Center for Computational Relativity and Gravitation

NUMERICAL GENERATION OF VECTOR POTENTIALS FOR GENERAL RELATIVITY SIMULATIONS

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MAGNETIC FIELDS IN THE UNIVERSE VI

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NUMERICAL RELATIVITY

HARM3D

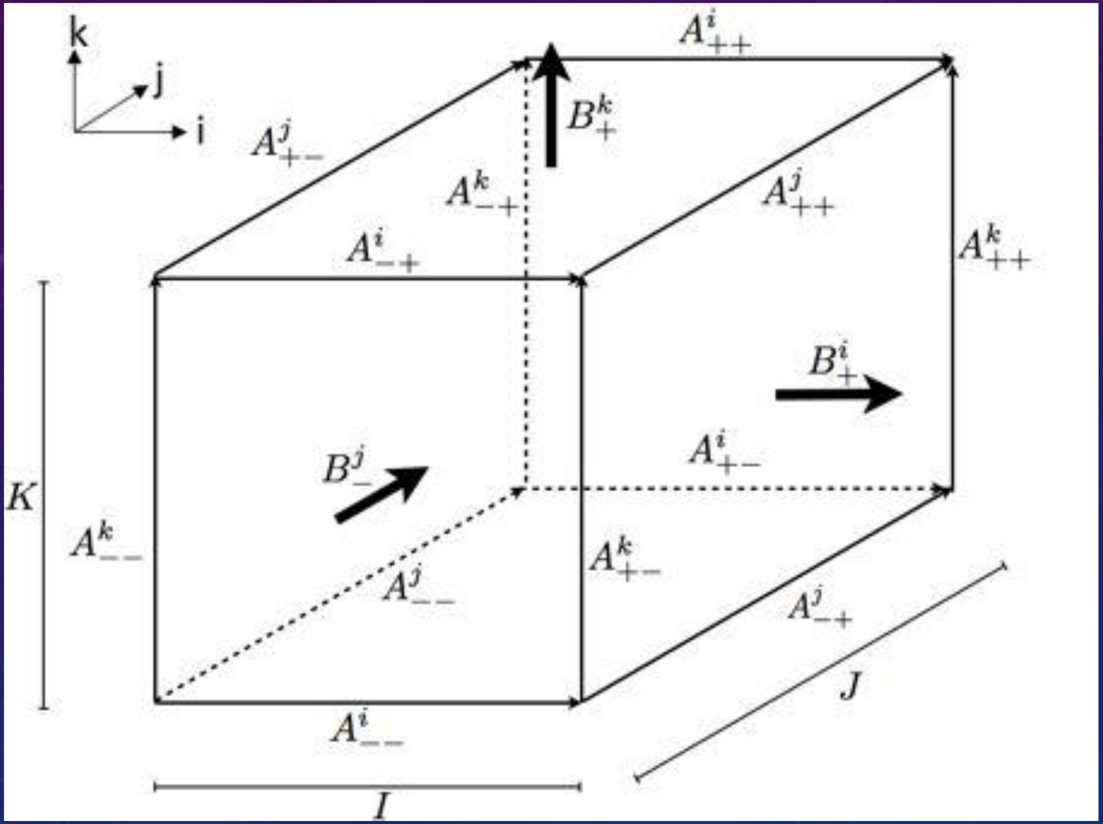
- Initial inspiral
- Approximations
- Evolves the magnetic field B

IllinoisGRMHD

- Merger
- Full Relativity
- Evolves the magnetic vector potential A

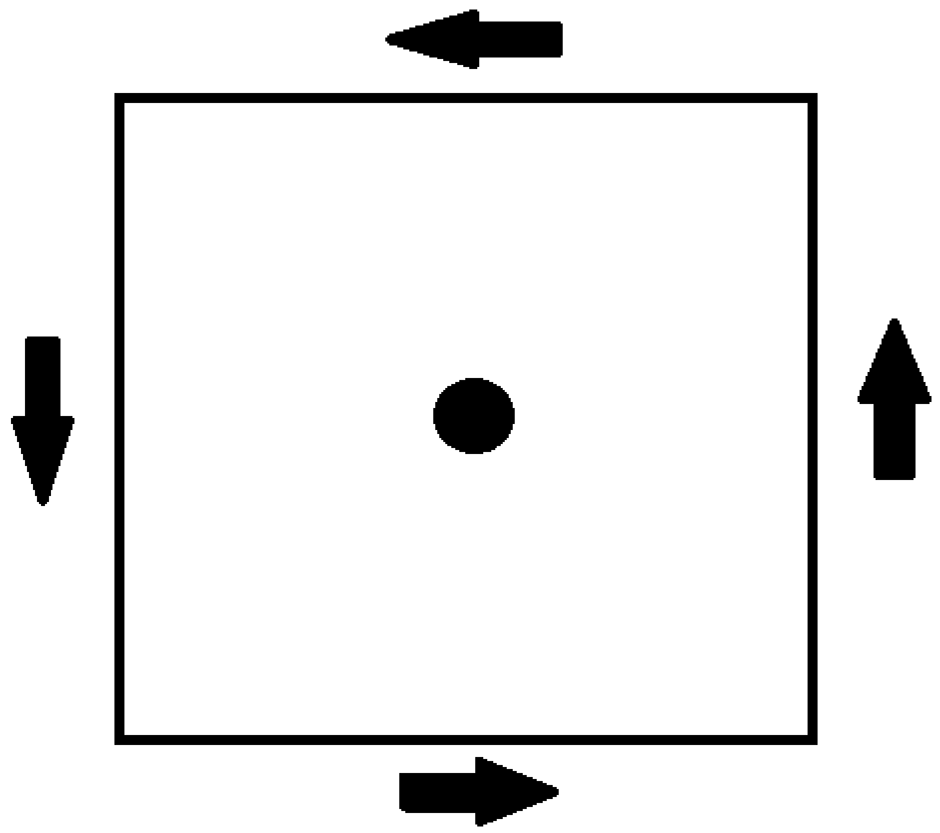
Gammie, McKinney, & Tóth 2003,
Etienne et al. 2012

A STAGGERED CELL



- Different quantities are defined at different locations in the grid
- Cell centers, face centers, edge centers, vertices

STAGGERED GRID



CALCULATING THE CURL

- I use a *centered* finite-differencing stencil to calculate the curl:

$$B^i_{\pm} = \frac{A^k_{\pm+} - A^k_{\pm-}}{J} - \frac{A^j_{+\pm} - A^j_{-\pm}}{K}$$

- with equivalent expressions for the other components of the magnetic field

CELL-BY-CELL GENERATION

- Calculate the 12 A-field values of a cell using
  - the 6 B-field values in that cell
  - any A-field values from previously determined cells
- Seek to maximize symmetry of solution
- In cases where multiple solutions exist, attempt to minimize cell-to-cell variation in A

RIGHT NOW

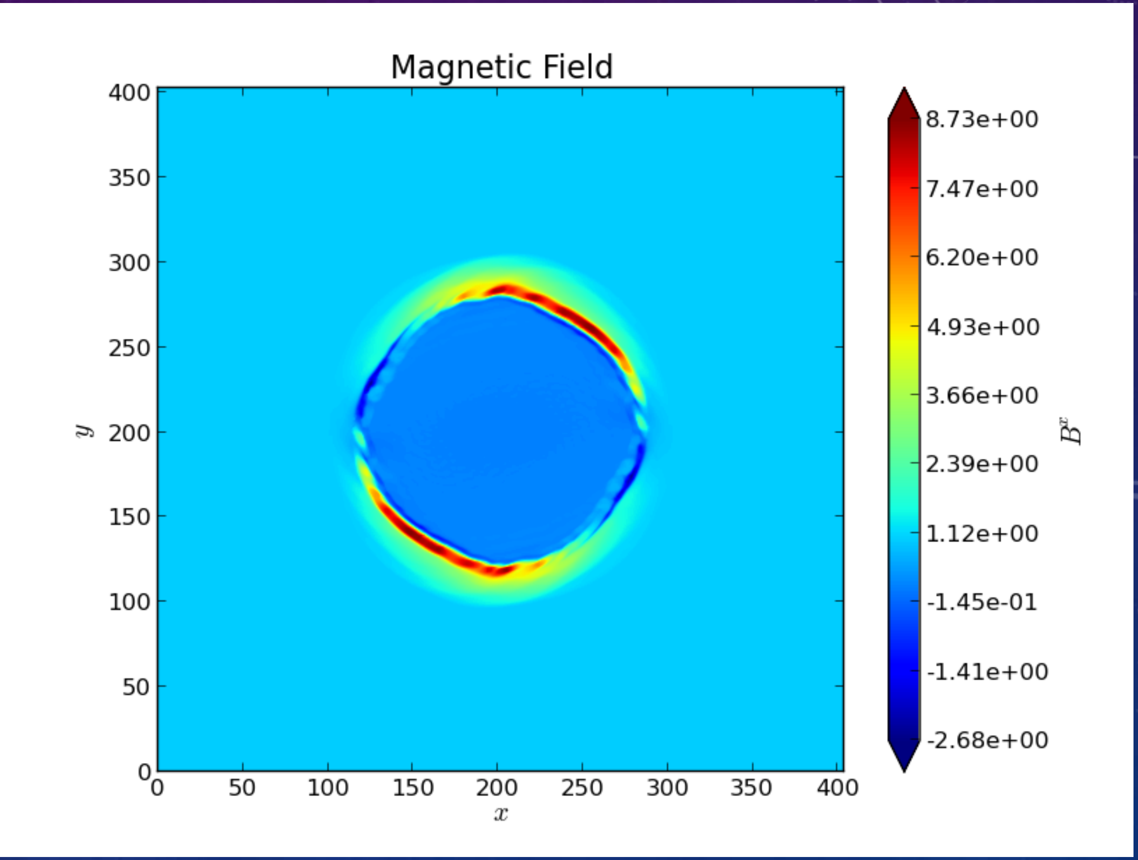
- Interpolates the data into my staggered coordinates
- Removes any divergence in B, if any exists
- Builds the A field
- Transforms into Coulomb gauge: $\nabla \cdot A = 0$
- Performs smoothness tests
- Ensures that the curl of A is B
- It works!

GLOBAL LINEAR ALGEBRA

- Treat the curl operator as a matrix
- Include Coulomb gauge conditions
- More symmetric than the cell-by-cell method
- Also works!

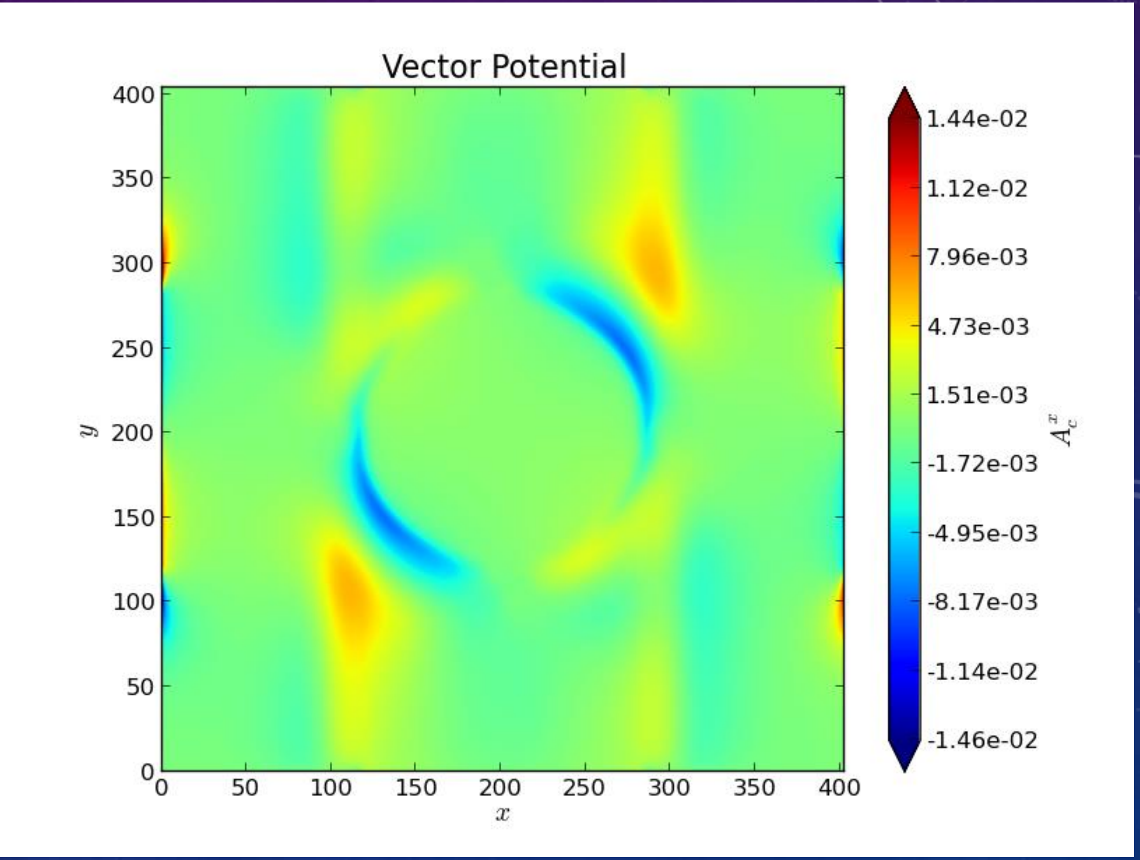
ROTOR TEST

- Initial B-field along x
- Edge of disk moving at 0.995c
- t = 0.2

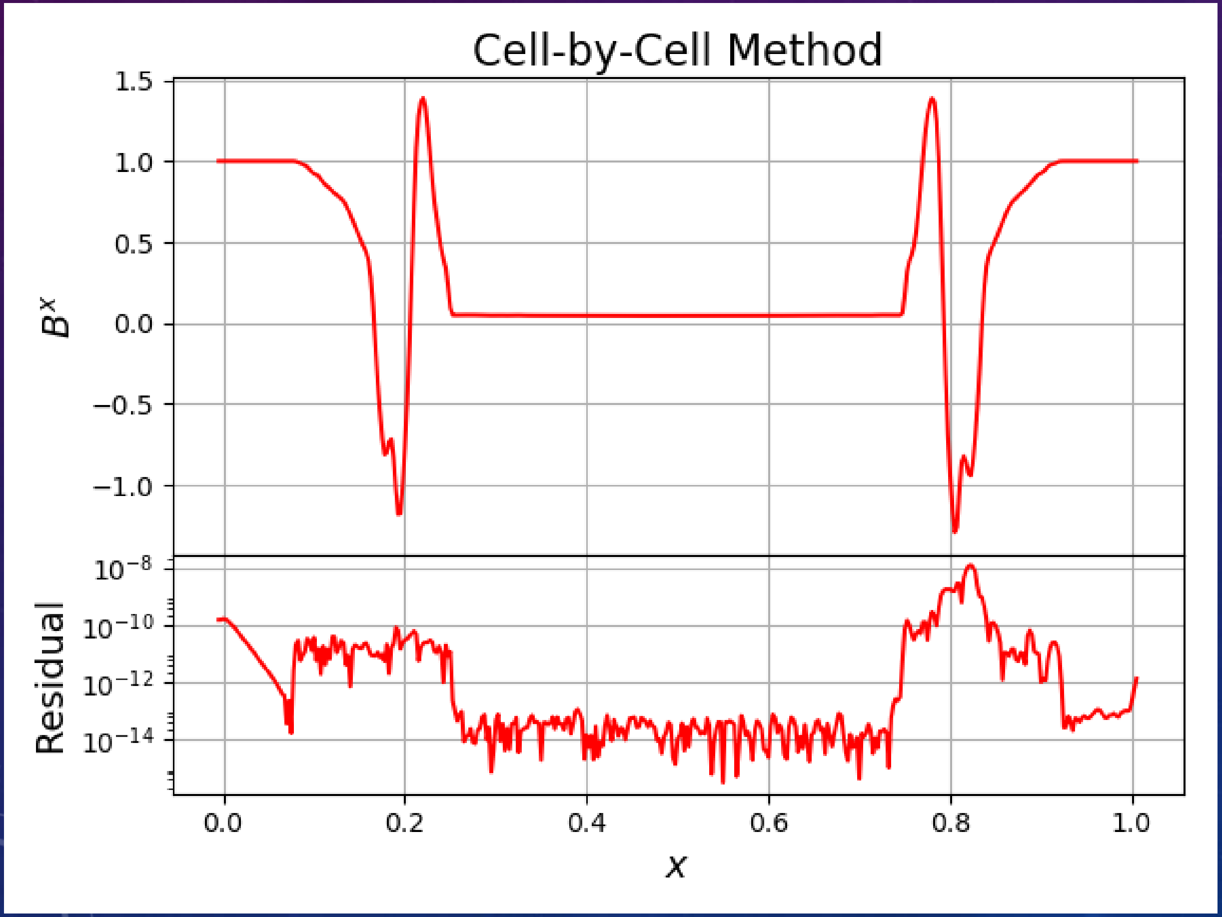


ROTOR TEST

- Initial B-field along x
- Edge of disk moving at 0.995c
- t = 0.2

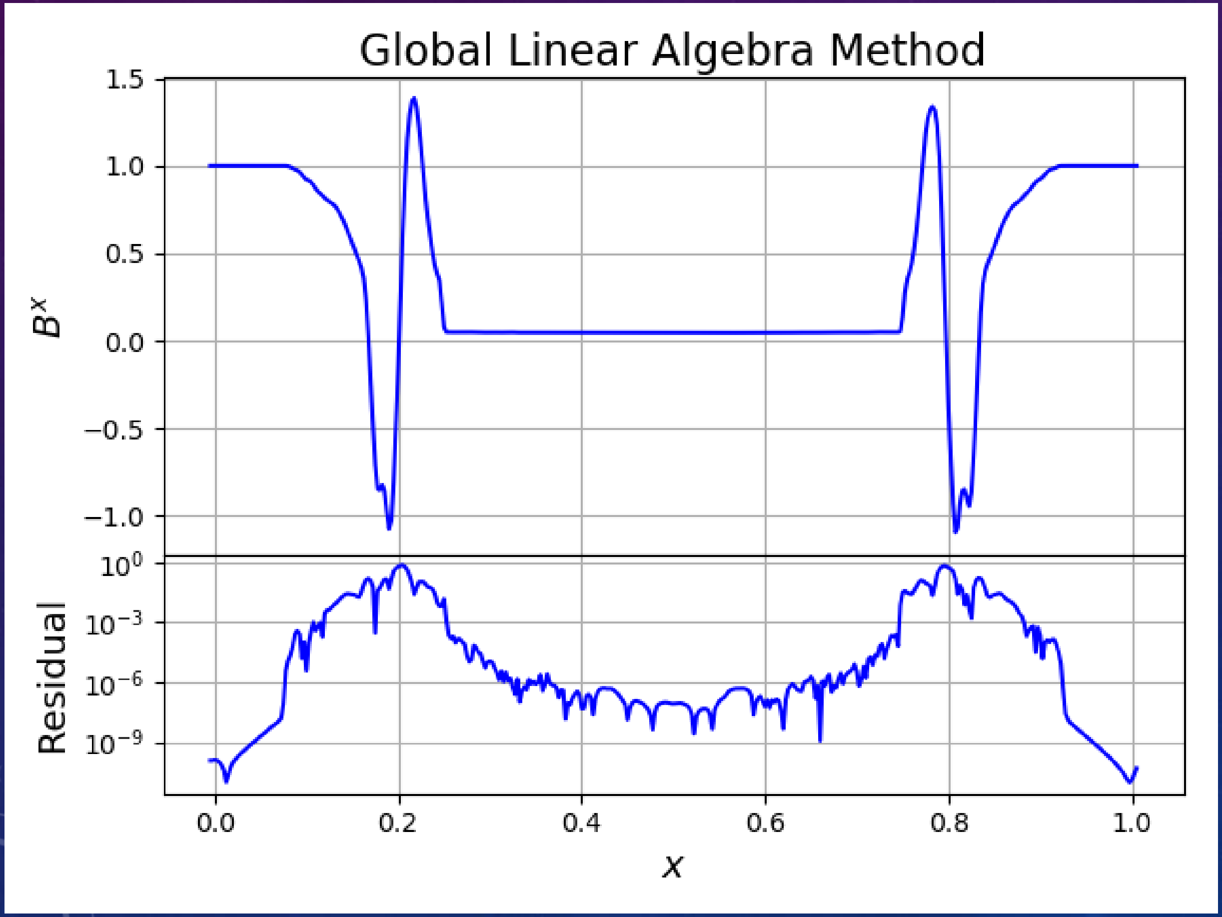


ROTOR TEST



- Final time t=0.4
- Run 1:
  - t=0 to t=0.4
- Run 2:
  - t=0 to t=0.2
  - B->A
  - t=0.2 to t=0.4

ROTOR TEST



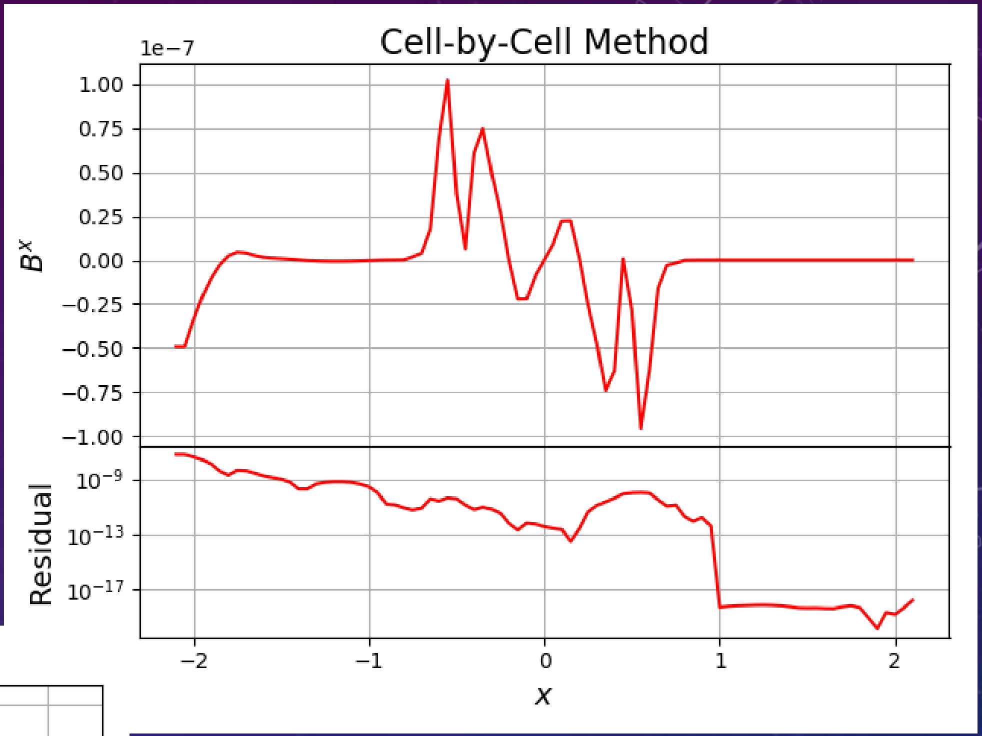
- Final time t=0.4
- Run 1:
  - t=0 to t=0.4
- Run 2:
  - t=0 to t=0.2
  - B->A
  - t=0.2 to t=0.4

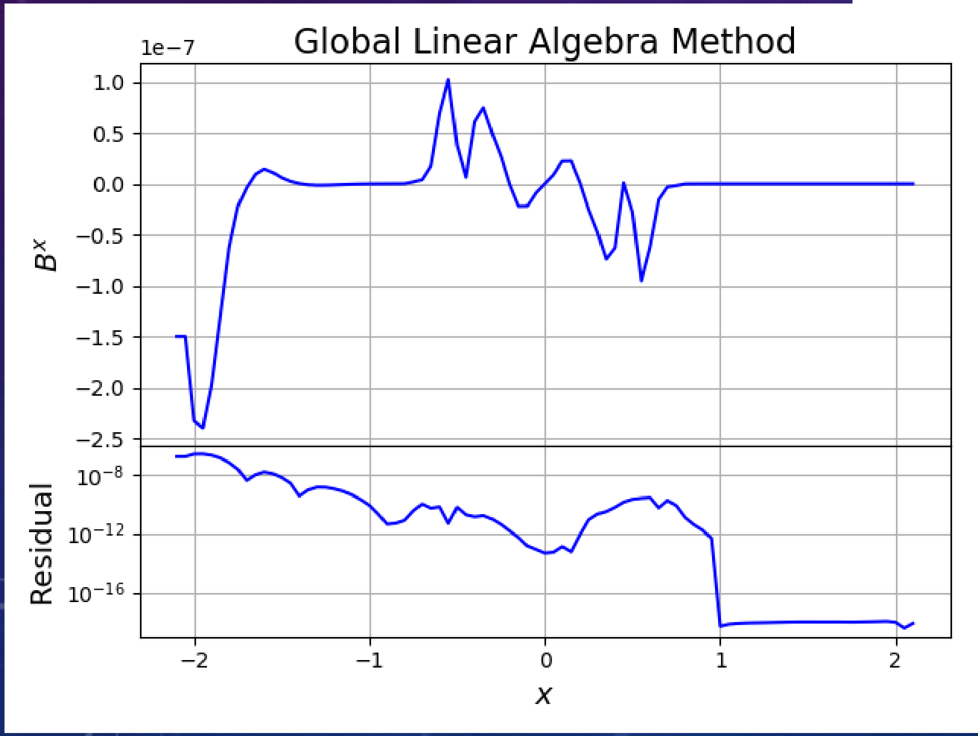
TOLMAN-OPPENHEIMER-VOLKOFF EQUATION

$$\frac{dP(r)}{dr} = -\frac{G}{r^2}\left[\rho(r) + \frac{P(r)}{c^2}\right]\left[M(r) + 4\pi r^3 \frac{P(r)}{c^2}\right]\left[1 - \frac{2GM(r)}{c^2 r}\right]^{-1}$$

- Structure of a body with:
  - Spherical symmetry
  - Isotropic material
  - Static gravitational equilibrium

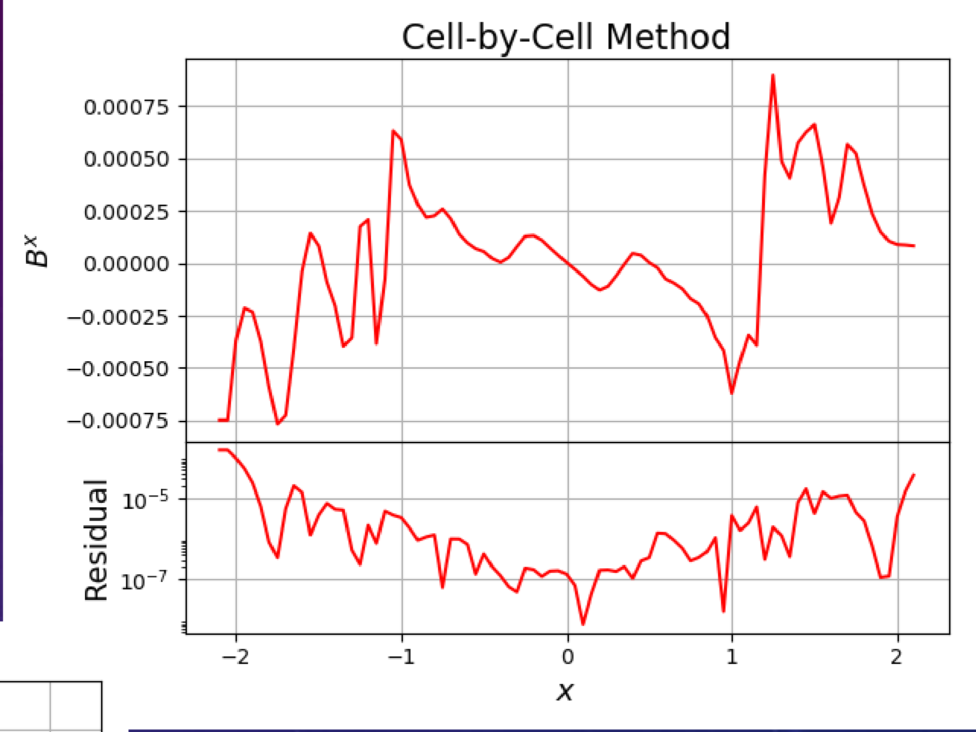
TOV STARS: STABLE

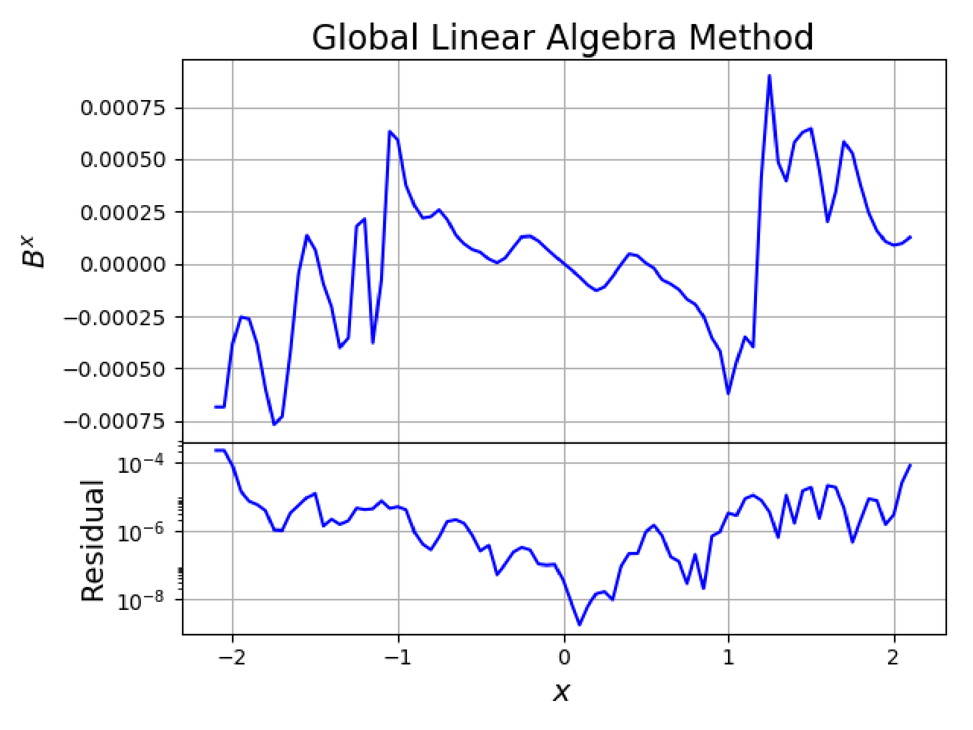




- Final time: $4t_{dyn} \approx 11.2$
- Restart Time: $t_{dyn} \approx 2.8$

TOV STARS: UNSTABLE





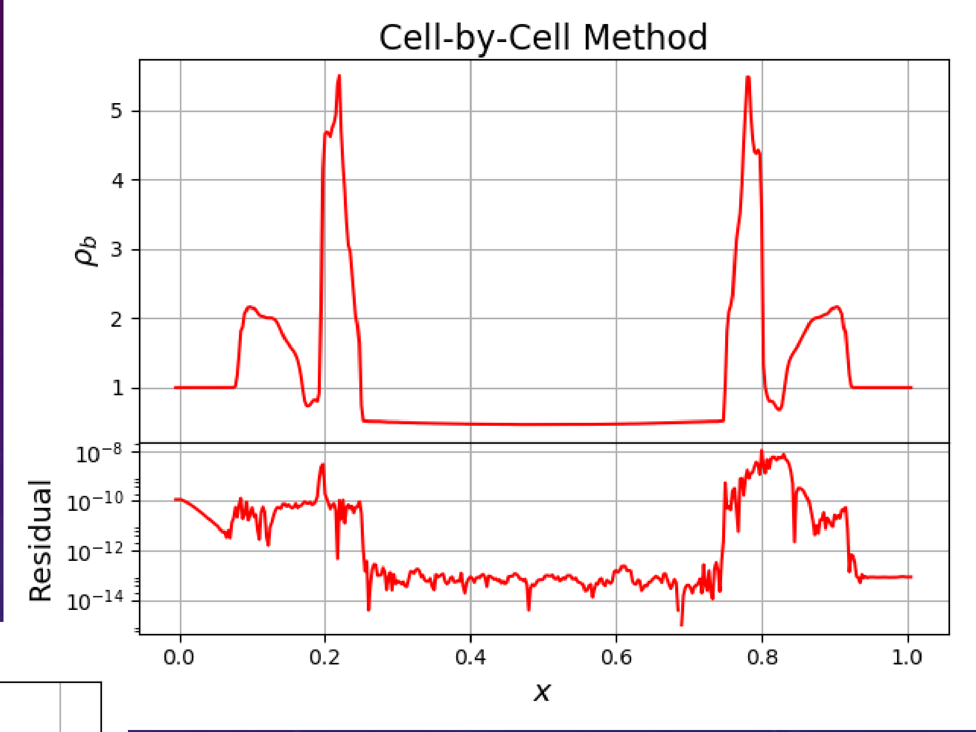
- Final time: $4t_{dyn} \approx 11.2$
- Restart Time: $t_{dyn} \approx 2.8$

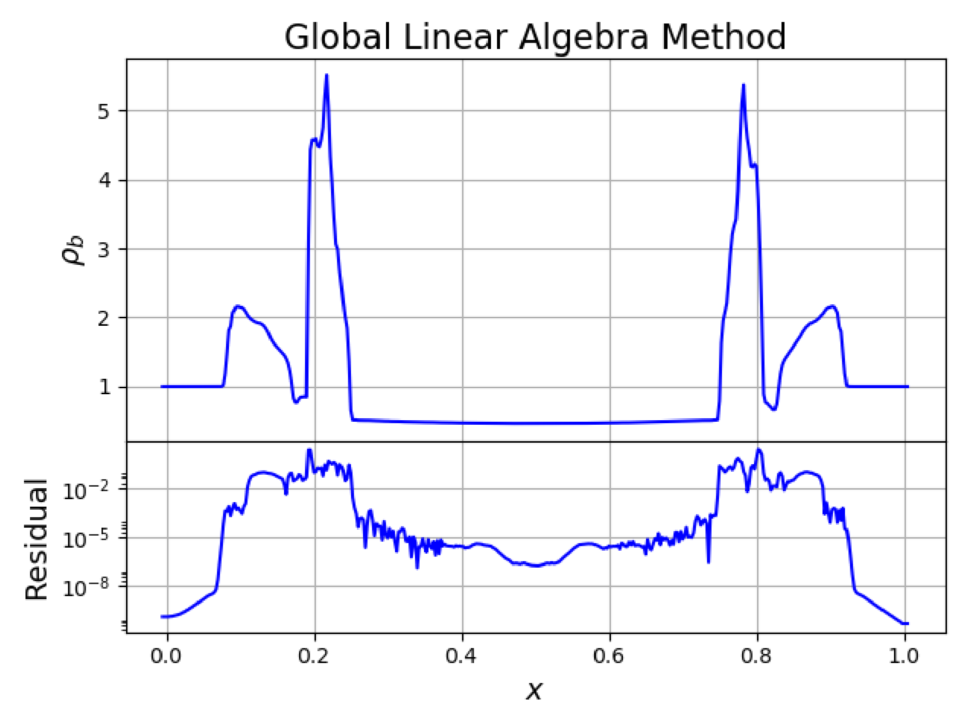
THE FUTURE

- More Evolution Tests
  - Continue taking A-field data for other configurations and evolve in time using the numerical relativity codes
- Mesh Refinement
  - Create multiple levels of data, where for example the grid spacing of one level is half that of another
- Use with HARM3D and IllinoisGRMHD
  - Run full binary black hole simulations

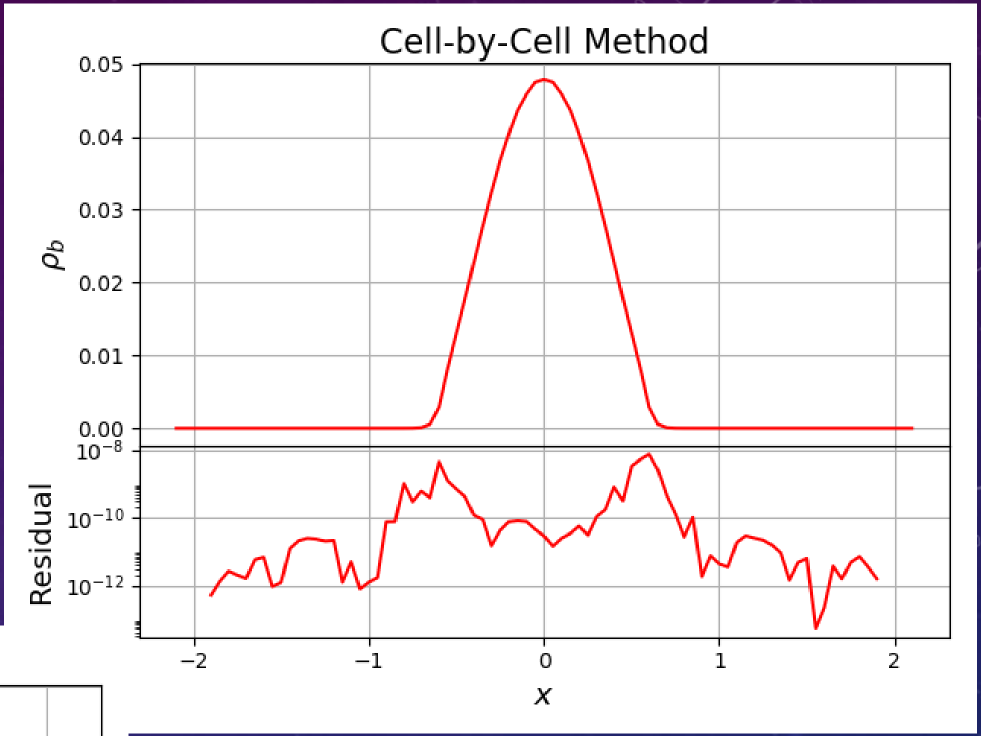
BONUS SLIDES

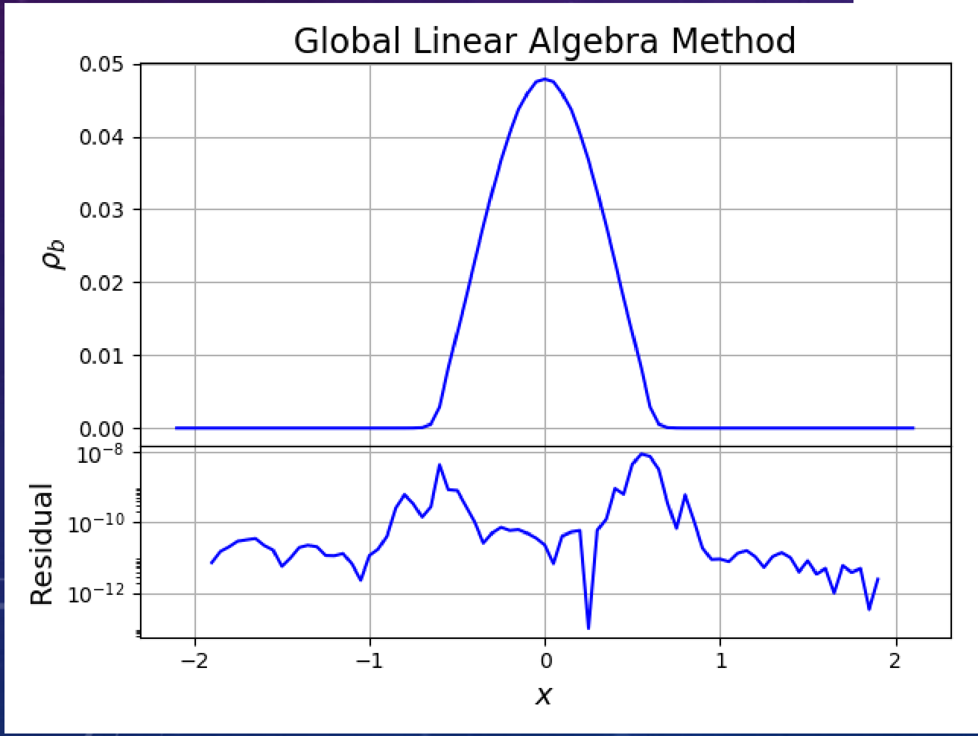
ROTOR DENSITY





TOV DENSITY: STABLE





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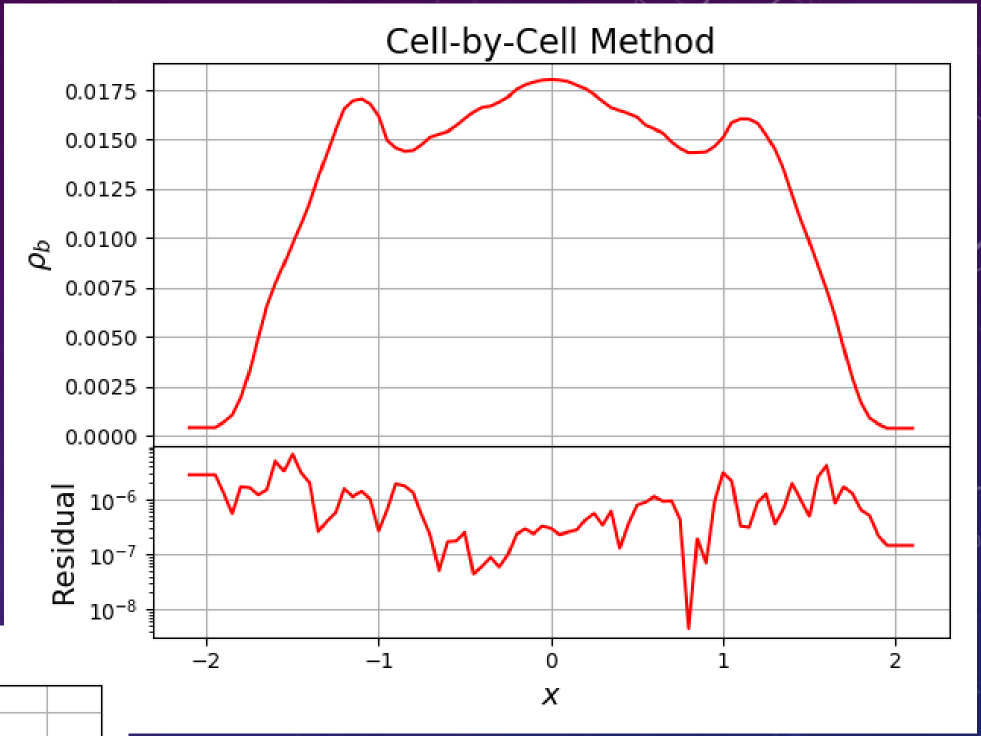
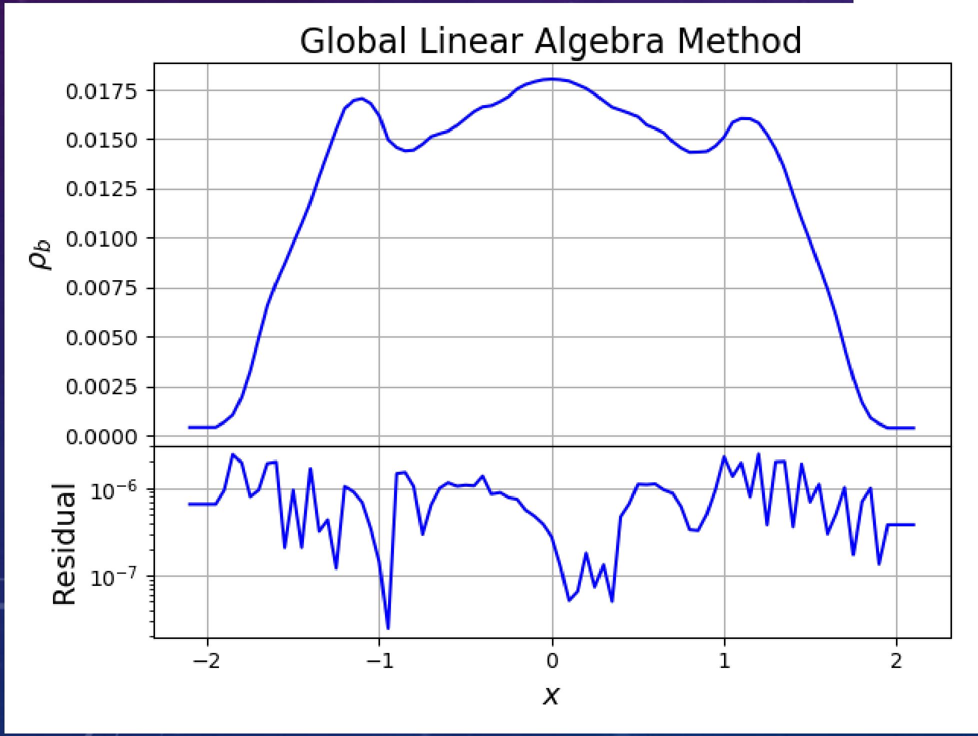
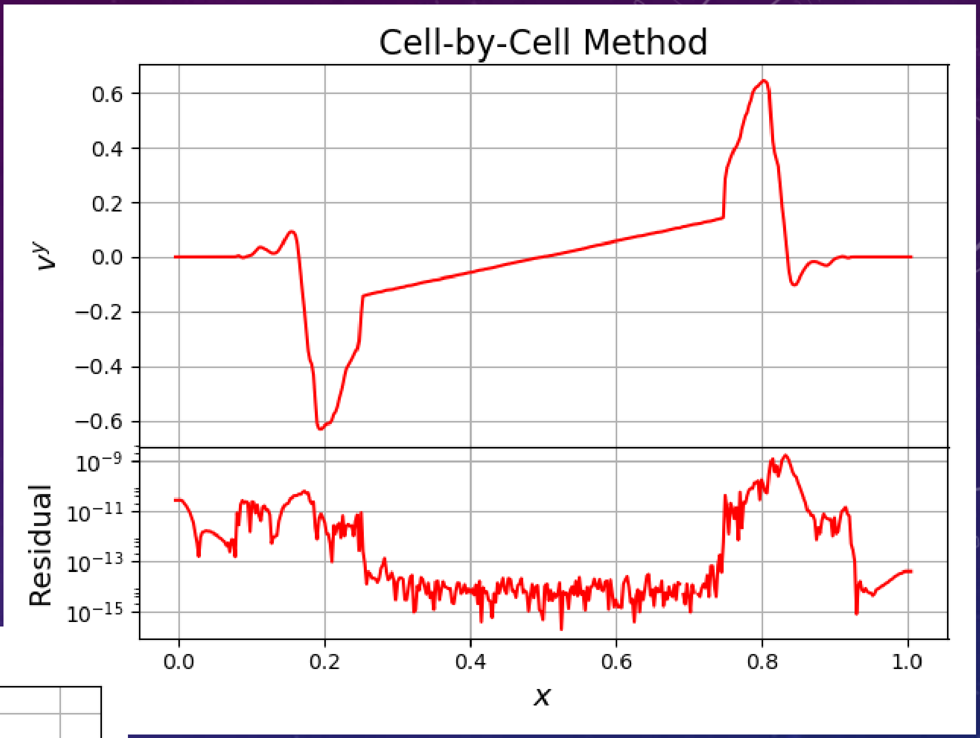
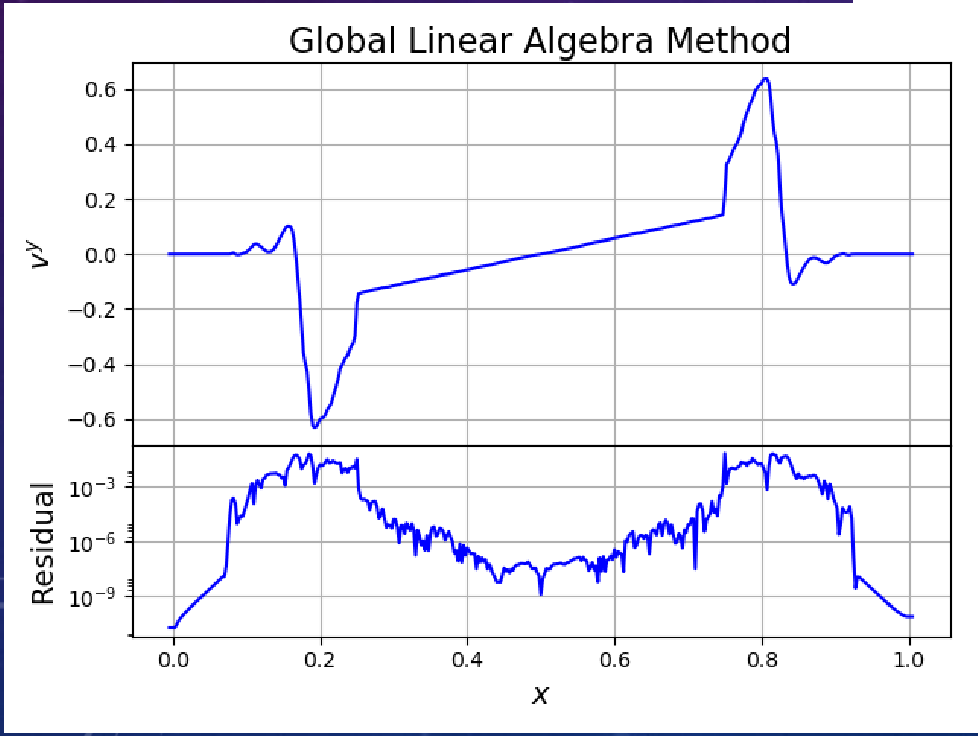




ROTOR VELOCITY





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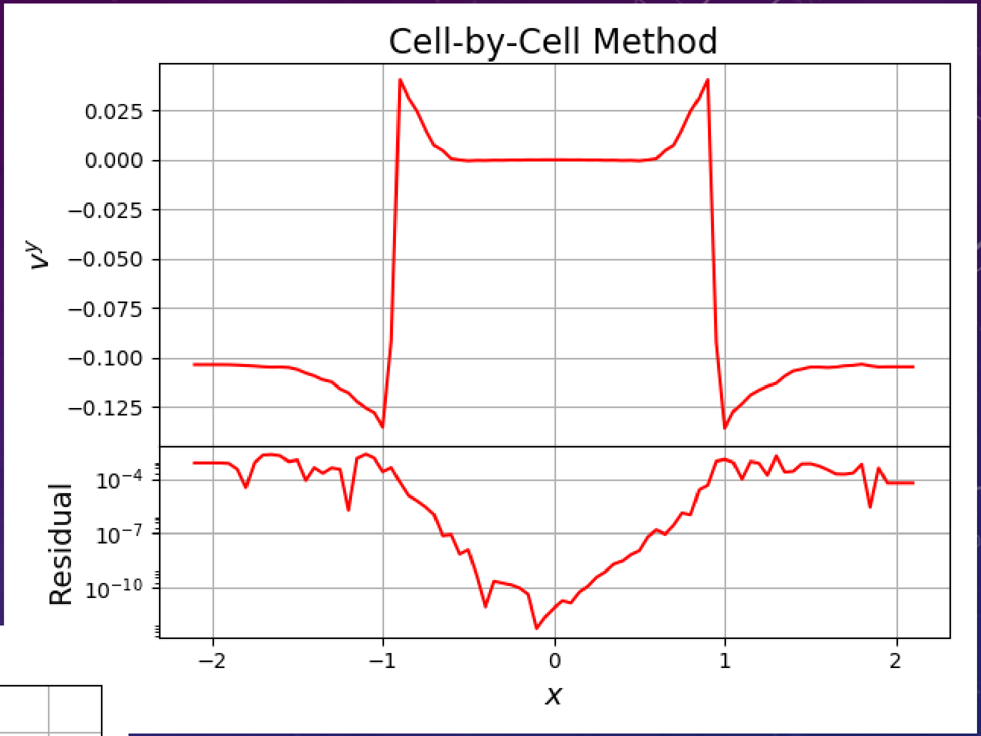

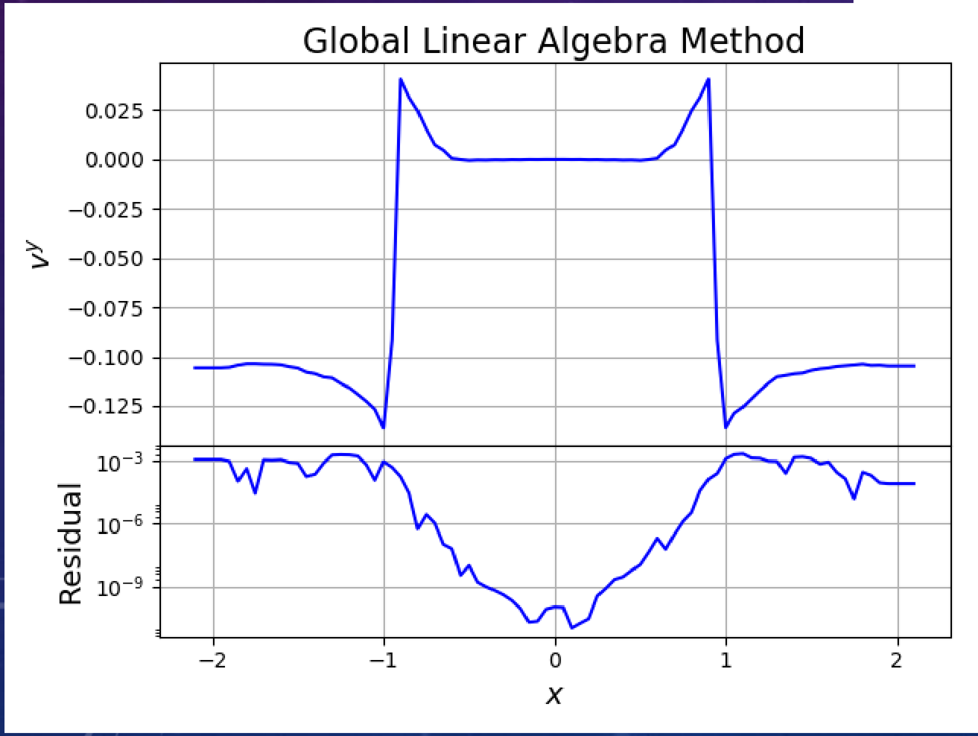

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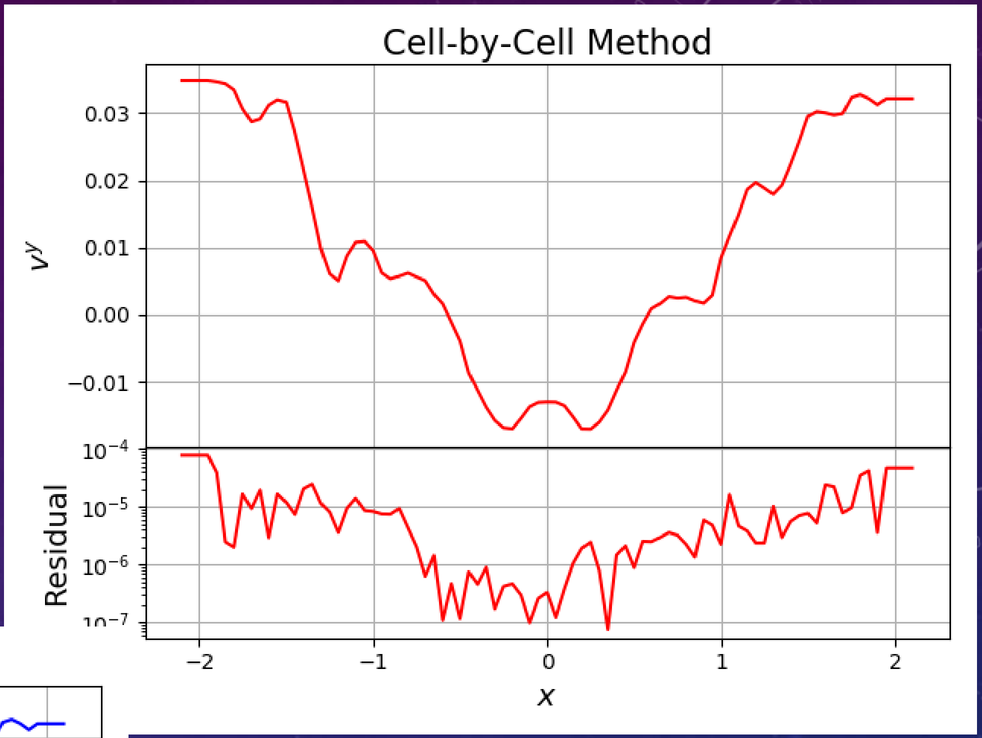
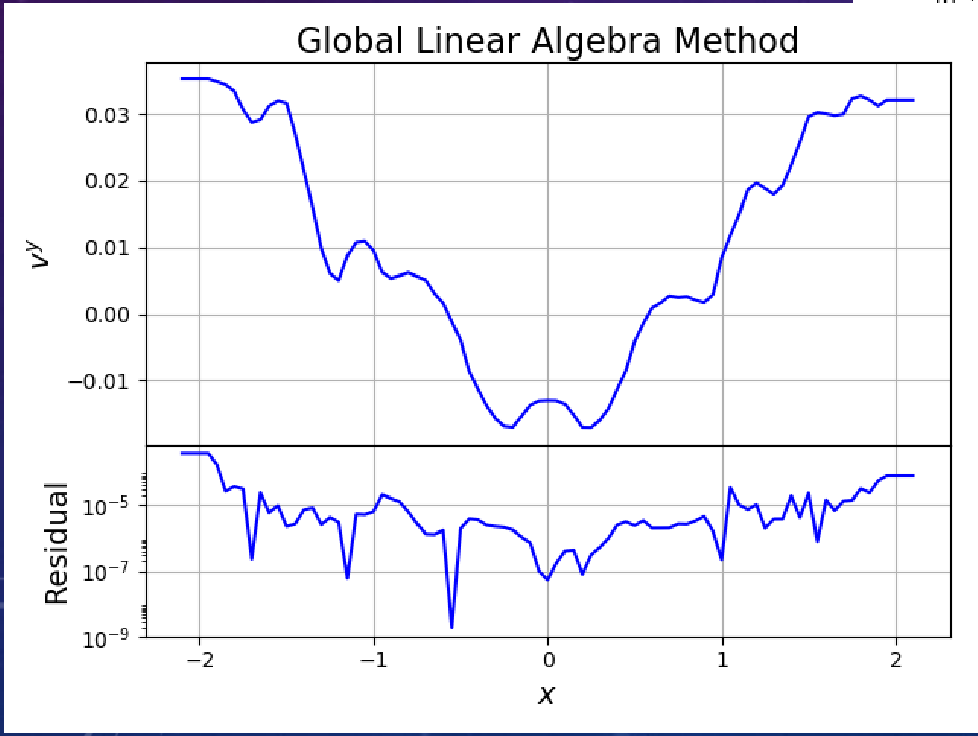




NO MONOPOLES

- The divergence-free nature of the magnetic field can be ensured by defining a vector potential. Maxwell's equations imply:

$$
\begin{aligned}
\nabla \cdot B &= 0 \\
B &= \nabla \times A
\end{aligned}
$$

- Getting from A to B is easy; getting from B to A is not!

DIVERGENCE REMOVAL

- The divergence is calculated for a cell by finite differencing, using the centered stencil

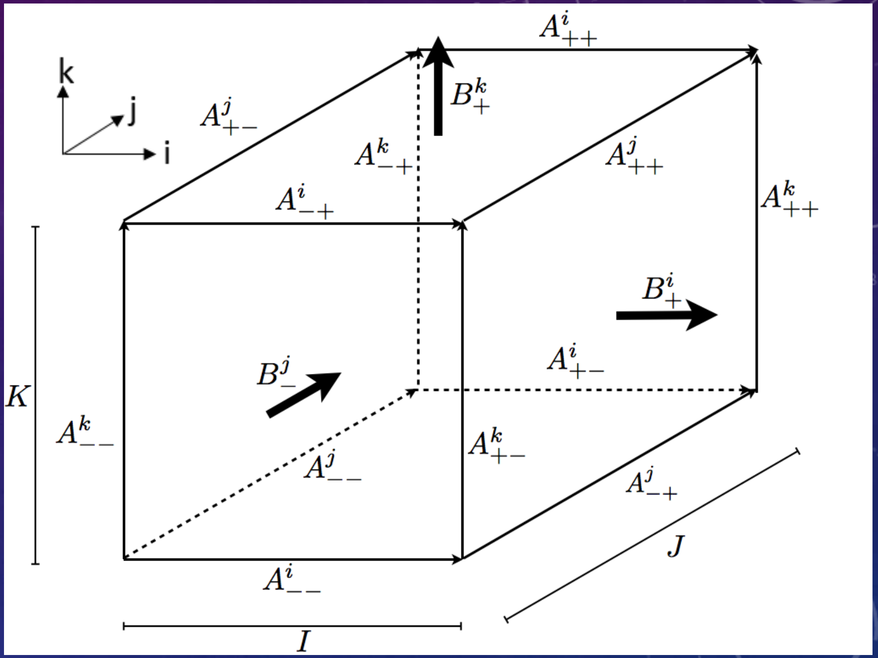
$$\nabla \cdot B = \frac{B^i_+ - B^i_-}{I} + \frac{B^j_+ - B^j_-}{J} + \frac{B^k_+ - B^k_-}{K}$$

- If this is non-zero, magnetic flux is symmetrically removed through the faces of the cell
- My A-field solver introduces no divergence by construction

Tóth 2000

SINGLE CELL SYMMETRY CONSIDERATIONS

- Each edge takes values based on:
  - The two neighboring faces
  - The two opposite faces
  - NOT the perpendicular faces!



$$
\begin{aligned}
A^k_{--} &= \alpha(-JB^i_- + IB^j_-) + \beta(-JB^i_+ + IB^j_+)\\
A^k_{-+} &= \alpha(-JB^i_+ - IB^j_-) + \beta(-JB^i_- - IB^j_+)\\
A^k_{++} &= \alpha(JB^i_+ - IB^j_+) + \beta(JB^i_- - IB^j_-)\\
A^k_{+-} &= \alpha(JB^i_- + IB^j_+) + \beta(JB^i_+ + IB^j_-)
\end{aligned}
$$

GAUGE CONDITIONS

- My results are in an arbitrary gauge, dependent on the order I evaluate my cells
- Coulomb: $\nabla \cdot A = 0$
- There is a prescription to take an arbitrary gauge and produce a Coulomb gauge solution:

$$\begin{aligned} \text{Let } A_c &= A - \nabla\phi_C \\ 0 = \nabla \cdot A_c &= \nabla \cdot A - \nabla^2\phi_C \end{aligned}$$