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TWO-BODY RANDOM HAMILTONIAN AND LEVEL DENSITY

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VALIDITY OF RANDOM MATRIX THEORIES FOR MANY-PARTICLE SYSTEMS *

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Universal Quantum-Critical Dynamics of Two-Dimensional Antiferromagnets

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(Received 13 April 1992)

The universal dynamic and static properties of two-dimensional antiferromagnets in the vicinity of a zero-temperature phase transition from long-range magnetic order to a quantum-disordered phase are studied. Random antiferromagnets with both Néel and spin-glass long-range magnetic order are considered. Explict quantum-critical dynamic scaling functions are computed in a $1/N$ expansion to two-loop level for certain nonrandom, frustrated square-lattice antiferromagnets. Implications for neutron scattering experiments on the doped cuprates are noted.

PACS numbers: 75.10.Jm, 05.30.Fk, 75.50.Ee

Sachdev-Ye-Kitaev Model

A model of *N* randomly interacting *Majorana* fermions

$$\boxed{\hat{H} = \sum_{ijkl}^{N} J_{ijkl}\, \chi_i \chi_j \chi_k \chi_l}$$

SYK model

where the interaction constants are static and random,

$$\langle |J_{ijkl}|^2 \rangle = \frac{6J^2}{N^3}$$

high energy scale

Two perspectives:

- strong correlation physics
- holography

SYK model cont'd

$$\boxed{\hat{H} = \sum_{ijkl}^{N} J_{ijkl}\, \chi_i \chi_j \chi_k \chi_l}$$

'infinite range'

amenable to large *N* mean field methods

chaotic

SYK model cont'd

$$\hat{H} = \sum_{ijkl}^{N} J_{ijkl}\, \chi_i \chi_j \chi_k \chi_l$$

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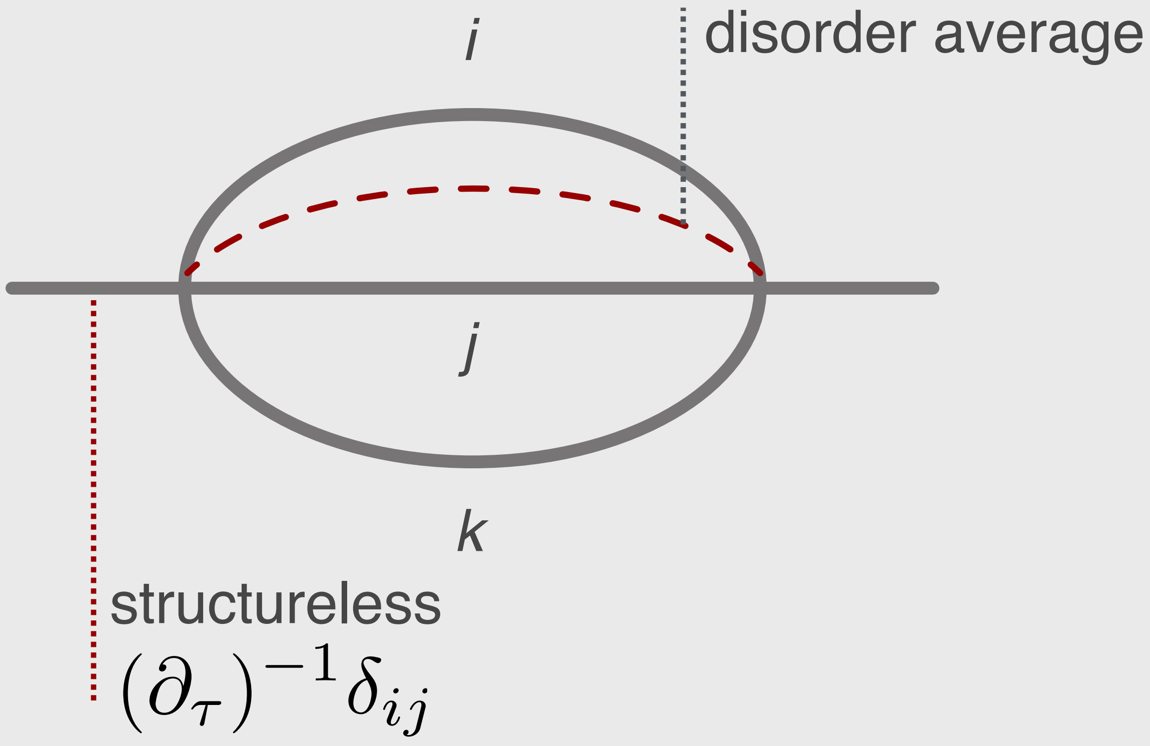
diagrammatic expansion of Majorana propagator



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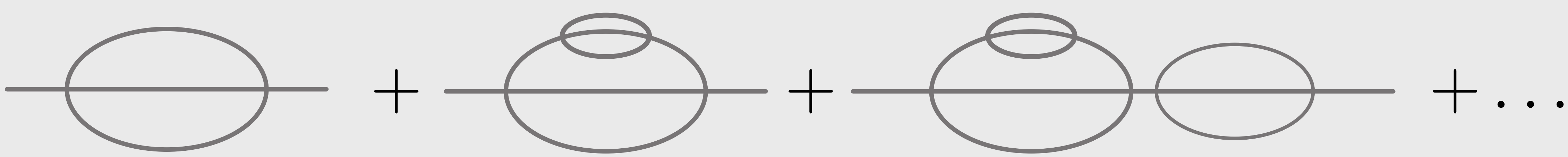

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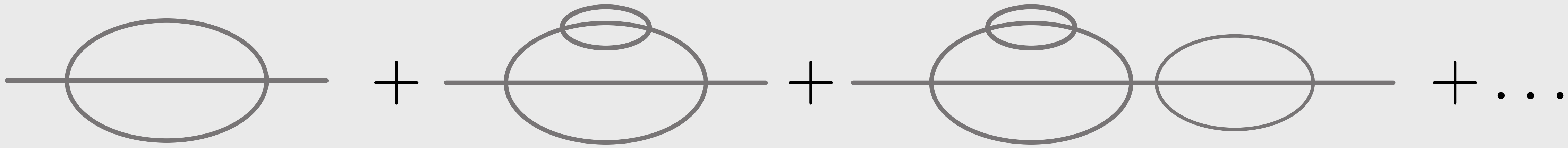

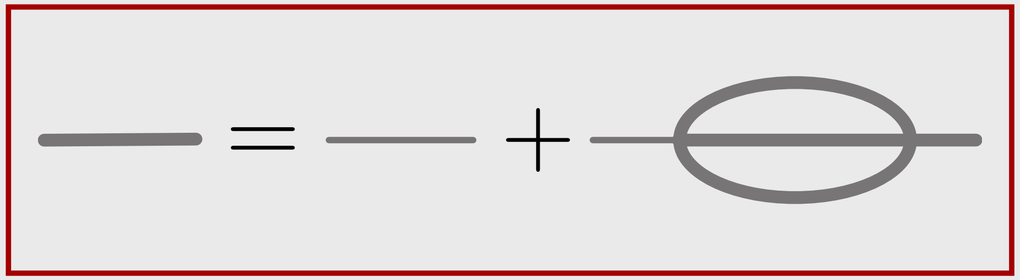

path integral approach

standard imaginary time coherent state field integral construction followed by disorder average leads to

$$Z = \int D[G, \Sigma] \exp(-S[G, \Sigma])$$

replica
matrix fields

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$$Z = \int D[G, \Sigma] \, \exp(-S[G, \Sigma])$$

replica
matrix fields

large N

self energy

$$S[\Sigma, G] = -\frac{N}{2} \int\int_{-\beta/2}^{\beta/2} d\tau d\tau' \left[\mathrm{Tr} \log(\partial_\tau \delta^{ab} + \Sigma^{ab}_{\tau,\tau'}) + \frac{J^2}{4} \left[G^{ab}_{\tau,\tau'} \right]^4 + \Sigma^{ba}_{\tau',\tau} G^{ab}_{\tau,\tau'} \right]$$

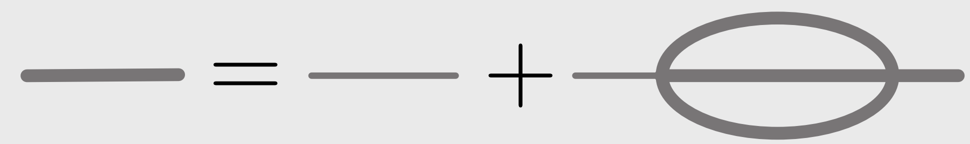
Green function

stationary phase

$$S[\Sigma, G] = -\frac{N}{2}\int\int_{-\beta/2}^{\beta/2} d\tau d\tau' \left[\mathrm{Tr}\log(\partial_\tau \delta^{ab} + \Sigma^{ab}_{\tau,\tau'}) + \frac{J^2}{4}\left[G^{ab}_{\tau,\tau'}\right]^4 + \Sigma^{ba}_{\tau',\tau} G^{ab}_{\tau,\tau'}\right]$$

variational equations

$$-(\partial_\tau + \Sigma)\cdot G = 1; \qquad \Sigma = J^2\left[G\right]^3$$



with solutions $(\partial_\tau \ll J)$

replica isotropy

$$G^{ab}(\tau - \tau') = -\frac{b}{J^{1/2}}\,\frac{\delta^{ab}\,\mathrm{sgn}(\tau - \tau')}{|\tau - \tau'|^{1/2}}$$

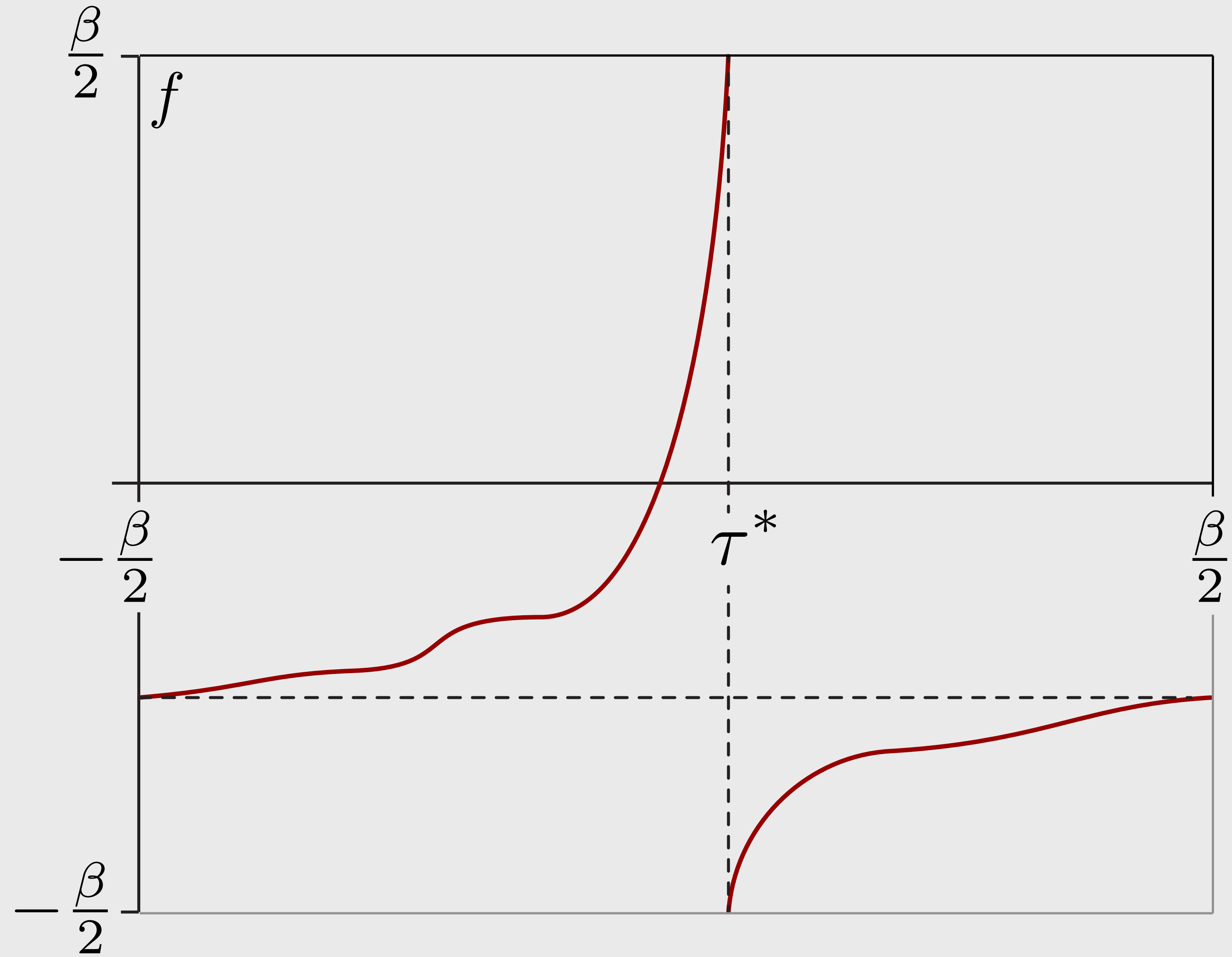
$$\Sigma^{ab}(\tau - \tau') = -b^3 J^{1/2}\,\frac{\delta^{ab}\,\mathrm{sgn}(\tau - \tau')}{|\tau - \tau'|^{3/2}}$$

numerical factor

Symmetries

Action (neglecting time derivatives) invariant under reparameterization of time

$$f : S^1 \to S^1, \tau \mapsto f(\tau),$$
$$f \in \mathrm{Diff}(S^1)$$



$$G(\tau,\tau') \to f'(\tau)^{1/4} G\big(f(\tau) - f(\tau')\big) f'(\tau')^{1/4},$$
$$\Sigma(\tau,\tau') \to f'(\tau)^{3/4} \Sigma\big(f(\tau) - f(\tau')\big) f'(\tau')^{3/4}$$

Elements of the diffeomorphism manifold describe reparameterizations of time. Infinitesimally: generated by Virasoro algebra. Weakly broken by time derivatives — problem has NCFT_1 symmetry (Maldacena and Stanford, 15).

Symmetry of the mean field

$$G^{ab}(\tau-\tau') = -\frac{b}{J^{1/2}}\,\frac{\delta^{ab}\,\mathrm{sgn}(\tau-\tau')}{|\tau-\tau'|^{1/2}}$$

$$\Sigma^{ab}(\tau-\tau') = -b^3 J^{1/2}\,\frac{\delta^{ab}\,\mathrm{sgn}(\tau-\tau')}{|\tau-\tau'|^{3/2}}$$

invariance under conformal transformations $\tau \to \dfrac{a\tau+b}{d\tau+c} \in \mathrm{SL}(2,R) \subset \mathrm{Diff}(S^1)$

each $f: S^1 \to S^1, \tau \mapsto f(\tau), \quad f \in \mathrm{Diff}(S^1)/\mathrm{SL}(2,R)$ generates new solution

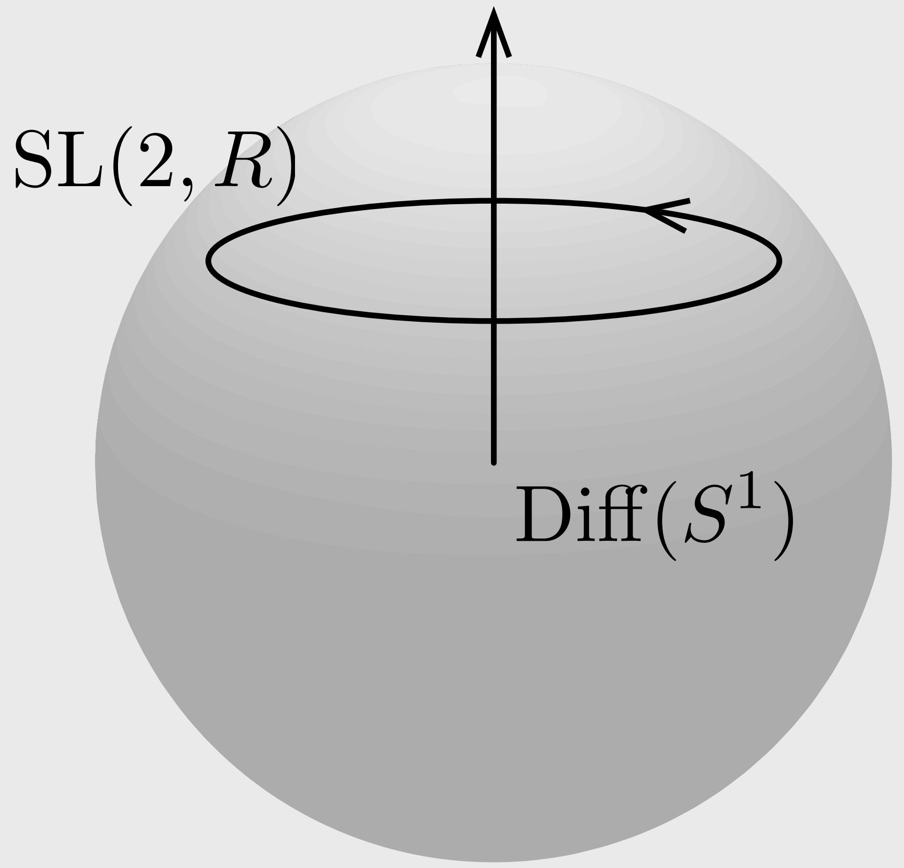
$$G([f],\tau,\tau') = f'(\tau)^{1/4} G\big(f(\tau)-f(\tau')\big) f'(\tau')^{1/4} = -\frac{b}{J^{1/2}}\mathrm{sgn}(\tau-\tau')\,\frac{f'(\tau)^{1/4} f'(\tau')^{1/4}}{|f(\tau)-f(\tau')|^{1/2}},$$

$$\Sigma([f],\tau,\tau') = f'(\tau)^{3/4} \Sigma\big(f(\tau)-f(\tau')\big) f'(\tau')^{3/4} = -b^3 J^{1/2}\mathrm{sgn}(\tau-\tau')\,\frac{f'(\tau)^{3/4} f'(\tau')^{3/4}}{|f(\tau)-f(\tau')|^{3/2}}$$

Goldstone mode manifold

emergence of infinite dimensional Goldstone mode manifold

$$\boxed{\mathrm{Diff}(S^1)/\mathrm{SL}(2,R)}$$



Large conformal Goldstone mode fluctuations in the SYK model

Natal, Mar. 2017

Alexander Altland, Dmitry Bagrets (Cologne), Alex Kamenev (Minnesota)

holographic analogies

conformal symmetry & Liouville quantum mechanics

quantum chaos & OTO correlation functions

Nucl. Phys. B **911**, 191 (2016)
arXiv:1702.08902

holographic interpretation

(amateur perspective)

Holographic interpretation (Maldacena & Stanford, 16; Almheiri & Polchinksi, 16)

Consider 2d Einstein-Hilbert action

also constant

positive cosmological constant

$$S = \frac{\phi_0}{16\pi G} \int \sqrt{g}(R + \Lambda)$$

Gravitational constant

action invariant under conformal deformations of 2d space (because it is topological)

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boundary
(where SYK lives)

compactified space

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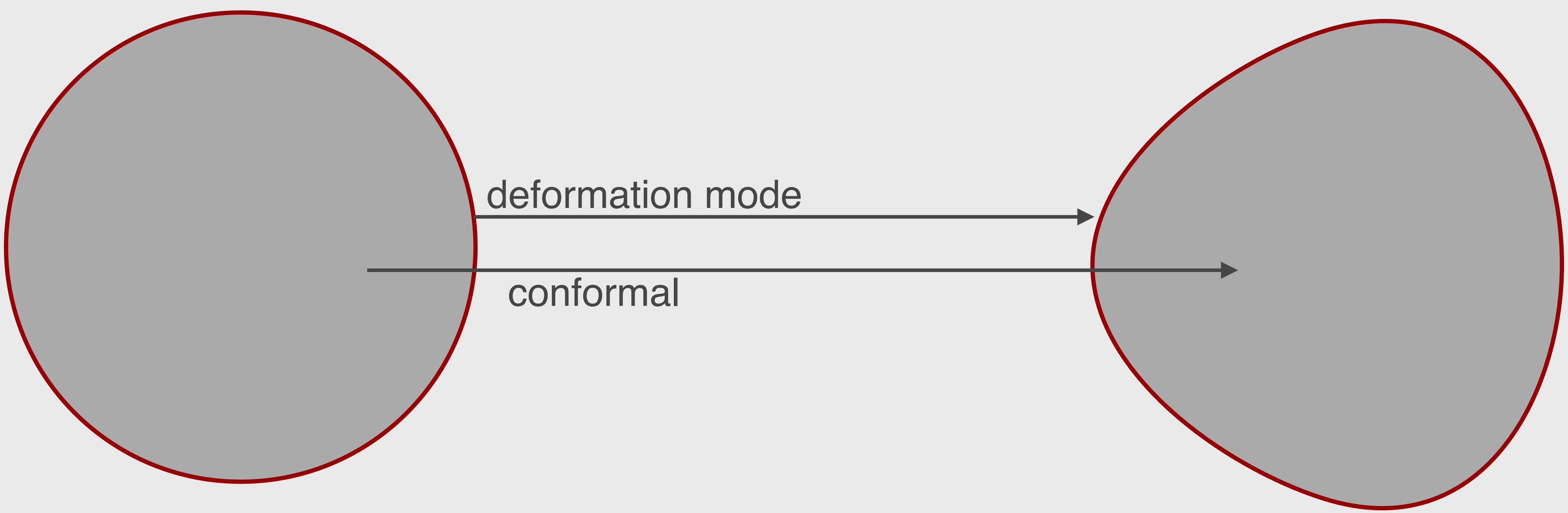

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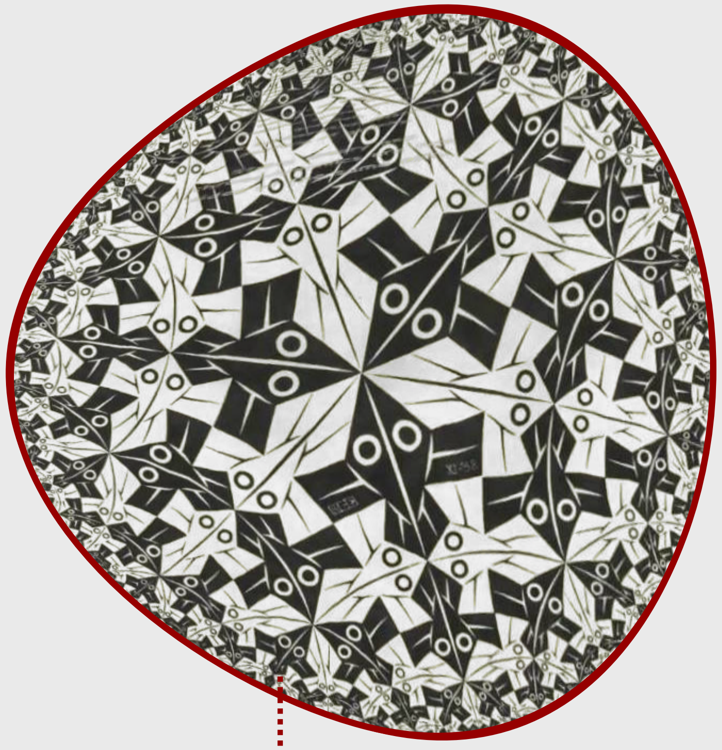

AdS metric

Holographic interpretation (continued)

AdS metric (spontaneously) breaks symmetry to *SL(2,R).* Reparameterization Goldstone modes without action.

Holographic interpretation (continued)

AdS metric (spontaneously) breaks symmetry to *SL(2,R).* Reparameterization Goldstone modes without action.

Improve situation by upgrading pure gravity action to dilaton action

now a field

$$S = \frac{\phi_0}{16\pi G}\int \sqrt{g}(R+\Lambda) \longrightarrow \frac{1}{16\pi G}\int \sqrt{g}\phi(R+\Lambda) + \dots$$

Jackiw Teitelboim gravity

This action (i) is non-topological, (ii) fluctuations of the dilaton field weakly break conformal symmetry (—> non-vanishing boundary action) and (iii) afford physical interpretation if AdS2 action is seen as boundary theory of higher dimensional extremal black hole.

Combination (i-iii) motivates boundary with conformal invariance breaking and signatures of quantum chaos.

conformal symmetry &
Liouville quantum mechanics

reparameterization action

Goal: construct effective ("magnon") action describing cost of reparameterization fluctuations.

Expand

!

$$S[\Sigma, G] = -\frac{N}{2} \int_{-\beta/2}^{\beta/2} d\tau d\tau' \left[\mathrm{Tr} \log(\partial_\tau + \Sigma_{\tau,\tau'}) + \frac{J^2}{4} \left[G_{\tau,\tau'} \right]^4 + \Sigma_{\tau',\tau} G_{\tau,\tau'} \right]$$

$$\to \frac{N}{4} \mathrm{Tr}(\partial_\tau G \partial_\tau G) = -\frac{b^2 N}{16J} \iint d\tau d\tau' \, \frac{f'(\tau)^{3/2} f'(\tau')^{3/2}}{|f(\tau) - f(\tau')|^3} \, .$$

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UV regularization at ~J

$$\to S[f] = \frac{M}{2} \int d\tau \left(\frac{f''(\tau)}{f'(\tau)} \right)^2 \qquad M = \frac{b^2}{32 J} N \log(N)$$

Goldstone mode action

time scale at which fluctuations become strong

Form of the action suggested by Maldacena *et al.* 16, present derivation (Bagrets *et al.* *16*) identifies *M*.

Low energy theory

$$Z = \int \mathcal{D}f \, \exp(-S[f]), \qquad S[f] = \frac{M}{2} \int d\tau \left(\frac{f''(\tau)}{f'(\tau)} \right)^2$$

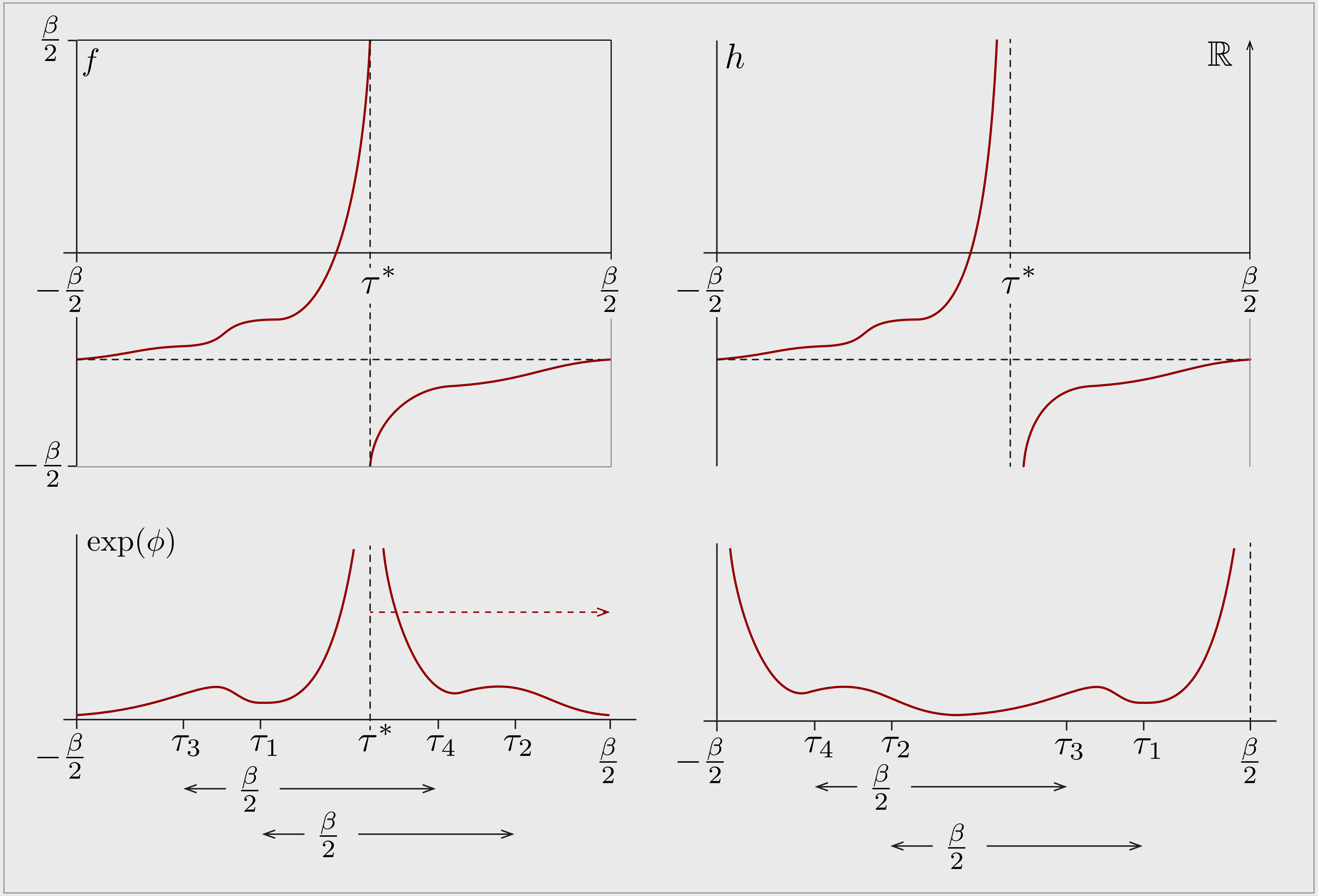
left invariant measure
involves functional determinant

Integral over left invariant measure (Bagrets *et al. 16)* $\mathcal{D}(f \circ g) = \mathcal{D}f$ not innocent (Witten & Stanford 17, Kitaev unpublished) as including integration over non-compact symmetry $\mathrm{SL}(2, R)$.

reparameterization freedom

creatively use freedom of reparameterization to obtain user friendly representation of field integral.

$$f(\tau) \to h(\tau) \equiv \tan(\pi T f(\tau)) \to \phi(\tau) \equiv \ln(h'(\tau))$$



Reparameterization mapping to Liouville Quantum mechanics

$$Z = \int \mathcal{D}\varphi \exp(-S[\varphi]), \qquad S[\varphi] = M \int d\tau \left(\frac{1}{2}(\varphi')^2 + 2e^{-\varphi} \right)$$

flat measure

action of Liouville QM

effect of low energy Goldstone mode fluctuations encapsulated in Liouville QM.
Universal feature (Shelton, Tsvelik 98): all operator correlation functions decay as

$$\langle \mathcal{O}(\tau)\mathcal{O}(\tau') \rangle \sim |\tau - \tau'|^{-3/2}$$

Sanity check I: Green function

path integral representation of Green function

$$G([f],\tau,\tau') = -\frac{b}{J^{1/2}} \left\langle \frac{f'(\tau)^{1/4} f'(\tau')^{1/4}}{|f(\tau)-f(\tau')|^{1/2}} \right\rangle_f$$

$$f(\tau) \to h(\tau) \equiv \tan(\pi T f(\tau)) \to \phi(\tau) \equiv \ln(h'(\tau))$$

$$= -\frac{b}{\sqrt{\pi} J^{1/2}} \left\langle e^{\frac{1}{4}(\phi(\tau_1)+\phi(\tau_2))} \int_0^\infty \frac{d\alpha}{\sqrt{\alpha}}\, e^{-\alpha \int_{\tau_1}^{\tau_2} ds\, e^{\phi(s)}} \right\rangle_\phi$$

quench potential

time local operator

Sanity check I: Green function

$$G(\epsilon) = -\frac{ib}{\sqrt{J}} \left(\frac{2}{\pi M} \right)^{1/2} \int_0^{+\infty} dk \, \frac{k \sinh(2\pi k)}{2\pi^2} \; \Gamma^2\left(\tfrac{1}{4} + ik\right) \Gamma^2\left(\tfrac{1}{4} - ik\right) \; \frac{2\epsilon}{E_k^2 + \epsilon^2},$$

$$E_k = k^2/2M$$

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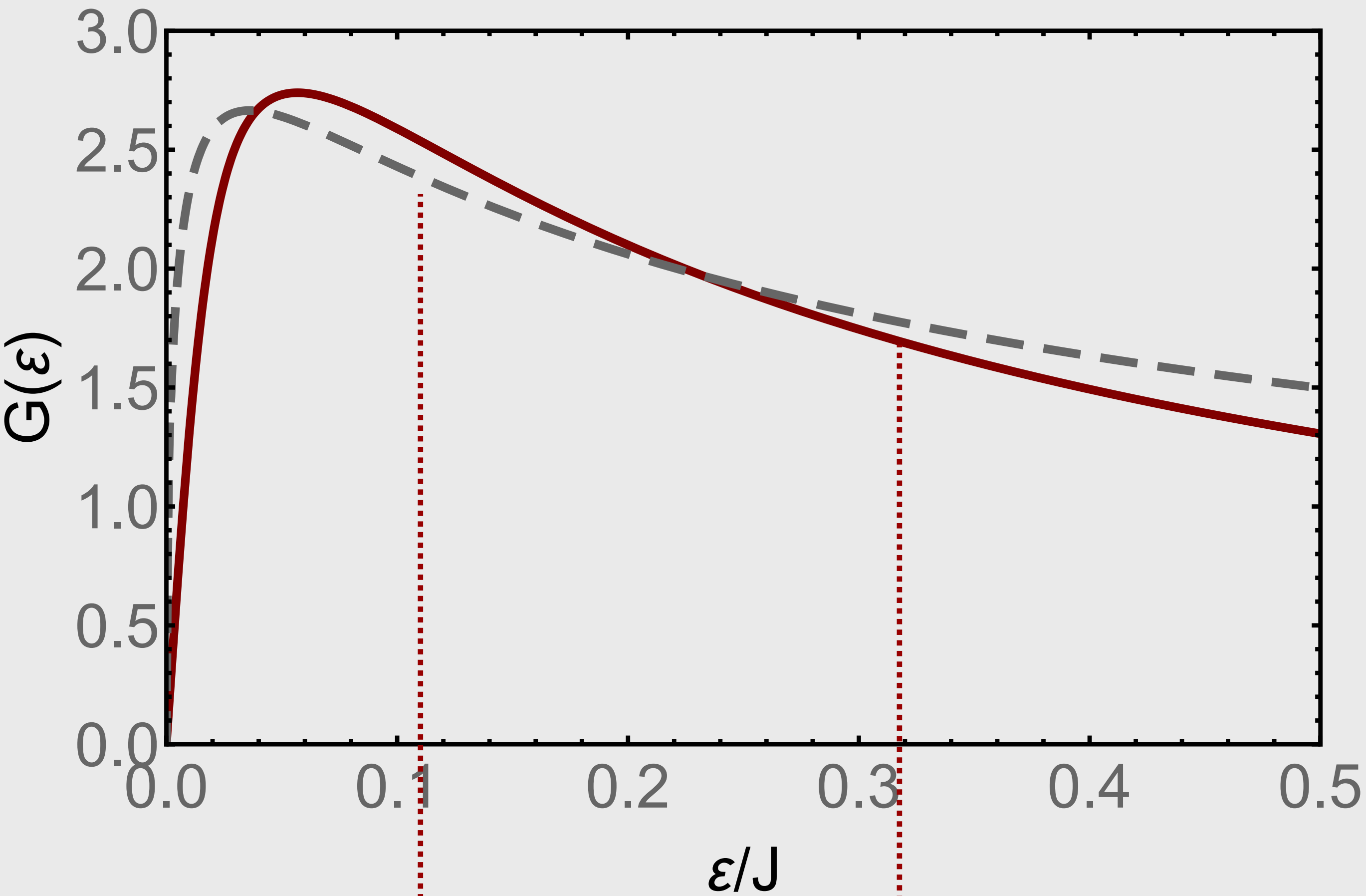
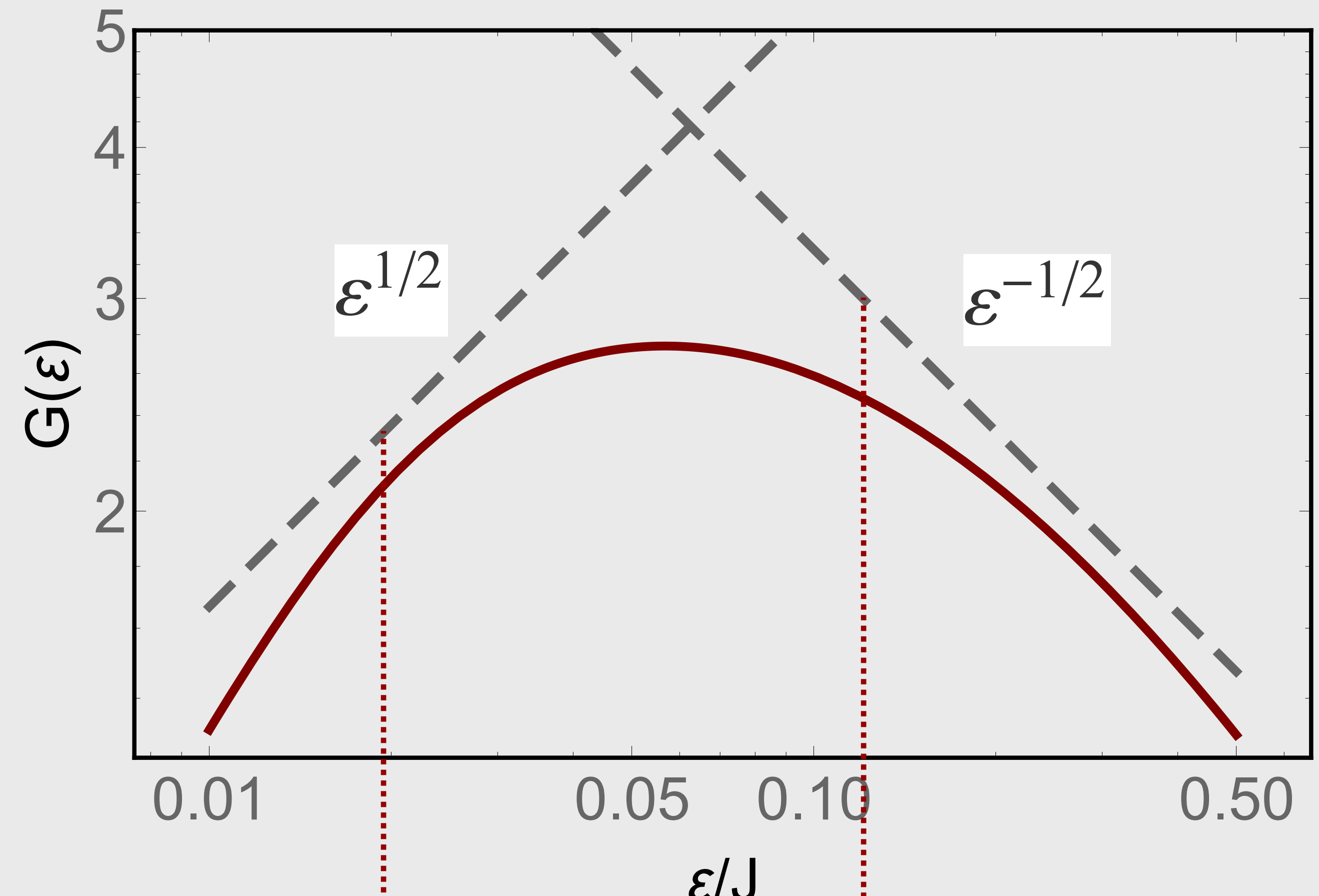
SYK Green function beyond mean field: resurrection of full symmetry at small energies



analytical GF

numerical (N=24) GF



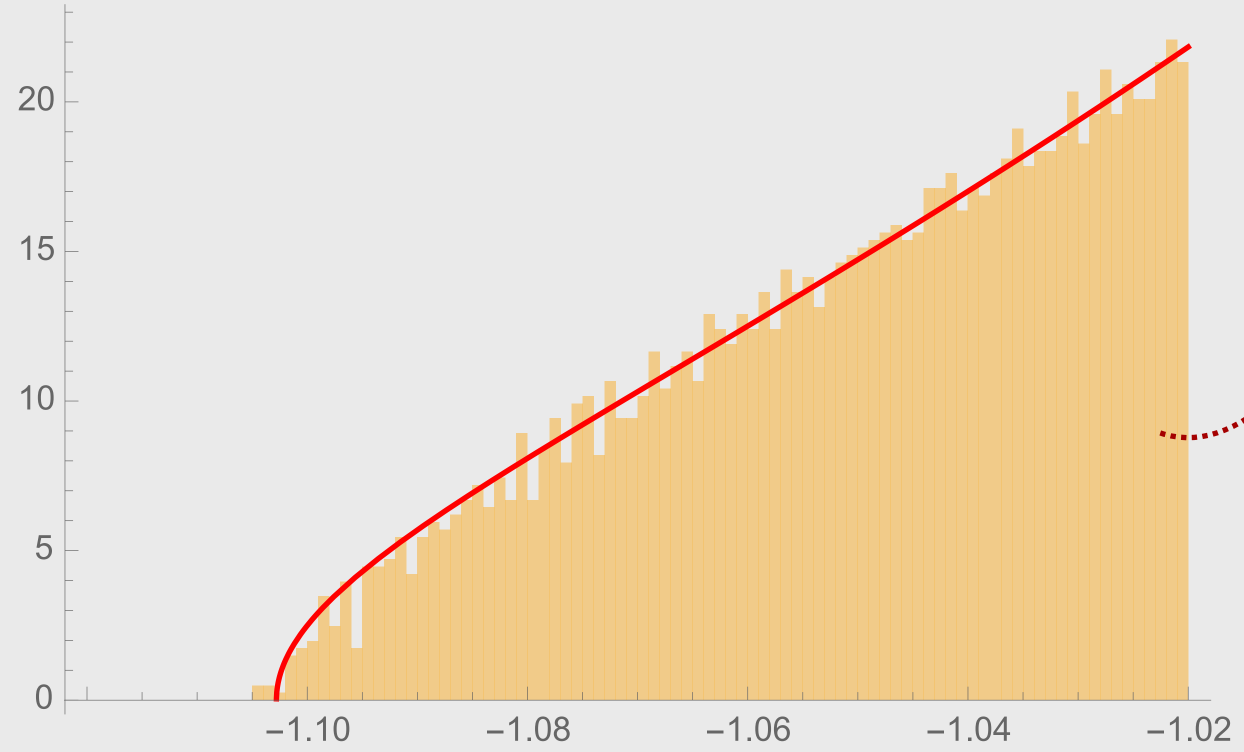
mean field

strong Goldstone mode fluctuations

sanity check II: SYK partition sum

$$Z = \langle \mathrm{tr}(e^{-\beta H}) \rangle \simeq \int \mathcal{D}\varphi \exp(-S[\varphi]) = \cdots =$$

$$= \frac{1}{\Gamma} \int_0^\infty d\epsilon\, \rho(\epsilon)\, e^{-\beta\epsilon}, \qquad \rho(\epsilon) \propto \sinh(2\pi\sqrt{M\epsilon})$$



N=34 data courtesy Garcia-Garcia

$\rho(\epsilon)$ is many body density of states above ground state. Previously obtained by combinatorial methods (Verbaarschot, Garcia-Garcia, 16), and within the limiting approximation of an q-body interaction model (Cotler et al. 16)

Note: field integral for partition sum is semiclassically exact (Stanford & Witten, 17).

chaos and OTO correlation functions

OTO correlation function

Out of time order (OTO) correlation function: a tool for diagnosing early stages of quantum chaotic dynamics (Larkin, Ovchinikov 69):

$$F(t) = \mathrm{tr}\left(e^{-\beta\hat{H}}\hat{X}\hat{Y}(t)\hat{X}\hat{Y}(t)\right)$$

X,*Y* one-body operators in many body context.

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Interpretation I: up to inessential terms, $F(t) = \langle [\hat{X}, \hat{Y}(t)]^2 \rangle$. For single particle system

$$\hat{X} = \hat{p}, \hat{Y} = \hat{q}, \quad F(t) = \langle (i\hbar\{p, q(t)\})^2 \rangle \propto \hbar^2 \langle (\partial_q q(t))^2 \rangle \propto \hbar^2 \exp(2\lambda t)$$

leading Lyapunov exponent

correlation function assumes sizable values at $t_E \equiv \lambda^{-1} \ln(\hbar)$, the Ehrenfest time.

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Interpretation II: for many (qubit) system, and $\hat{X} = \sigma_{z,i}, \hat{Y} = \sigma_{z,j}$, non-vanishing commutator builds up at times sufficiently large to entangle sites, *i,j.*

OTO correlation function continued

Interpretation III: $F(t) = \mathrm{tr}\left(e^{-\beta\hat{H}}\hat{X}\hat{Y}(t)\hat{X}\hat{Y}(t)\right)$ essentially equivalent to

$$F(t) = \mathrm{tr}\left(e^{-\frac{\beta\hat{H}}{4}}\hat{X}e^{-\frac{\beta\hat{H}}{4}}\hat{Y}(t)e^{-\frac{\beta\hat{H}}{4}}\hat{X}e^{-\frac{\beta\hat{H}}{4}}\hat{Y}(t)\right)$$

for low temperatures $T < \hbar\lambda$ growth rate of *F* set by chaos bound $T/\hbar$ (Maldacena & Stanford, 16)

OTO correlation function continued

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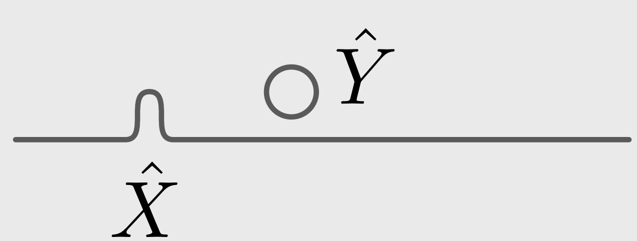
Interpretation IV: quantum butterfly effect



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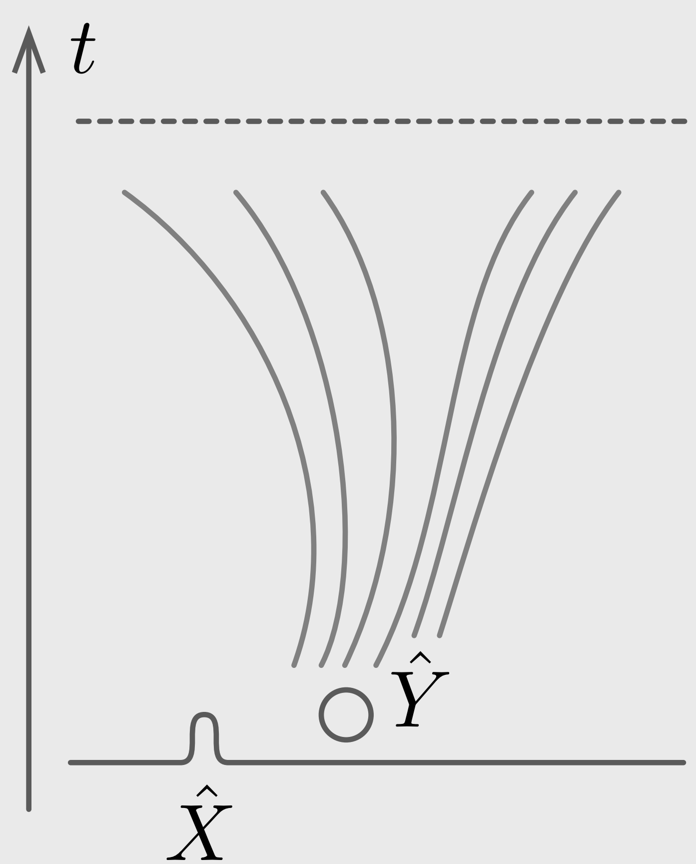

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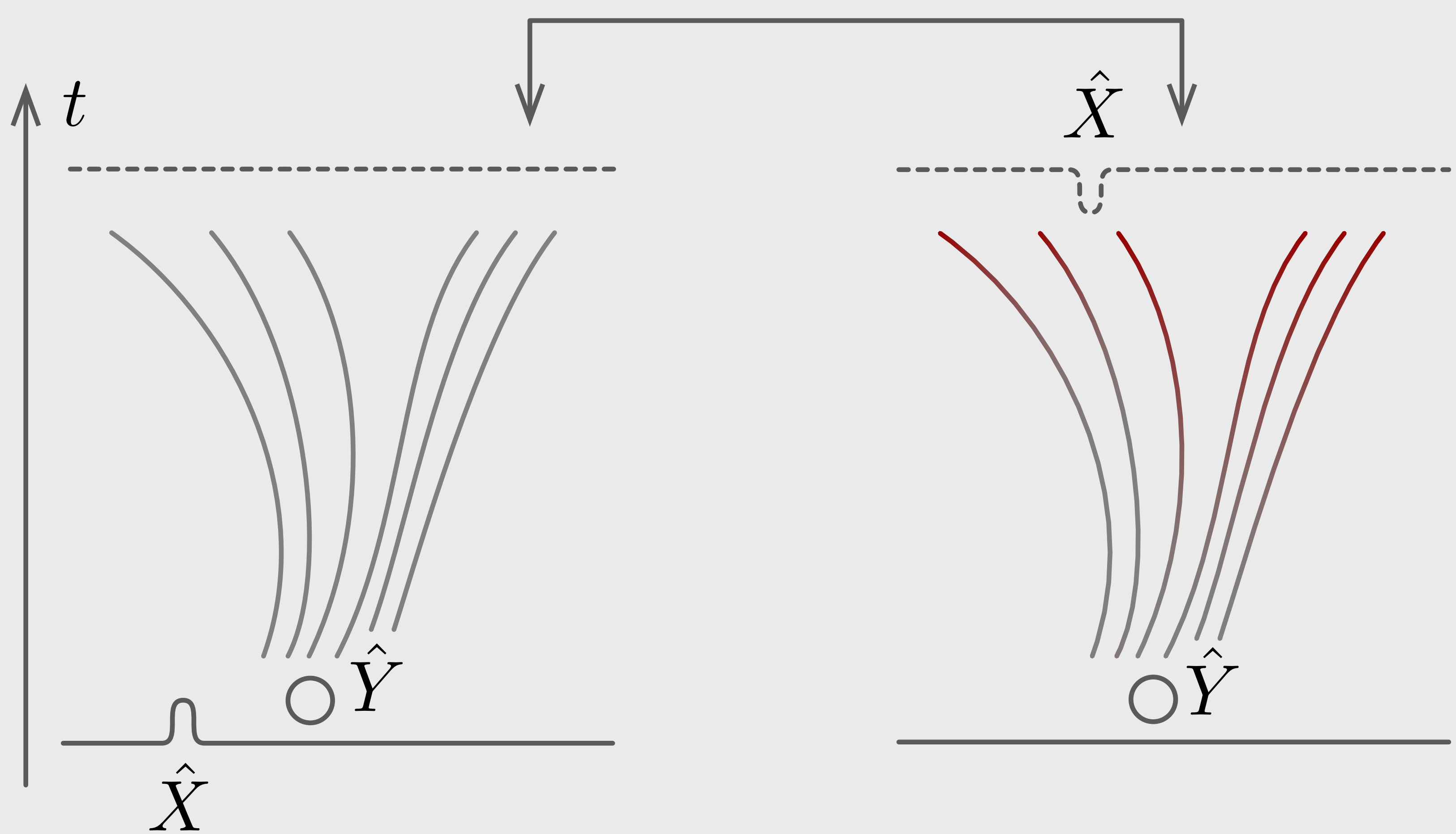
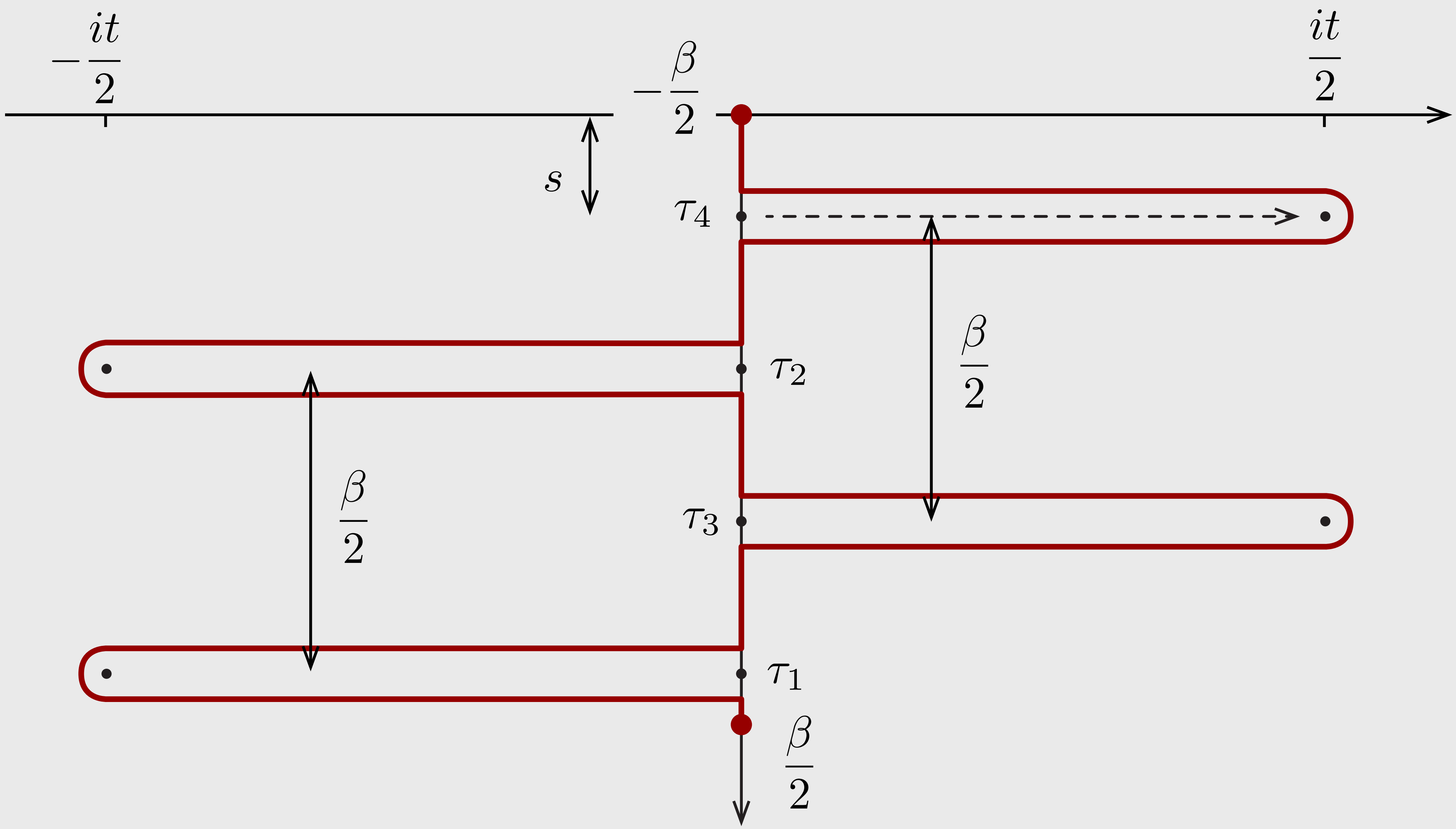


SYK OTO correlation function

obtained from contour-ordered four-point Green function

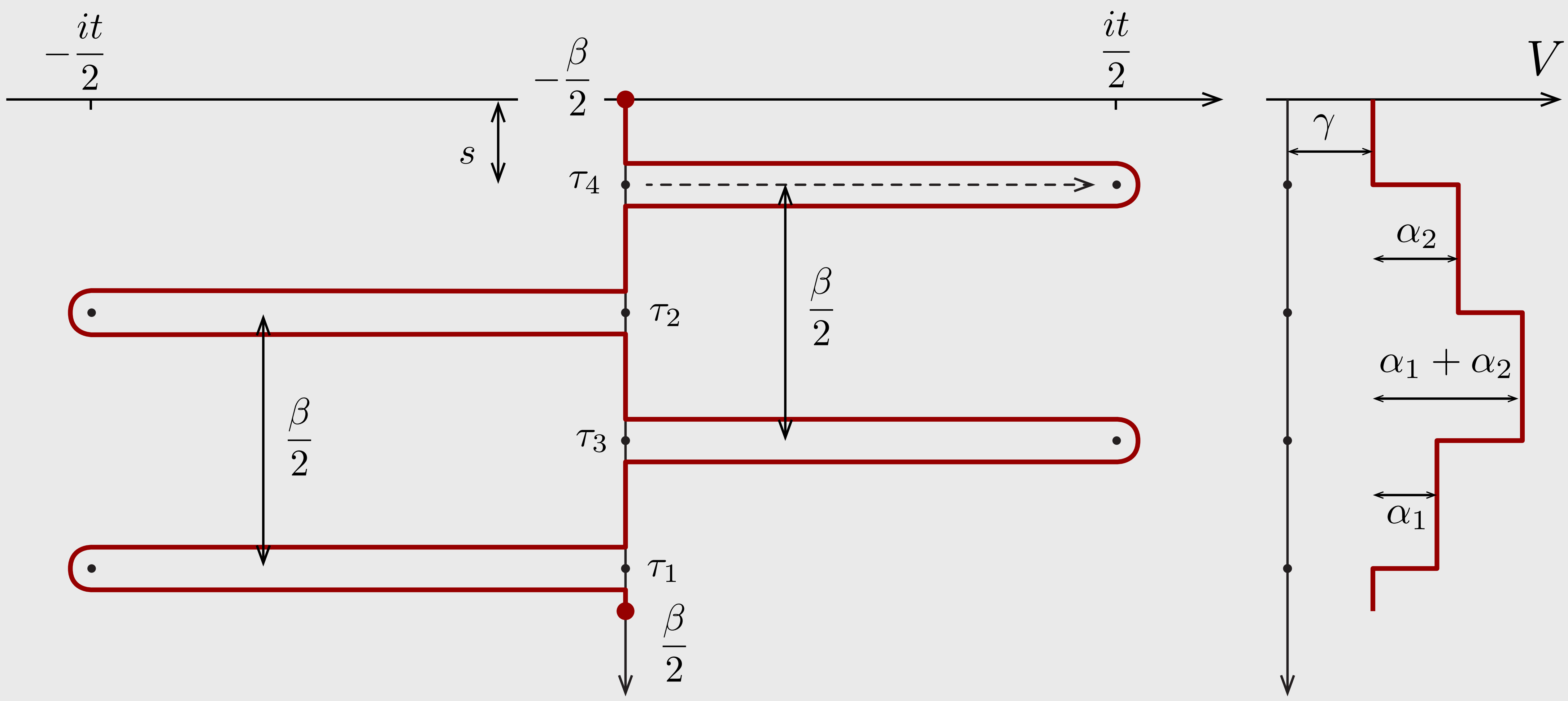
$$G_4(\tau_1, \tau_2, \tau_3, \tau_4) \equiv \frac{1}{N^2} \sum_{i,j} \langle T_\tau \chi_i(\tau_1)\chi_i(\tau_2)\chi_j(\tau_3)\chi_j(\tau_4)\rangle$$

after analytic continuation into complex plane



OTO correlation function con'd

substitution of path integral representation of Green functions generates double quantum quench protocol



Short time OTO: stationary phase

At short times large explicit symmetry breaking ‘magnon’ regime of Goldstone modes. Apply stationary phase method (neglecting quench potentials) to obtain

$$F(t) = 1 - \frac{\beta e^{2\pi t/\beta}}{64\pi M} + \mathcal{O}(e^{\pi t/\beta}/M)$$

in agreement with earlier results (Maldacena *et al.* 16)

Result can be trusted up to effective Ehrenfest time (chaos bound maxed out!)

$$t \sim t_E \equiv \frac{\ln(MT)}{2\pi T}$$

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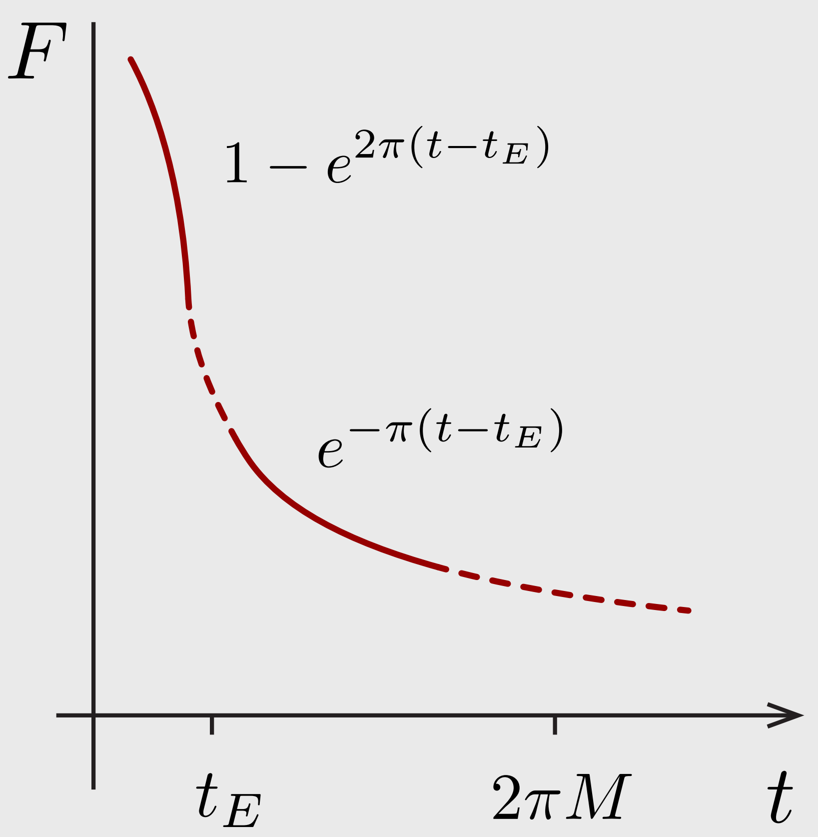
in agreement with earlier results (Maldacena *et al.* 16)

Result can be trusted up to effective Ehrenfest time (chaos bound maxed out!)

$$t \sim t_E \equiv \frac{\ln(MT)}{2\pi T}$$



At intermediate times $t_E < t < M$ stationary phase method including quench potentials yields

$$F(t) = \ln(MT)\, e^{-\pi T(t-t_E)}$$

Long time OTO: Liouville Schrödinger equation

At long times large Goldstone mode fluctuations suggest analysis of time dependent Schrödinger equation equivalent to path integral

Hamiltonian: $\hat{H}(t) = -\frac{\partial_\phi^2}{2M} + \gamma(t) e^{\phi}$

piecewise constant
quench potential

Eigenfunctions: $\langle \phi | k \rangle = \Psi_k(\phi) = \mathcal{N}_k K_{2ik}\left(2\sqrt{2M\gamma}\, e^{\phi/2}\right), \qquad \mathcal{N}_k = \frac{2}{\Gamma(2ik)}$

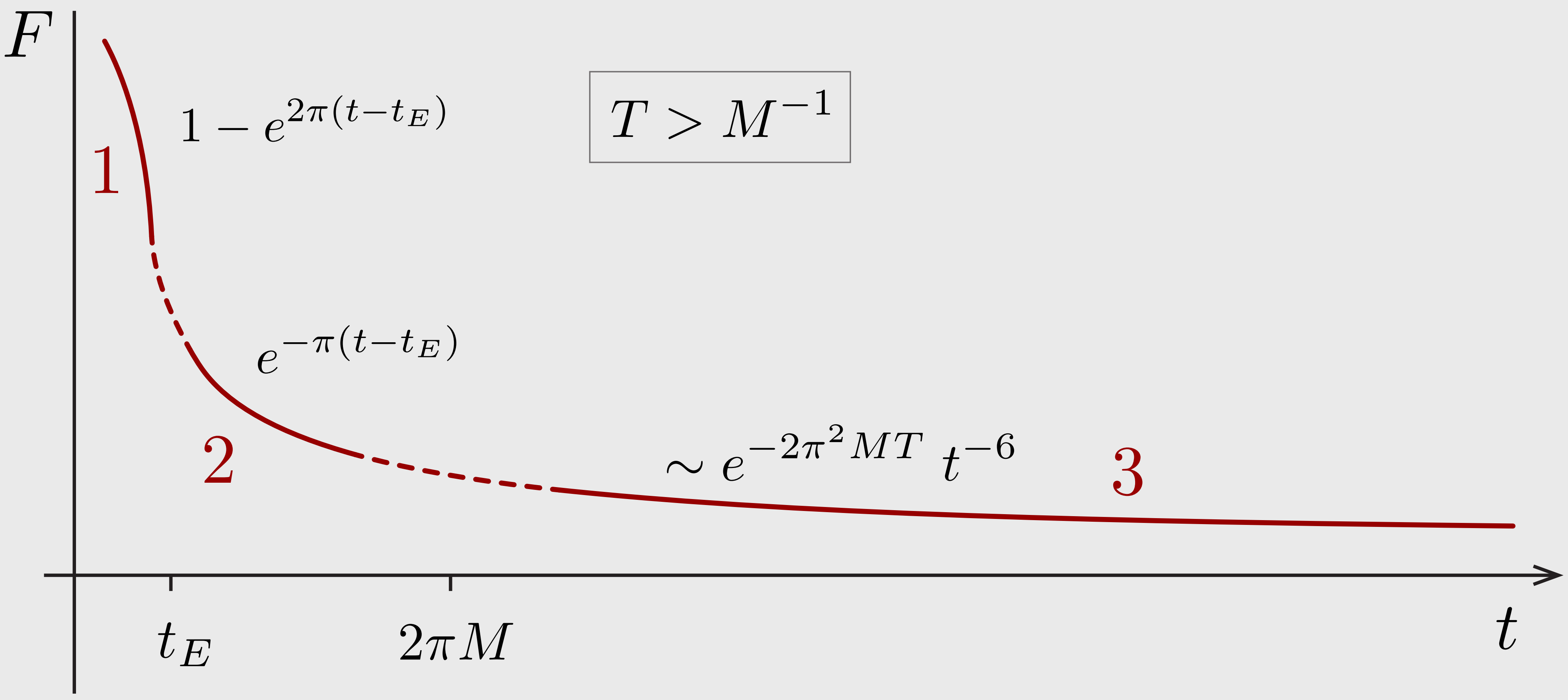
'momentum'

Eigenvalues: $\epsilon_k = \frac{k^2}{2M}$ (independent of potential strength)

Spectral decomposition of 4-point function leads to

$$\boxed{F(t) \sim e^{-2\pi^2 M/\beta} \left(\frac{\beta}{M}\right)^{3/2} \left(\frac{M}{t}\right)^6 \propto t^{-6}}$$

OTO result



Interpretation of the power law

Interpretation I: consequence of gapless dispersion of Liouville momentum, k.

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evaluated on correlation function on four time contours, implies -6=4x(-3/2) power law.

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Interpretation III: Lehmannize original expression

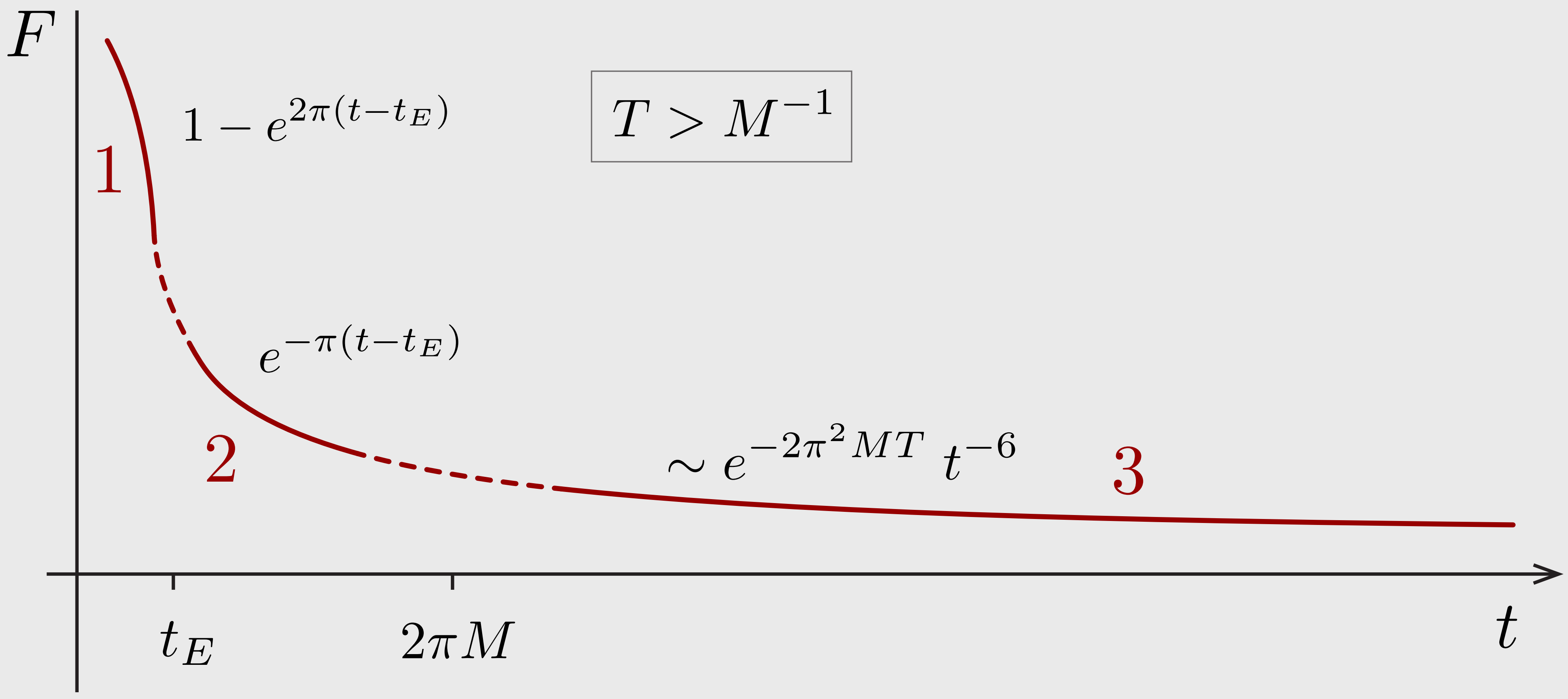
$$G_4(\tau_1,\tau_2,\tau_3,\tau_4) \equiv \frac{1}{N^2}\sum_{i,j}\langle T_\tau \chi_i(\tau_1)\chi_i(\tau_2)\chi_j(\tau_3)\chi_j(\tau_4)\rangle$$

$$= \frac{1}{N^2}\sum_{ij,m_i}\left\langle \langle m_1|\chi_i|m_2\rangle\langle m_2|\chi_j|m_3\rangle\langle m_3|\chi_i|m_4\rangle\langle m_4|\chi_j|m_1\rangle e^{-\left(\frac{\beta}{4}+it\right)\epsilon_{m_1}-\left(\frac{\beta}{4}-it\right)\epsilon_{m_2}-\left(\frac{\beta}{4}+it\right)\epsilon_{m_3}-\left(\frac{\beta}{4}-it\right)\epsilon_{m_4}}\right\rangle$$

(random) many body matrix elements

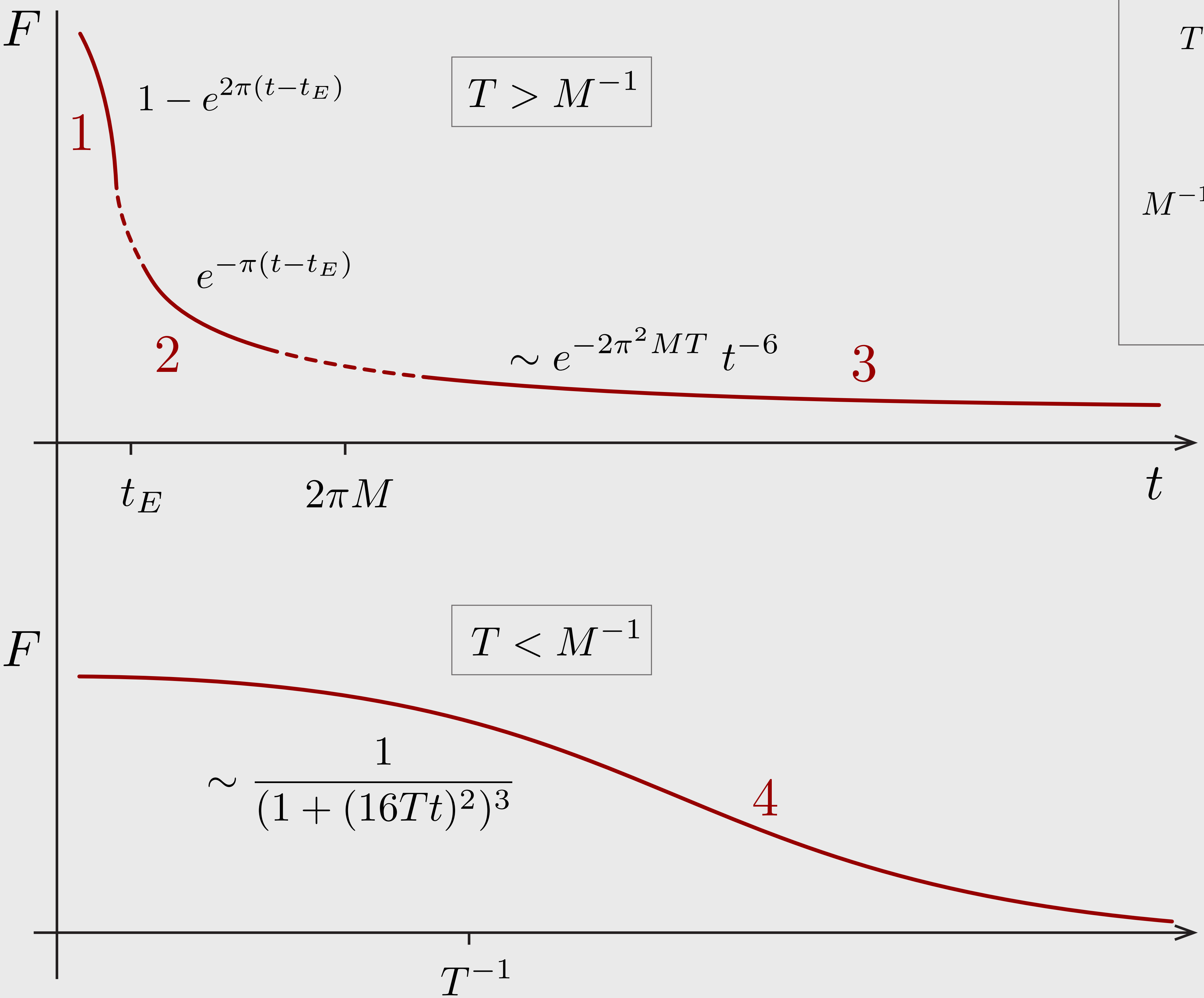
$$\sim \left(\int_0^\infty d\epsilon\,\rho(\epsilon)e^{-(\beta/4+it)\epsilon}\right)^4 \sim t^{-6}$$

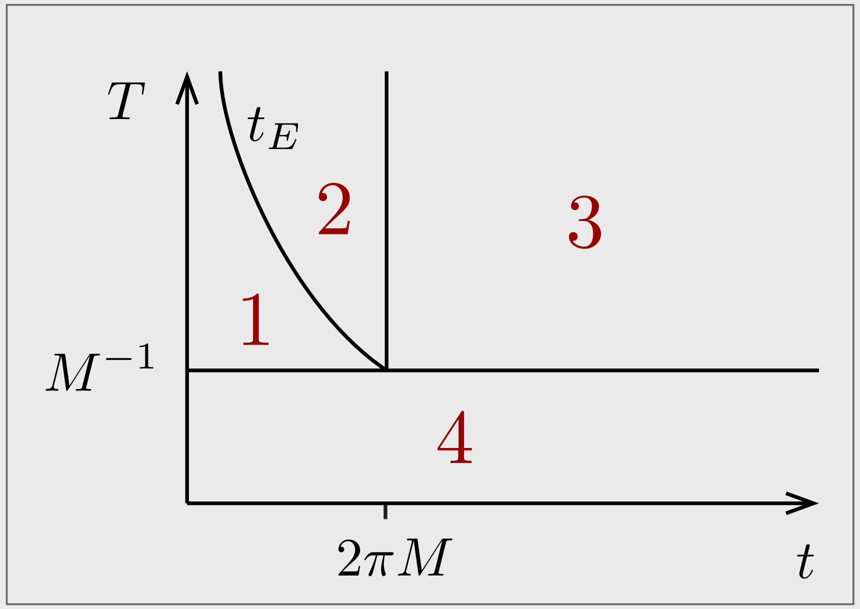
OTO result (including low temeratures, *T<1/M*)



Interpretation IV: At time scales t>M the system looses its semiclassical character

OTO result (including low temeratures, *T<1/M*)





Interpretation IV: At time scales t>M the system looses its semiclassical character

summary

conformal symmetry breaking in SYK model leads to large Goldstone mode fluctuations

fluctuations qualitatively affect physics at large time scales, $t>N/J$, and

modify correlation functions.

But what is the holographic interpretation?