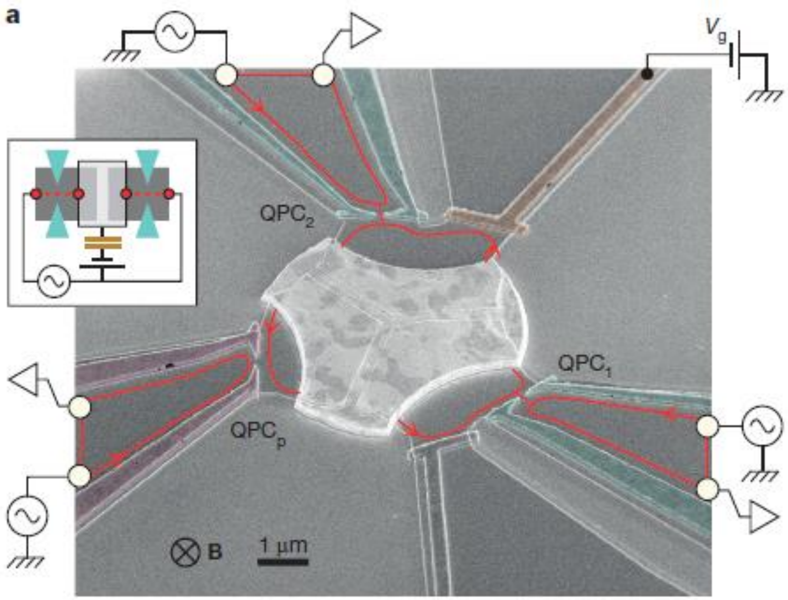
Stable

Top. Kondo Vs. fine-tuned non-Fermi liquids





Potok *et al*., Nature 446, 167 (2007)



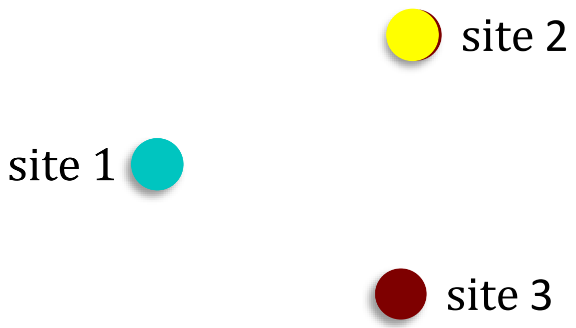
Iftikhar et al, Nature 2015

Altland and Egger (PRL 2013)
Beri (PRL 2013)

Statistical Transmutation

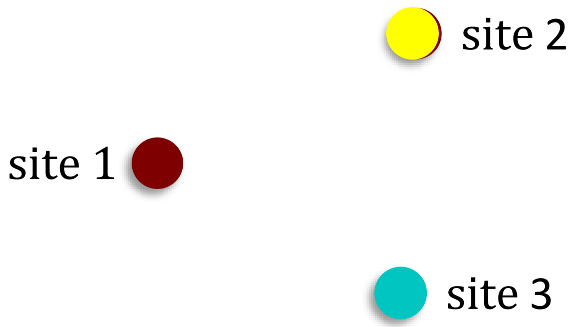
Exchange two identical fermions

$$|\text{initial}> = c_1^{\dagger} c_2^{\dagger} |0>$$



Exchange two identical fermions

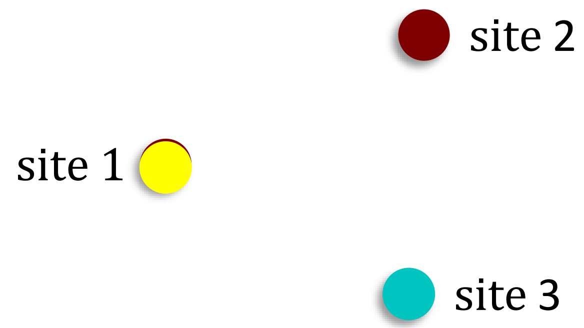
$$|\text{initial} >= c_1^\dagger c_2^\dagger |0>$$



$$c_3^\dagger c_1 \ |\text{initial} >$$

Exchange two identical fermions

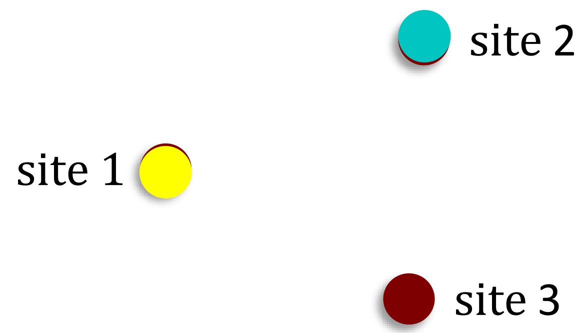
$$|\text{initial}> = c_1^\dagger c_2^\dagger |0>$$



$$c_1^\dagger c_2 \; c_3^\dagger c_1 \; |\text{initial}>$$

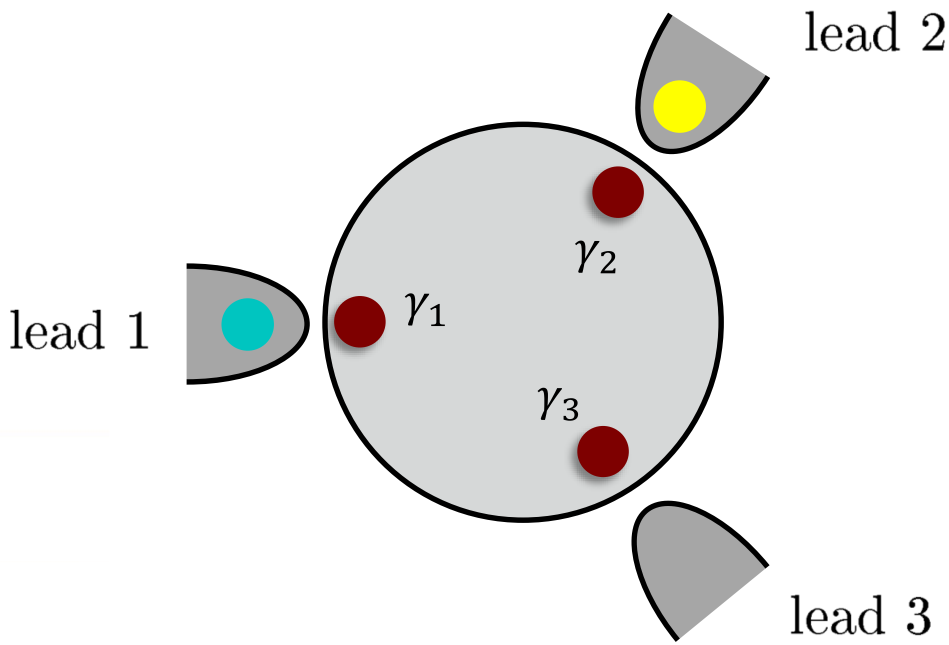
Exchange two identical fermions

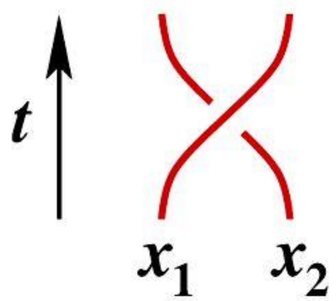
$|\text{initial}> = c_1^\dagger c_2^\dagger |0>$



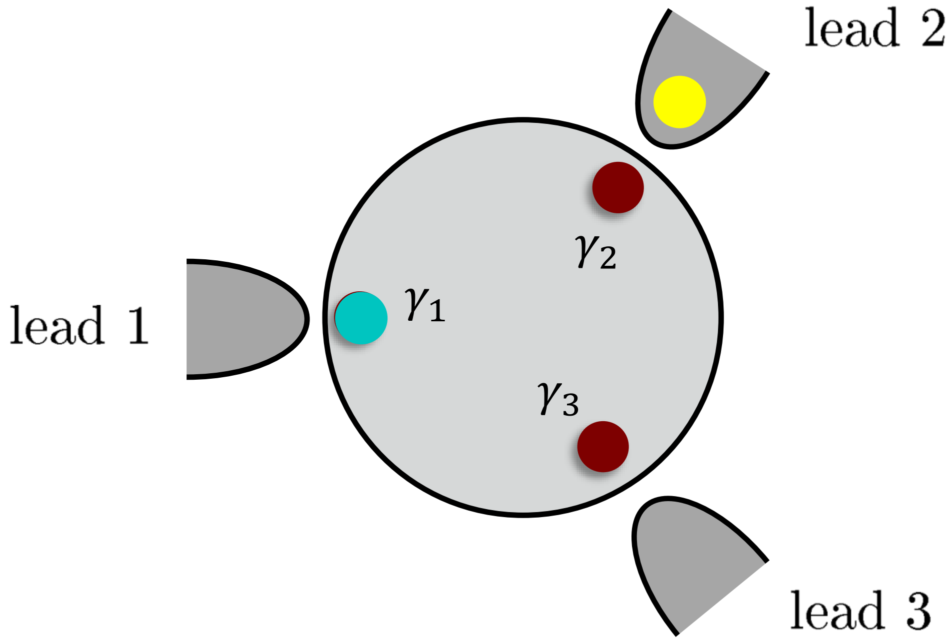
$c_2^\dagger c_3 \; c_1^\dagger c_2 \; c_3^\dagger c_1 \; |\text{initial}> = - \, |\text{initial}>$

Exchange two identical fermions via Majorana box

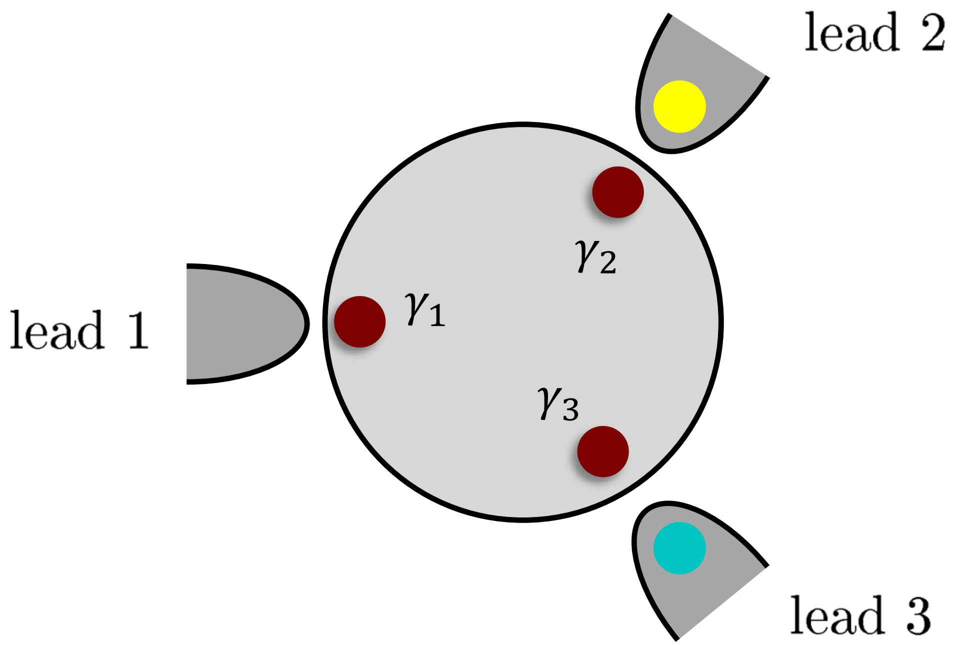




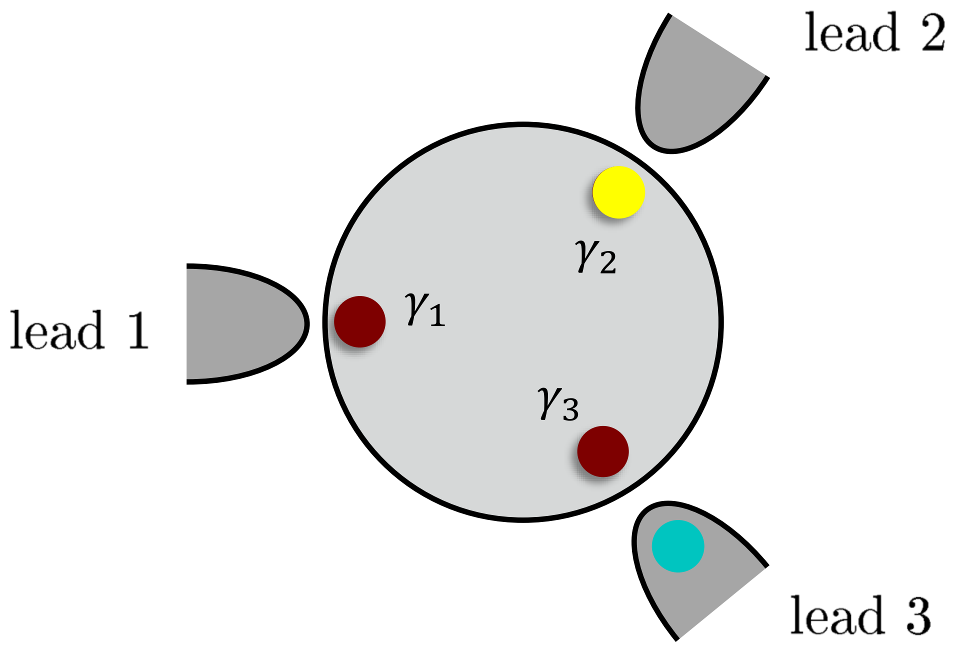
Statistical transmutation



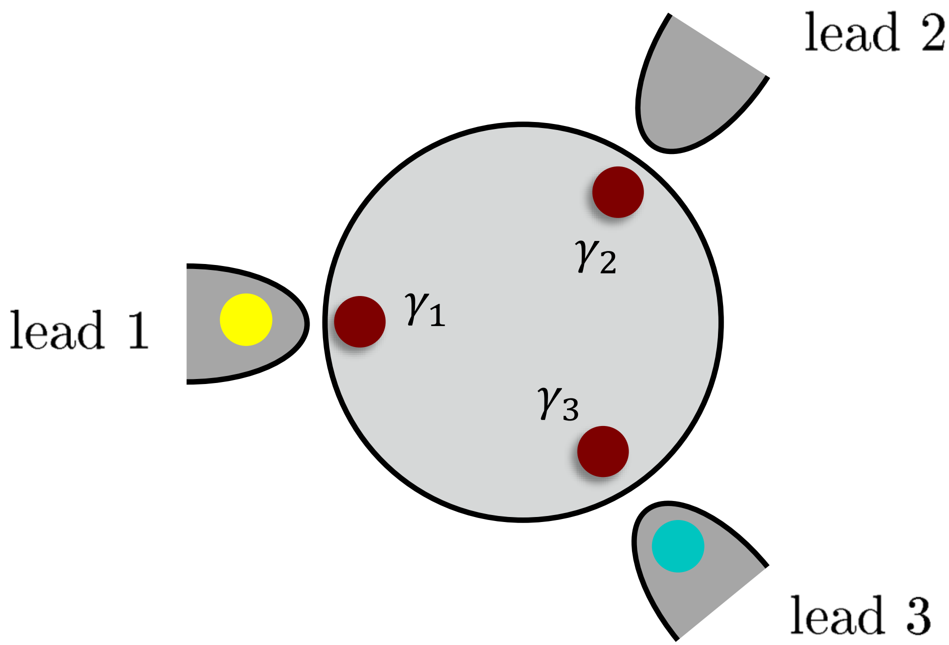
Statistical transmutation



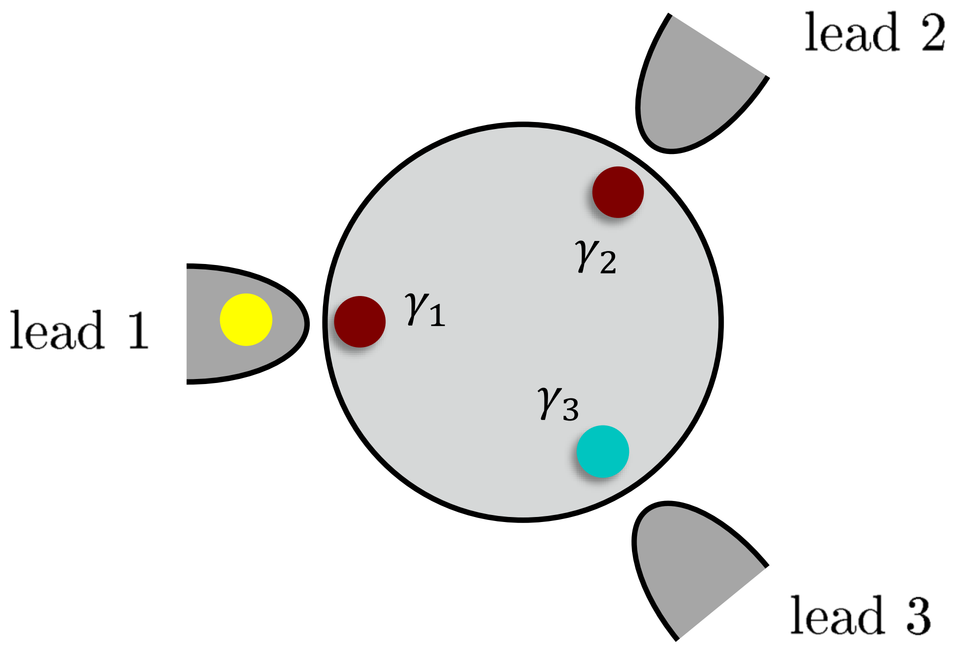
Statistical transmutation



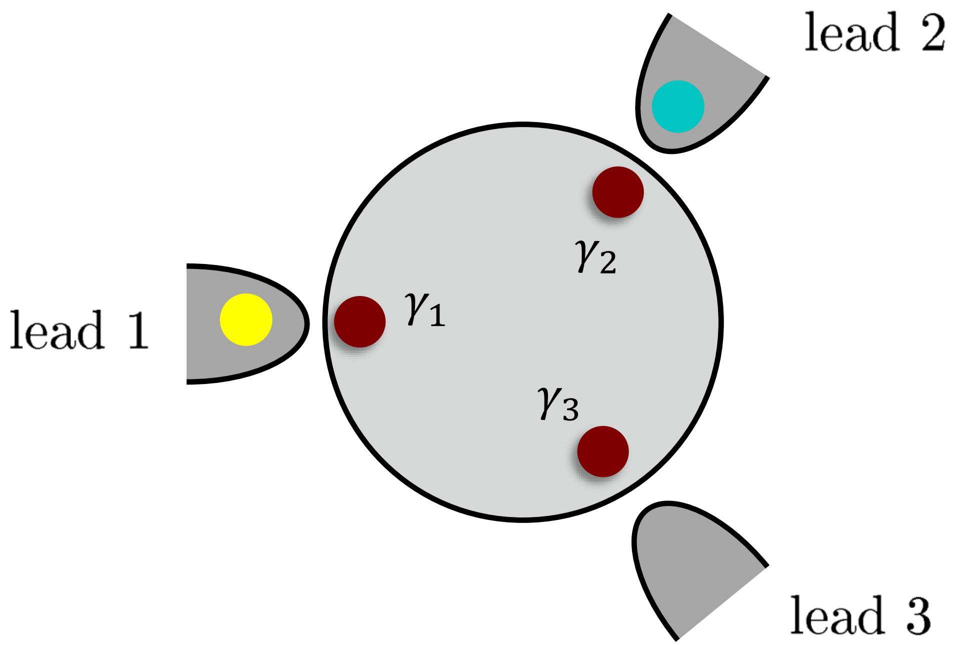
Statistical transmutation



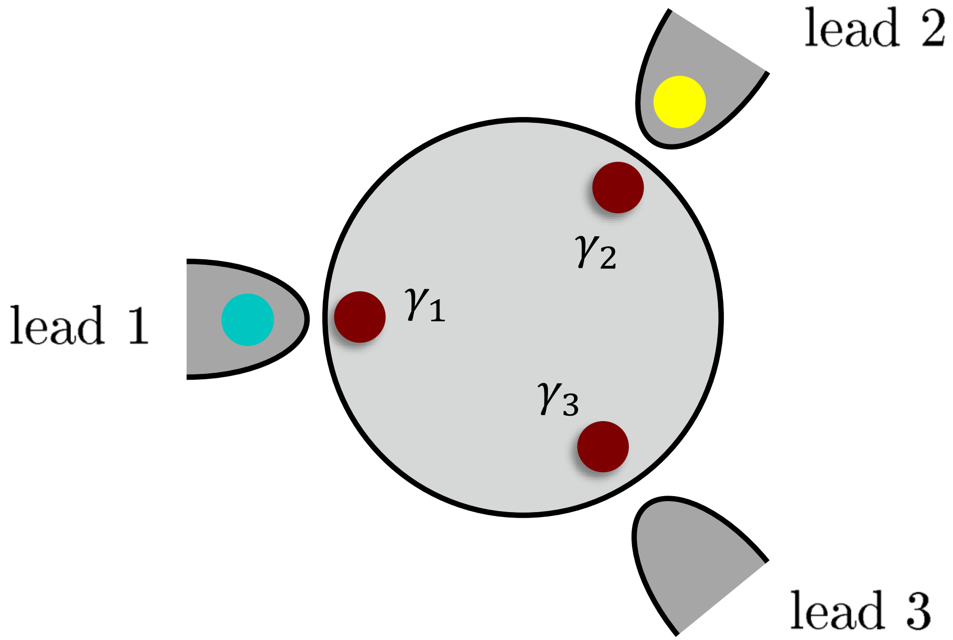
Statistical transmutation



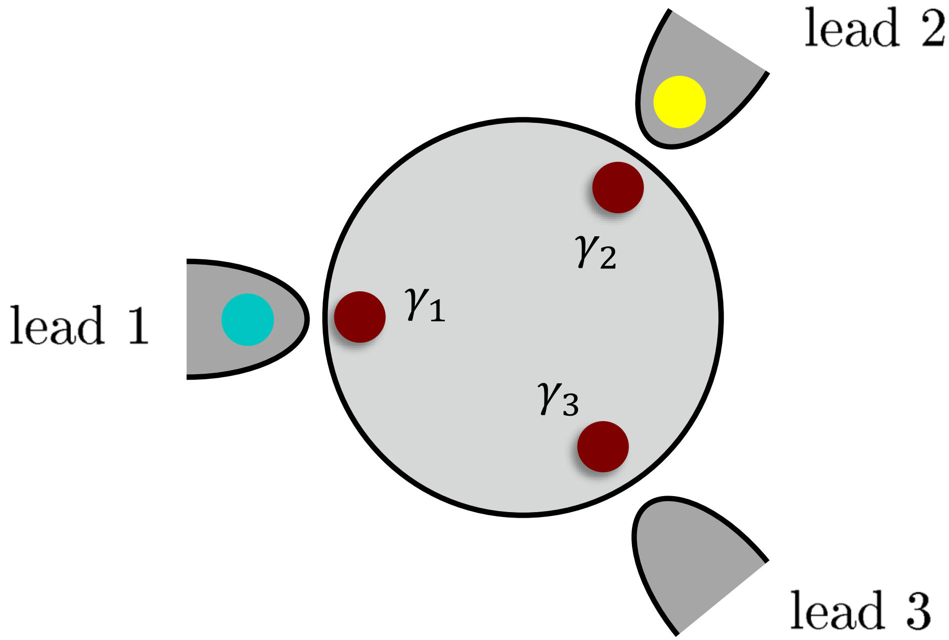
Statistical transmutation



Statistical transmutation



Statistical transmutation



$$\psi_2^\dagger \gamma_2 \gamma_3 \psi_3 \psi_1^\dagger \gamma_1 \gamma_2 \psi_2 \psi_3^\dagger \gamma_3 \gamma_1 \psi_1 \,|\text{initial}> = +\,|\text{initial}>$$

Majorana fermions have totally disappeared -
Instead the tunneling electrons become bosons

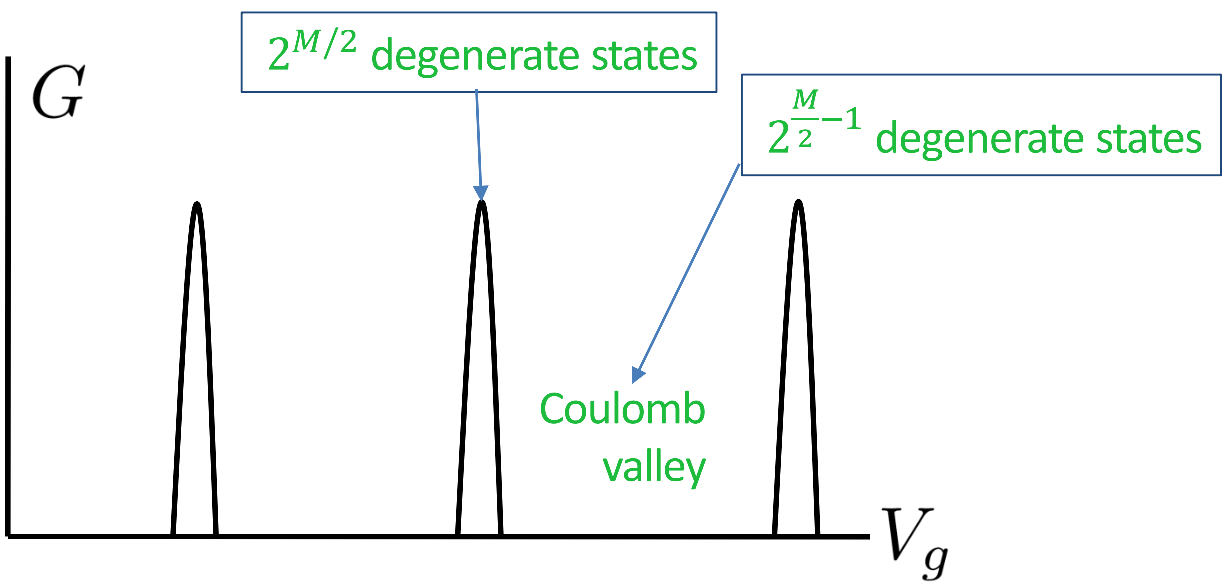
Altland and Egger (PRL 2013)
Beri (PRL 2013)

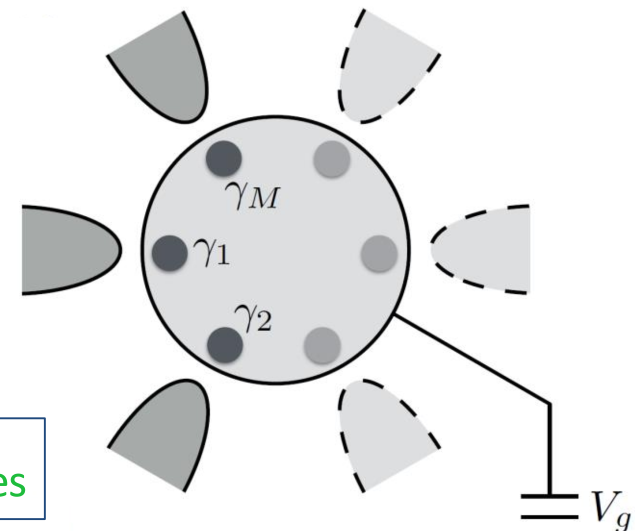
Michaeli, Landau, E.S, Fu, arXiv:1608.00581

Topological Kondo effect at charge degeneracy

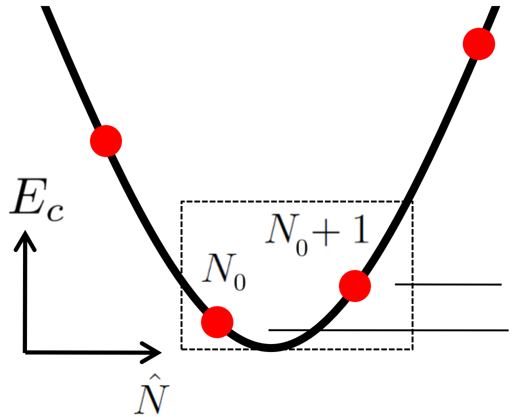
Majorana Physics at the Coulomb peak

- Extra charge degeneracy
- New type of Kondo interaction
- Most relevant experimentally

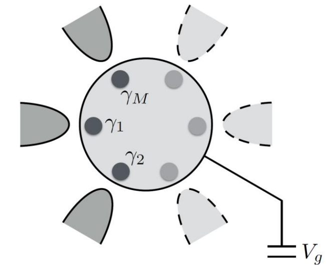




Charge-Majorana Kondo model



$$\Delta_g = E_c(N_0 - n_g + 1/2)$$



large E_c $\longrightarrow$ projecting to 2 states: $\sigma_z = +1 \longrightarrow N_0 + 1$

$$\sigma_z = -1 \longrightarrow N_0$$

$$H = \sum_{i=1}^{M} t_i \psi_i^\dagger \gamma_i \sigma^- + \text{h.c} + \frac{\Delta_g}{2}\sigma_z$$

Results at charge degeneracy

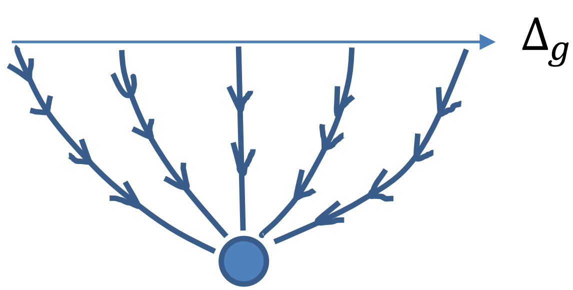
- We solved the model via mapping to a quantum brownian motion in a honeycomb potential

- T=0 conductance: $$G_{i\neq j} = \frac{2e^2}{h}\frac{1}{M}$$

- Low T conductance fractional power law $$G_{ij} = \frac{e^2}{h}\frac{2}{M}\left[1 + CT^{2\left(1-\frac{1}{M}\right)}\right]$$

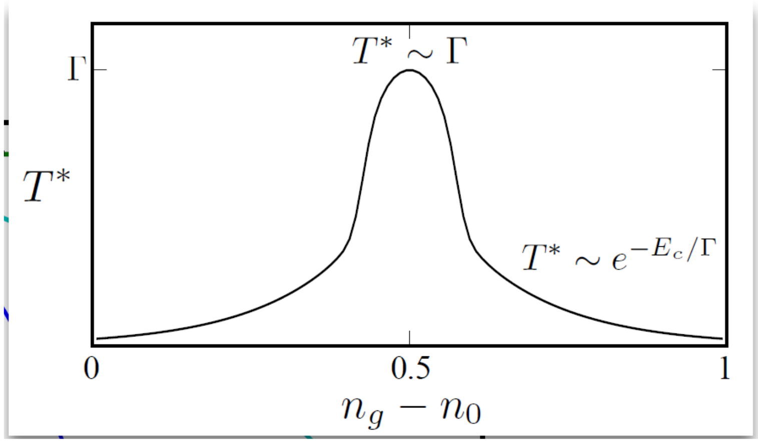
- The same result of topological Kondo away from charge degeneracy

- Conclude: system flows to one fully stable strong coupling fixed point

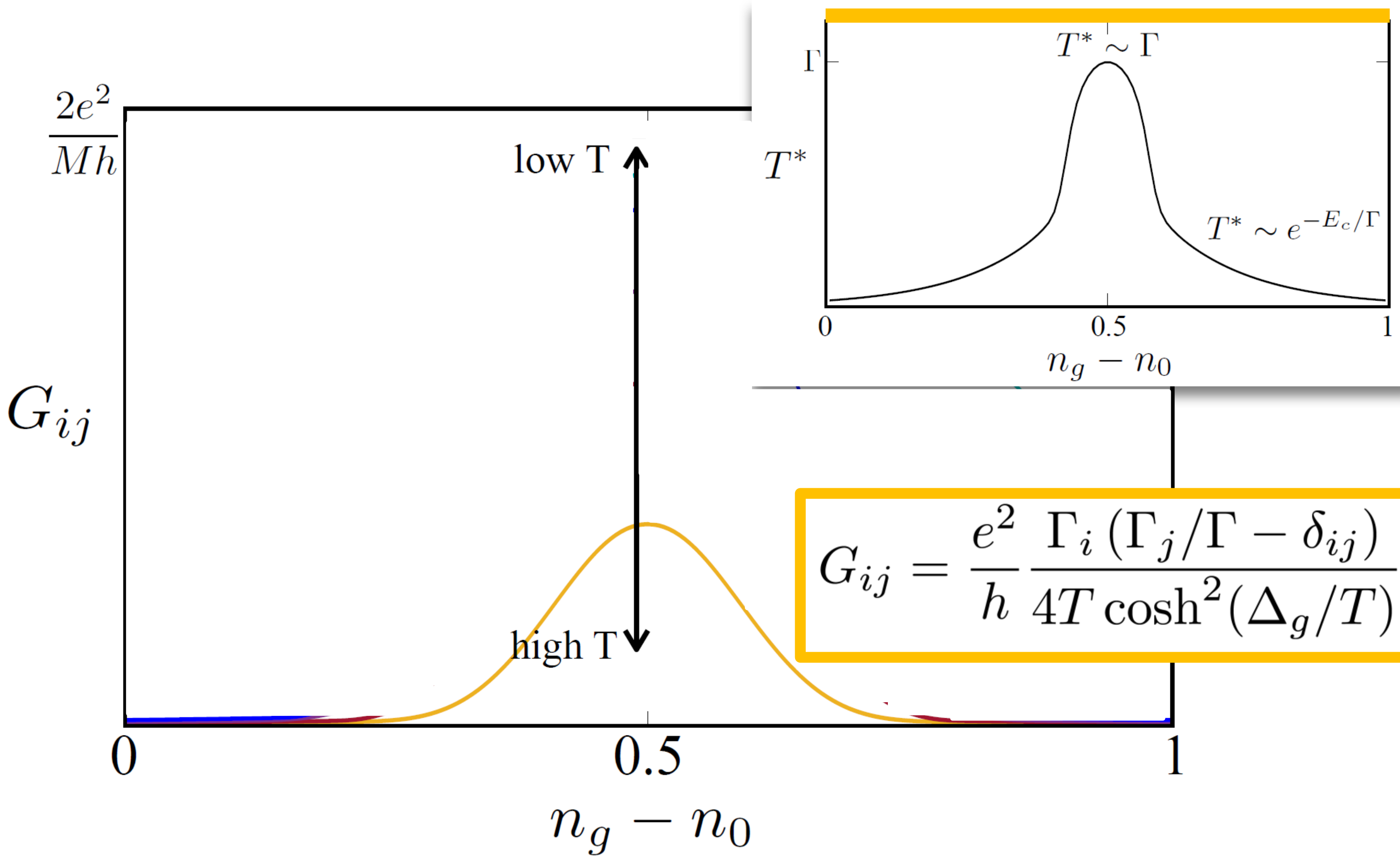


- Crucial advantage: high Kondo temperature.

Kondo energy scale

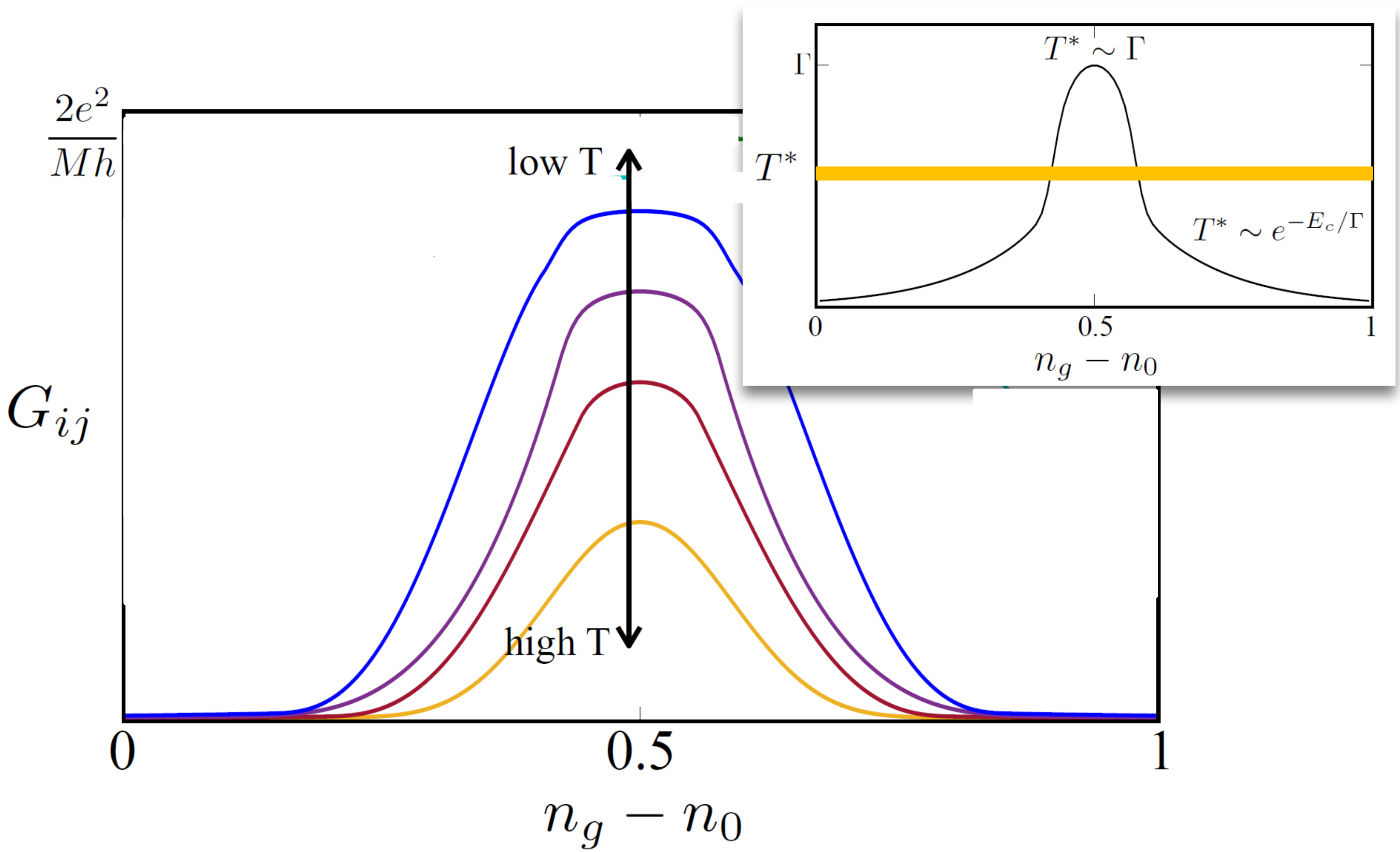


Development of topological Kondo state



$$G_{ij} = \frac{e^2}{h} \frac{\Gamma_i \left(\Gamma_j/\Gamma - \delta_{ij}\right)}{4T \cosh^2(\Delta_g/T)}$$

Development of topological Kondo state



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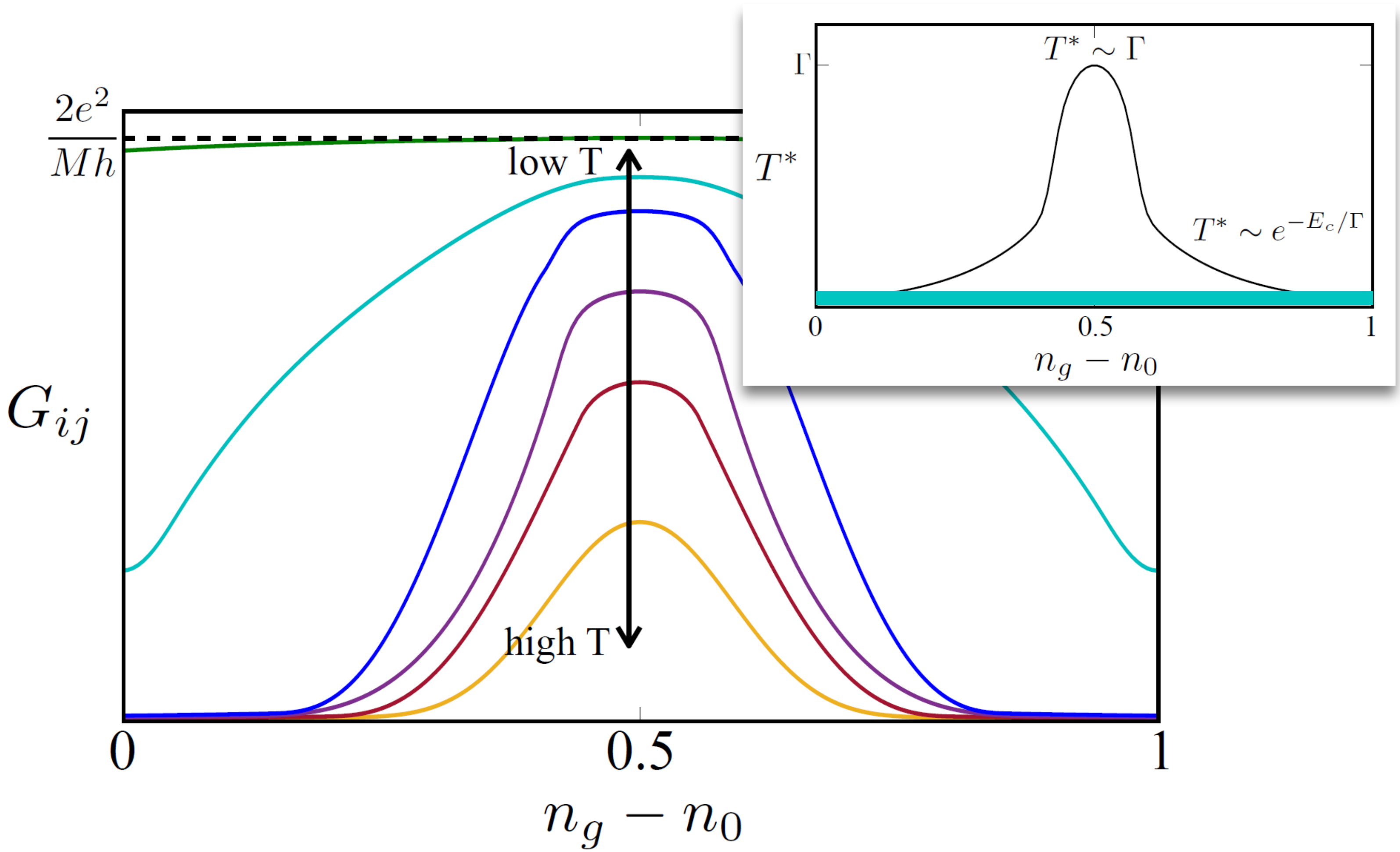


Landau, E.S PRB 2017

Coulomb Blockade vs. Josephon Tunneling

Floating vs. Grounded top. superconductors

SCIENCE VOL 336 25 MAY 2012

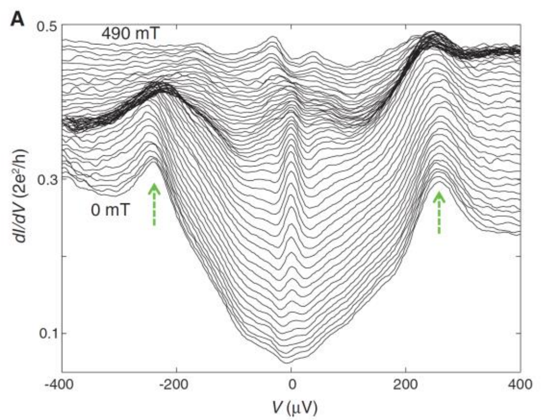
Signatures of Majorana Fermions in Hybrid Superconductor-Semiconductor Nanowire Devices

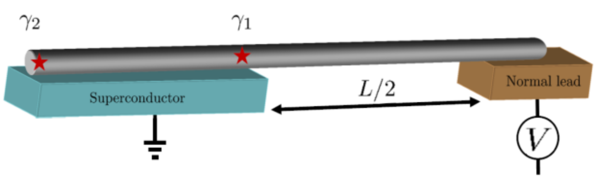
V. Mourik,[1*] K. Zuo,[1*] S. M. Frolov,[1] S. R. Plissard,[2] E. P. A. M. Bakkers,[1,2] L. P. Kouwenhoven[1]†

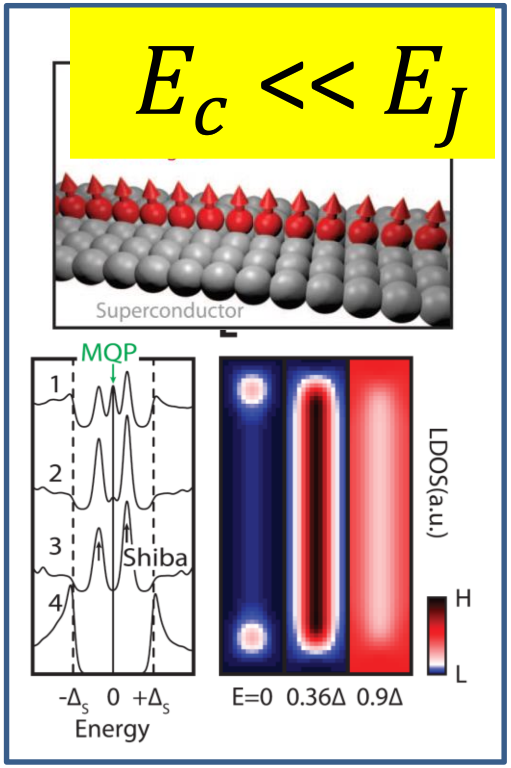
nature physics ARTICLES
PUBLISHED ONLINE: 11 NOVEMBER 2012 | DOI: 10.1038/NPHYS2479

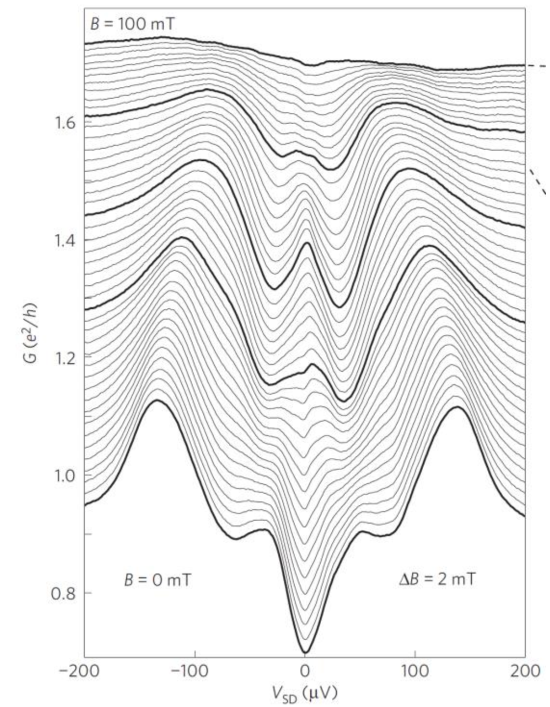
Zero-bias peaks and splitting in an Al-InAs nanowire topological superconductor as a signature of Majorana fermions

Anindya Das†, Yuval Ronen†, Yonatan Most, Yuval Oreg, Moty Heiblum* and Hadas Shtrikman





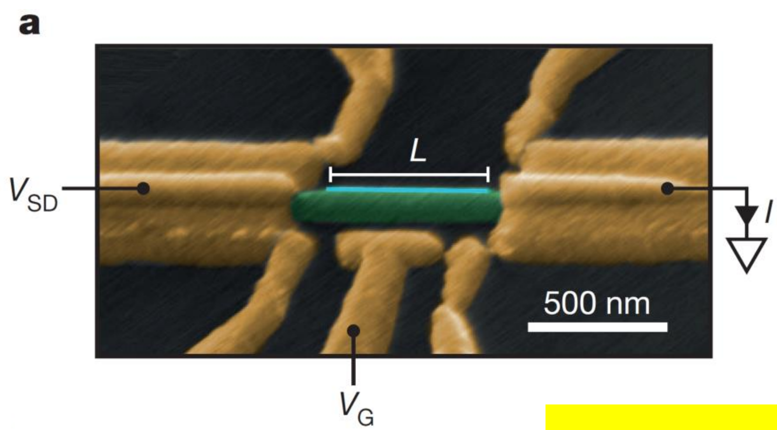


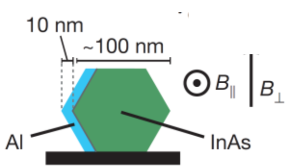


Exponential protection of zero modes in Majorana islands

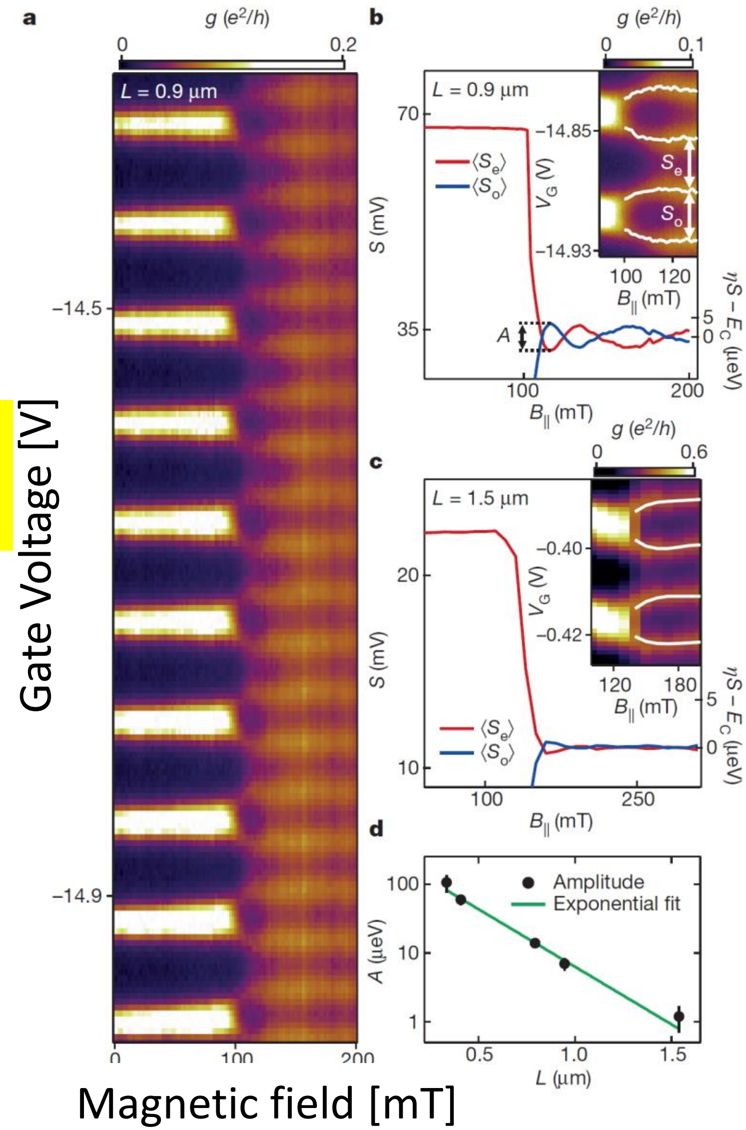
Nature 2016

S. M. Albrecht[1]*, A. P. Higginbotham[1,2]*, M. Madsen[1], F. Kuemmeth[1], T. S. Jespersen[1], J. Nygård[1], P. Krogstrup[1] & C. M. Marcus[1]





$$E_c >> E_J$$

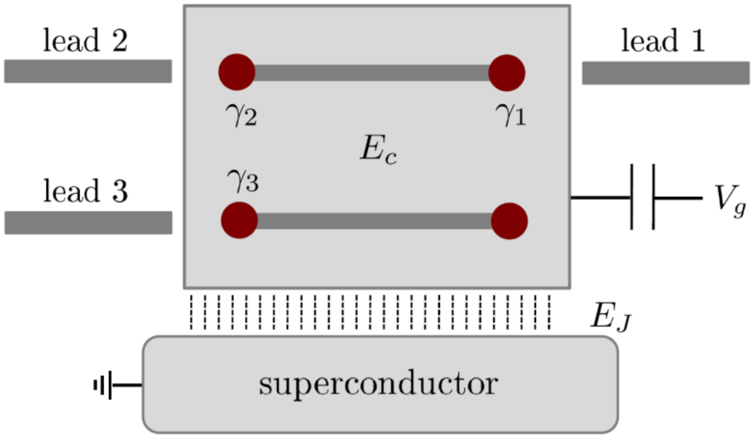


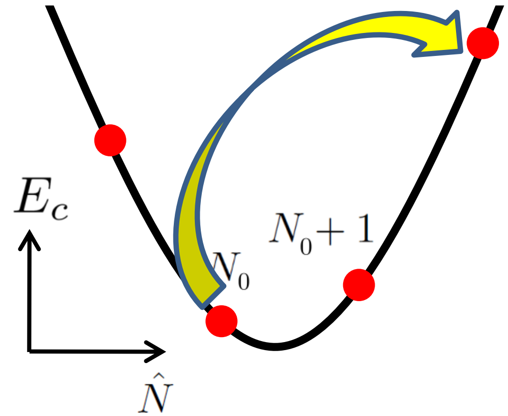
Mid-way between grounded superconductors and floating superconductors?

New states?

Zazunov et. al. PRL 2014

Josephson-Majorana box model





$$H_c = E_c\left(\mathcal{N} - n_g\right)^2 \qquad H_\mathrm{J} = -E_J \cos(\phi)$$

$$H_\mathrm{T} = \sum_{j=1}^{M} t_j \psi_j^\dagger(0)\gamma_j e^{-i\phi/2} + \mathrm{H.c.},$$

Altland Egger. PRL 2013

Topological Kondo — Intermediate phase? — Free Majorana 0-bias peak

$E_J = 0$ ⟶ $E_J >> E_c$

Floating — **Grounded**

Electronic processes & intermediate phase

cotunneling

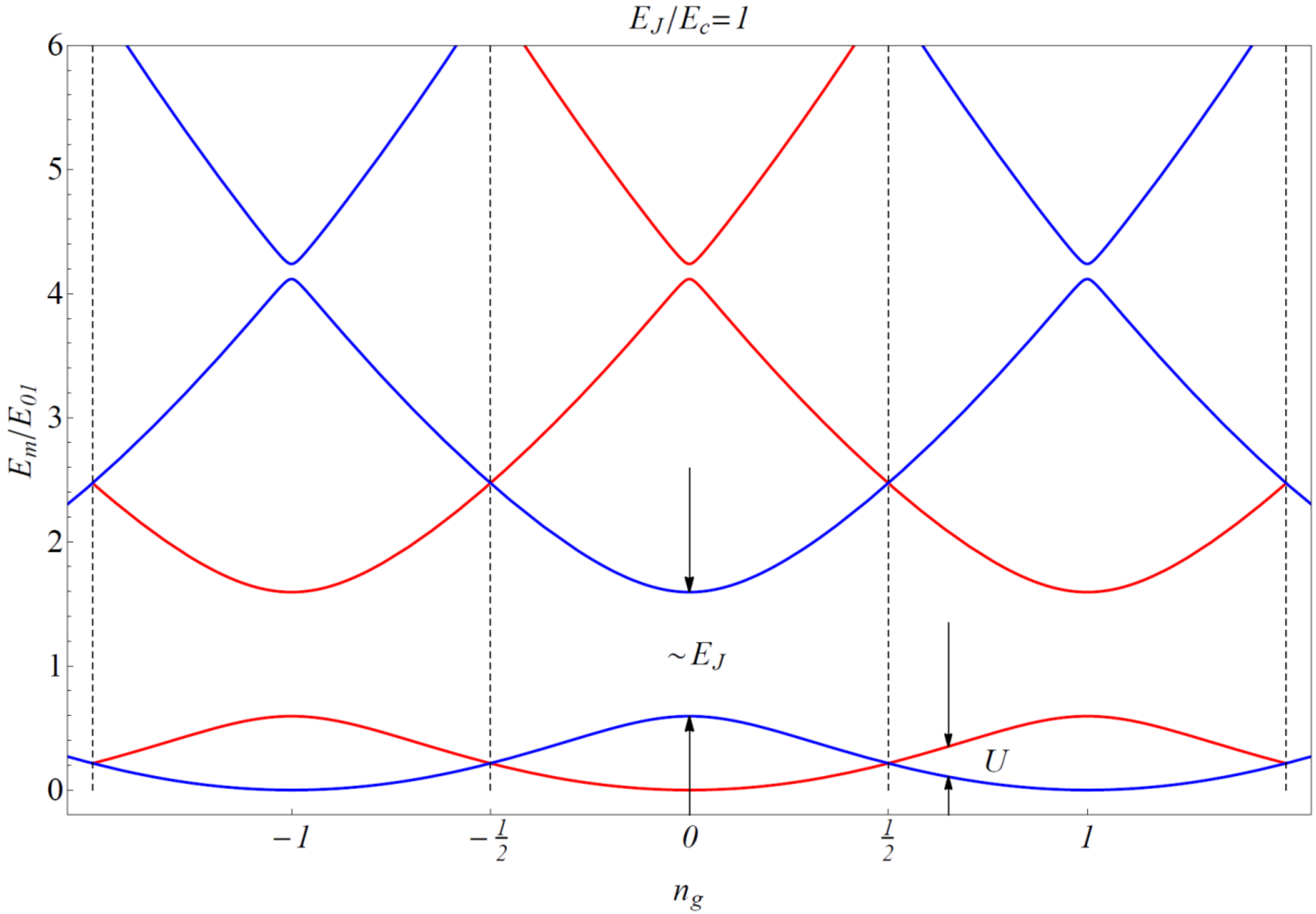
non-local Andreev processes

$$H_{\text{eff}} = \frac{t^2}{E_c} \sum_{i \neq j} \gamma_j \gamma_i \left[\psi_i^\dagger \psi_j - \frac{3E_J}{2E_c} \psi_i^\dagger \psi_j^\dagger \right] + \text{h.c.}$$

- No Josephson coupling → regular cotunneling
- Josephson coupling → non-local Andreev processes
- Large Josephson coupling → parity eigenstates survive

Parity interaction

Eigenstates of $H_c + H_J$ $\quad H_c = E_c(\mathcal{N} - n_g)^2 \quad H_J = -E_J\cos(\phi)$



U: a residual interaction in the strongly charge fluctuating regime

Electronic processes & intermediate phase

cotunneling

non-local Andreev processes

$$H_{\text{eff}} = \frac{t^2}{E_c} \sum_{i \neq j} \gamma_j \gamma_i \left[\psi_i^\dagger \psi_j - \frac{3E_J}{2E_c} \psi_i^\dagger \psi_j^\dagger \right] + \text{h.c.}$$

- No Josephson coupling → regular cotunneling
- Josephson coupling → non-local Andreev processes
- Large Josephson coupling → parity eigenstates survive

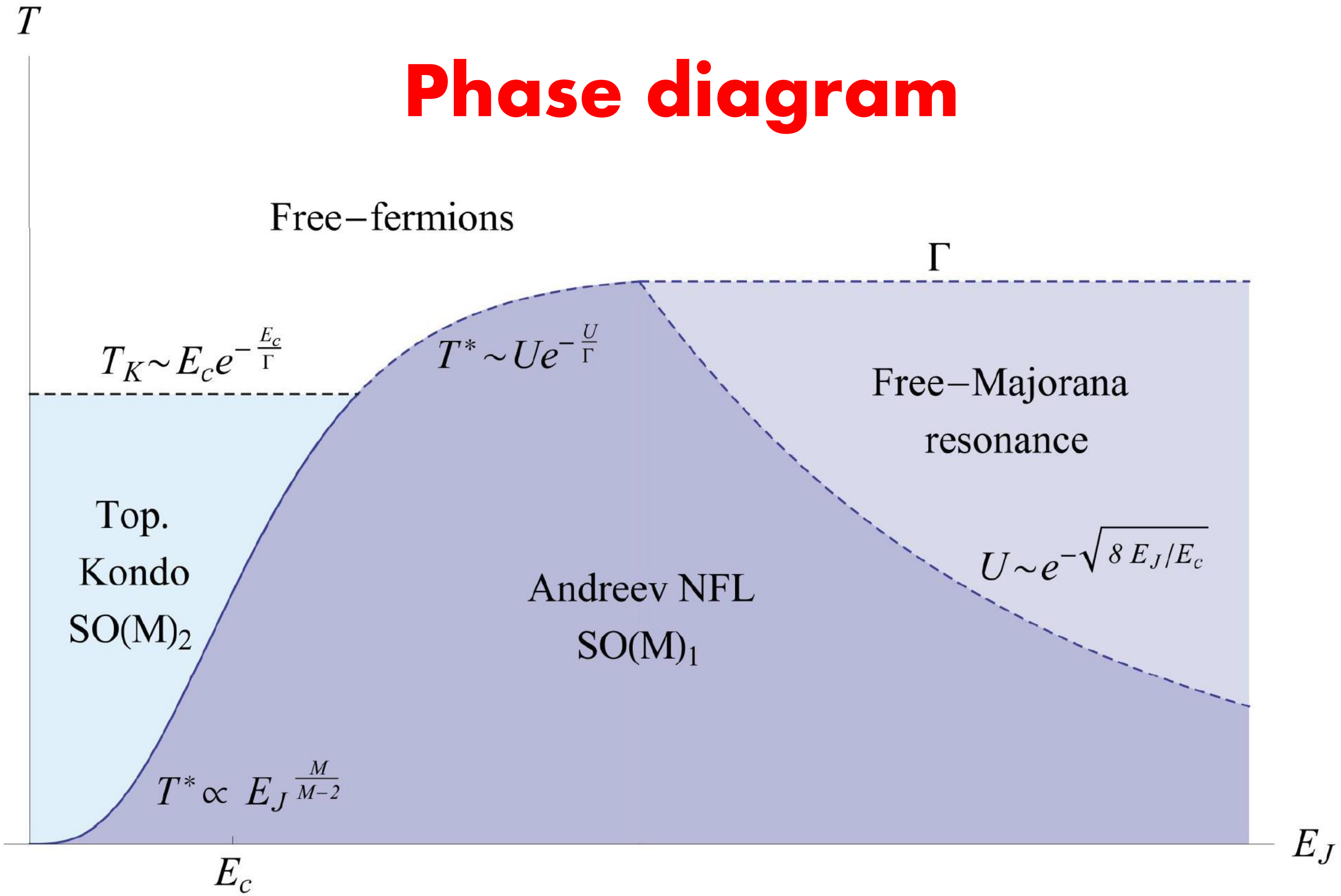
- Electronic process for T<< U are cotunneling of Majoranas
- $\rho_i = (\psi_i - \psi_i^\dagger)/\sqrt{2}$

$$H_{\text{eff}} = \sum_{i \neq j} J_{ij} \rho_i(0) \rho_j(0) \gamma_i \gamma_j$$

SO(M)$_1$ Kondo model

- Equivalent for M=3 to two-channel Kondo Tsvelik PRL 2013 , Zazunov 2014
- At very large Josephson coupling U is exponentially small → No interaction between different Majoranas

Phase diagram

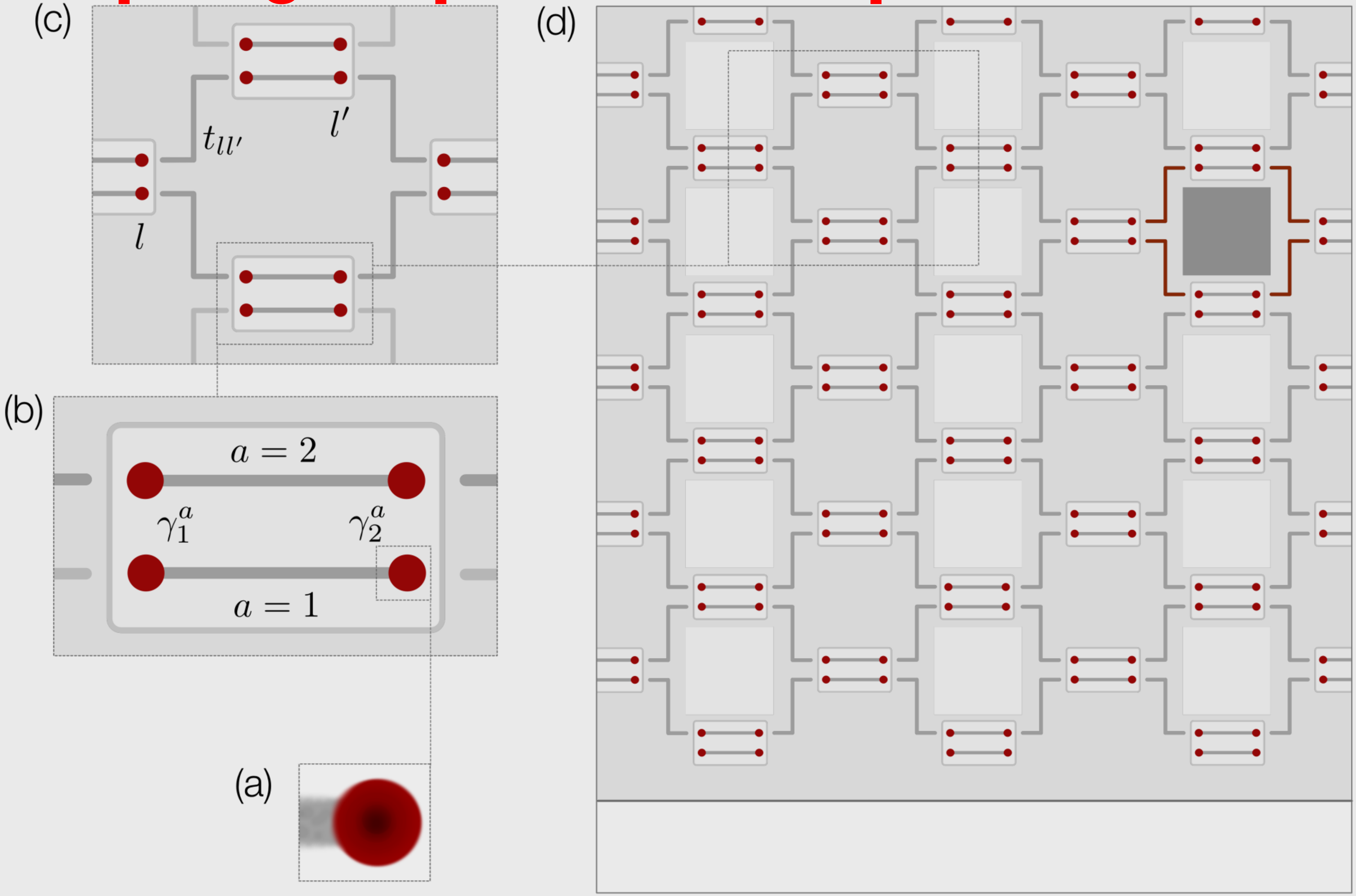


- Top. Kondo unstable to Josephson coupling
- New NFL state has different conductance $G_{ij} = \frac{2e^2}{h}(\delta_{ij} + A_{ij} T^{2\Delta_M - 2})$

$$\Delta_M = \frac{3}{2} \text{ for } M = 3, \text{ or } \Delta_M = 2 \text{ for } M > 3$$

Summary and Outlook

Topological phases of Majorana fermions



See talks by Karsten Flensberg and Stephan Plugge

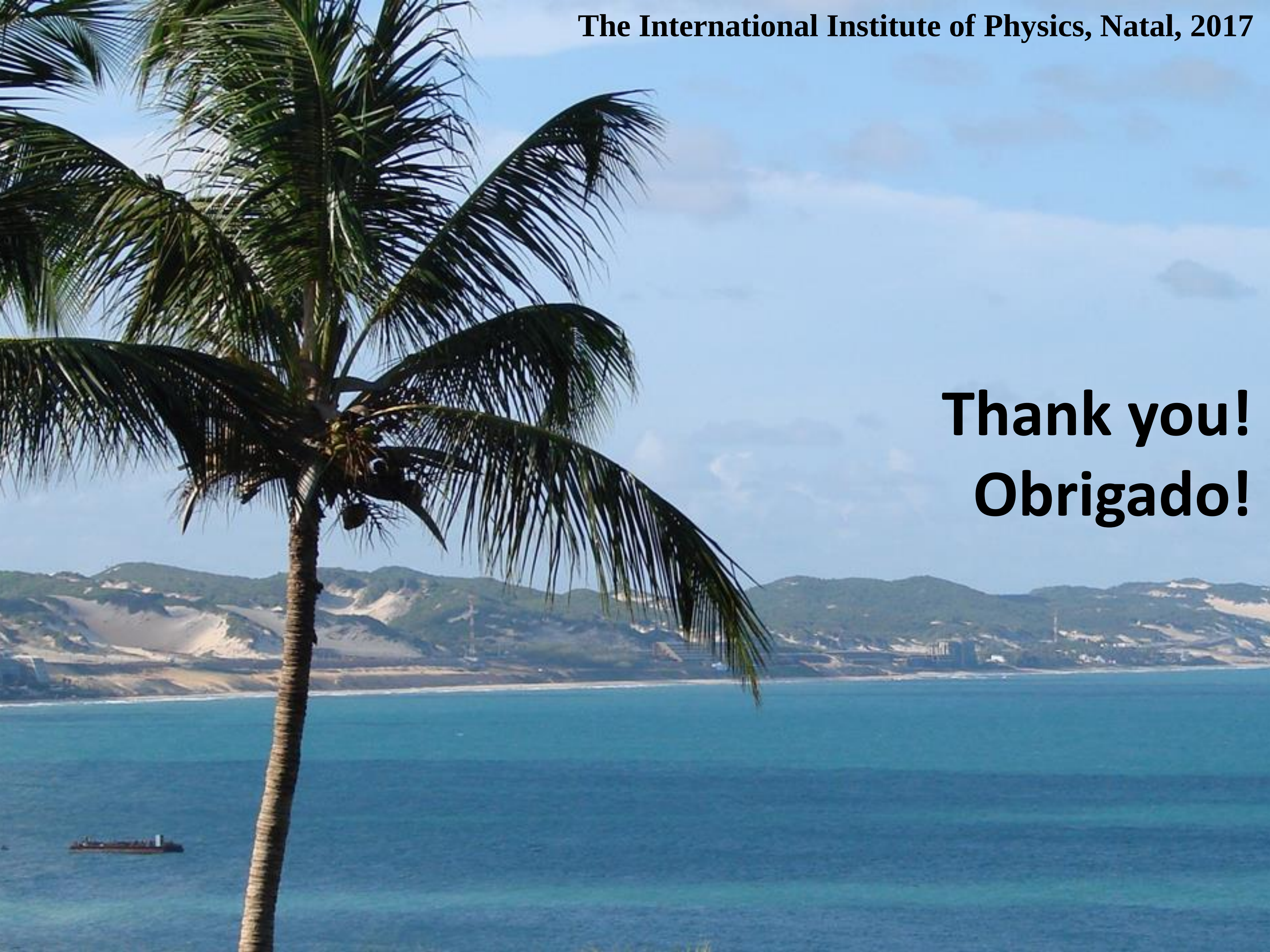
Summary

We might have Majorana fermions at our disposal -
It is now time to build interesting states!

- Strongly interesting topological Kondo states
  - non-Fermi liquid states
  - High energy scale
  - Highly stable
  - New phases SO(M)1 states facilitated by Josephson coupling.
- Strongly interacting bulk states and applications to quantum computation

L. A. Landau, S. Plugge, E. Sela, A. Altland, S. M. Albrecht, R. Egger, Phys. Rev. Lett. 116, 050501 (2016)
K. Michaeli, L. A. Landau, E. Sela, L. Fu, arXiv:1608.00581
L. A. Landau, E. Sela, PRB 2017

The International Institute of Physics, Natal, 2017



Thank you!
Obrigado!