# An alternative description of superconductivity through domain walls $^{1} \$

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### Summary

#### Fundamental concepts of superconductivity

- The Meissner effect
- London equation
- Ginzburg-Landau equation
- The BCS theory

#### 2 Superconductivity in field theory

- Type II domain walls Bloch walls
- The condensate at finite temperature
- Conductivity

#### Conclusions

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The Meissner effect London equation Ginzburg-Landau equation The BCS theory

Fundamental concepts of superconductivity

Zero electric resistence in some metals below a critical temperature is discovered in 1911 by Kamerlingh Onnes.

The Meissner effect London equation Ginzburg-Landau equation The BCS theory

#### The Meissner effect

- It was discovered in 1933.
- It manifests below a critical temperature  $(T_c)$ .
- It reveals the superconductor is a perfect diamagnetic.
- There exists a critical field.

Fundamental concepts of superconductivity

Superconductivity in field theory Conclusions

#### The Meissner effect

London equation Ginzburg-Landau equation The BCS theory

#### The Meissner effect



#### Figure: Levitation showing the Meissner effect.

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The Meissner effect London equation Ginzburg-Landau equation The BCS theory

#### Types of superconductors

- Type I It consists of pure specimens which typically has a low resistance to magnetic fields.
- Type II Consists of metallic alloys and transition metals.

The superconducting state in this case is stronger and more relevant to applications.

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#### London equation

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The London brothers formulated an equation to describe the Meissner effect, now known as the London equation was formulated in 1935.

#### London equation

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It is postulated that in the superconducting state the current density is directly proportional to the vector potential  $\vec{A}$ , making the constant of proportionality as  $\frac{-1}{\lambda^2}$ , we find

$$\vec{J} = -\frac{1}{\lambda_L^2} \vec{A}$$
(1)

or

$$\nabla \times \vec{J} = -\frac{1}{\lambda_L^2} \vec{B},\tag{2}$$

#### London equation

The Meissner effect London equation Ginzburg-Landau equation The BCS theory

or yet in a simpler way

$$\nabla^2 \vec{B} = \frac{\vec{B}}{\lambda_L^2}.$$
 (3)

The term  $\lambda_L$  is known as the penetration depth of London.

The Meissner effect London equation Ginzburg-Landau equation The BCS theory

#### Ginzburg-Landau equation

The most elegant phenomenological theory of superconductivity is the Ginzburg-Landau theory. In this theory it is defined the order parameter  $\Psi(\vec{r})$  with the following property:

$$\Psi^*(\vec{r})\Psi(\vec{r}) = n_s(\vec{r}), \qquad (4)$$

where  $n_s(\vec{r})$  is the total concentration of superconducting electrons.

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The Meissner effect London equation Ginzburg-Landau equation The BCS theory

#### Ginzburg-Landau equation

For temperatures T sufficiently near a critical temperature  $T_c$  and slow varying fields, the free energy function is defined by:

$$F_{s0} = F_{m0} + \alpha |\Psi|^2 + \frac{\beta}{2} |\Psi|^4 + \left(\frac{1}{2m}\right) \left| \left( -i\hbar\nabla - q\frac{\vec{A}}{c} \right) \Psi \right|^2 - \int_0^{B_a} \vec{M} \cdot d\vec{B_a}$$
(5)

The Meissner effect London equation Ginzburg-Landau equation The BCS theory

#### Ginzburg-Landau equation

By minimizing the free energy function with respect to  $\Psi$  we find

$$\left[\left(\frac{1}{2m}\right)\left(-i\hbar\nabla - q\frac{\vec{A}}{c}\right)^2 - \alpha + \beta |\Psi|^2\right]\Psi = 0, \quad (6)$$

that is the Ginzburg-Landau equation. Now minimizing with respect to  $\delta \vec{A}$  we find the supercurrent

$$\vec{J}_{s}(r) = -\left(\frac{iq\hbar}{2m}\right)\left(\Psi^{*}\nabla\Psi - \Psi\nabla\Psi^{*}\right) - \left(\frac{q^{2}}{mc^{2}}\right)\Psi^{*}\Psi\vec{A}.$$
 (7)

The BCS theory

The Meissner effect London equation Ginzburg-Landau equation The BCS theory

A very complete microscopic theory explaining superconductivity was proposed in 1957 by Bardeen, Cooper and Schrieffer. This theory has rendered to its authors the Nobel Prize for Physics in 1972. The theory that is now known as BCS theory has a good agreement with the experimental results.

#### The BCS theory

The Meissner effect London equation Ginzburg-Landau equation The BCS theory

The electron-lattice-electron interaction form Cooper pairs. The BCS theory shows that in the ground state of a superconductor all Cooper pairs occupy the same energy state. The most important result of this theory is the prediction that the first excited state above the ground state is separated by an energy gap.

#### Superconductivity in field theory

The superconducting domain walls solution developing a condensate in its core can be found through the following Lagrangian

$$\mathcal{L} = \frac{1}{2} \partial_{\mu} \phi \partial^{\mu} \phi + (\partial^{\mu} \chi + iq A^{\mu} \chi) (\partial_{\mu} \chi^* - iq A_{\mu} \chi^*) - V(\phi, |\chi|) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$$\mathcal{W}(\phi,|\chi|) = rac{1}{2} \left(rac{\partial W}{\partial \phi}
ight)^2 + rac{1}{2} \left|rac{\partial W}{\partial \chi}
ight|^2,$$

where we adopt the 'superpotential'

$$W(\phi, |\chi|) = \lambda \left(\frac{1}{3}\phi^3 - a^2\phi\right) + \mu\phi|\chi|^2.$$

We shall focus on two-dimensional domain walls such that the indices vary as  $\mu, \nu = t, x, y, r$ .

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#### Superconductivity in field theory

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(8)

We shall focus on two-dimensional domain walls such that the indices vary as  $\mu, \nu = t, x, y, r$ .

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#### Domain wall description of superconductivity

The real scalar field  $\phi$  develops a Z<sub>2</sub>-symmetry and is responsible for the formation of the domain wall, whereas the complex scalar field  $\chi$  develops a condensate inside the domain wall. This ensures a type II domain wall configuration known as Bloch wall.

#### Domain wall description of superconductivity

The equations of motion for the scalar fields coupled to electromagnetic field are given by

$$\Box \phi + \frac{\partial V}{\partial \phi} = 0, \tag{9}$$

$$\Box \chi + \frac{\partial V}{\partial \chi^*} - 2iqA_{\mu}\partial^{\mu}\chi - q^2A_{\mu}A^{\mu}\chi = 0, \qquad c.c., \quad (10)$$

$$\Box A_{\mu} + iq(\chi^* \partial_{\mu} \chi - \chi \partial_{\mu} \chi^*) + 2q^2 A_{\mu} |\chi|^2 = 0.$$
 (11)

From equation (11), we can define the supercurrent

$$-J_{\mu}^{s} = iq[\chi^{*}\partial_{\mu}\chi - \chi\partial_{\mu}\chi^{*}] + 2q^{2}A_{\mu}|\chi|^{2}.$$
(12)

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#### Obtaining the London equation

The equation of motion for the electromagnetic field becomes

$$\Box A_{\mu} = J^{s}_{\mu}. \tag{13}$$

Considering the vacuum configuration  $\left(0, \pm a \sqrt{\frac{\lambda}{\mu}}\right)$  onde  $|\chi|^2 = \frac{a^2 \lambda}{\mu}$  we find

$$J^{s}_{\mu} = -\frac{q^2 a^2 \lambda}{\mu} A_{\mu}. \tag{14}$$

The spatial part is just the famous London equation

$$\vec{J} = -\frac{1}{\lambda_L^2} \vec{A}.$$
 (15)

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#### Obtaining the London equation

Now rewriting this equation one finds

$$\nabla^2 \vec{B} = \frac{q^2 a^2}{2N^2} \vec{B}, \qquad N^2 = \frac{2\mu}{\lambda}.$$
 (16)

For a configuration of uniform magnetic field parallel to the domain wall in x-y-plane and transverse to the coordinate r, the general solution to (16) is given by

$$\vec{B} = \vec{B_0} \exp\left(-\sqrt{\frac{q^2 a^2}{2N^2}}r\right) + \vec{B_1} \exp\left(\sqrt{\frac{q^2 a^2}{2N^2}}r\right).$$
(17)

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#### Obtaining the London equation



Figure: The magnetic field behavior penetrating the superconductor.

# The electromagnetic field in the presence of background fields

For  $A_{\mu} = 0$  the equations of motion for static real scalar sector can be now reduced to the following first order equations:

$$\Box \phi + \frac{\partial V}{\partial \phi} = 0 \rightarrow \frac{d\phi}{dr} = W_{\phi}$$
$$\Box \chi + \frac{\partial V}{\partial \chi} = 0 \rightarrow \frac{d\chi}{dr} = W_{\chi}$$
(18)

which produce static domain wall solutions whose profiles are known as BPS solutions of the

Type I solution:

$$\chi = 0$$
 (orbit)

$$\phi(r) = -a anh(\lambda a r),$$
  
 $\chi = 0$ 
(19)
(19)
(19)

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# The electromagnetic field in the presence of background fields

Type II solution:  $\phi^2 + (\frac{\lambda}{\mu} - 2)^{-1}\chi^2 = a^2$  (orbit)

$$\phi(r) = -a \tanh(2\mu a r),$$
  

$$\chi(r) = \pm a \sqrt{\frac{\lambda}{\mu} - 2} \operatorname{sech}(2\mu a r),$$
(20)

where r is a coordinate transverse to the domain walls. As  $\lambda/\mu \leq 2$  the type II solution vanishes and the system is governed only by type I solution.

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# The electromagnetic field in the presence of background fields



Figure: Orbits for the Type I and Type II solutions: Solutions: Solution of superconductivity through domain

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# The electromagnetic field in the presence of background fields



Figure: The profile of the type II background solution

# The electromagnetic field in the presence of background fields

By introducing  $A_{\mu}(t,r) = A_{\mu}(r)e^{-i\omega t}$  e  $\chi(t,r) = \chi(r)e^{-i\theta t}$  into the equation of motion

$$\Box A_{\mu} + iq(\chi^* \partial_{\mu} \chi - \chi \partial_{\mu} \chi^*) + 2q^2 A_{\mu} |\chi|^2 = 0$$
(21)

we obtain the Schroedinger-like equation for  $A_x$  (or  $A_y$ ) in the form

$$-A_x'' + \frac{1}{4}\ell^2 \operatorname{sech}^2(\alpha r) A_x = \omega^2 A_x, \qquad (22)$$

where  $\ell = 2\sqrt{2}qa\sqrt{\frac{\lambda}{\mu}-2}$  and  $\alpha = 2\mu a$ .

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# The electromagnetic field in the presence of background fields

This is a well-known Schroedinger problem for a sech-type barrier potential, which solution is given by

$$A_{x}(\omega, \alpha, \ell, r) = \left(\operatorname{sech}\left(\alpha r\right)\right)^{-\frac{i\omega}{\alpha}} {}_{2}F_{1}\left[a1, a2; a3; \frac{1}{2}(1-\tanh\left(\alpha r\right))\right] (23)$$

where  $_2F_1$  is a hypergeometric function with parameters defined in the form

$$a1 = \frac{1}{2} \frac{-2i\omega + \alpha + \sqrt{-\ell^2 + \alpha^2}}{\alpha},$$

$$a2 = -\frac{1}{2} \frac{2i\omega - \alpha + \sqrt{-\ell^2 + \alpha^2}}{\alpha},$$

$$a3 = -\frac{i\omega - \alpha}{\alpha}.$$
(24)
$$a3 = -\frac{i\omega - \alpha}{\alpha}.$$
(25)
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#### The condensate at finite temperature

In the orbits above followed by the BPS solutions into the  $(\phi, \chi)$ -plane, the type II kink solutions are following *accelerating* trajectories.

See figure below for a Lorentzian signature in  $(\phi, \chi)$ -plane by making the change  $\phi \rightarrow i\phi$  and  $r \rightarrow i\tilde{r}$  in the first order equations.

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#### The condensate at finite temperature



Figure: The trajectories with constant accelerations in Lorentzian signature.

#### The condensate at finite temperature

The soliton in this sense is a Rindler observer experiencing a thermal bath of bosonic and fermionic modes that are distributed, respectively, according to Bose-Einstein and Fermi-Dirac statistics (Unruh effect):

$$< n_{\Omega} > = rac{1}{\exp\left(rac{2\pi\Omega}{a_c}
ight) \mp 1},$$
(25)

where the constant acceleration  $a_c$  is promptly identified with the Unruh temperature  $T = \frac{a_c}{2\pi}$  and  $\Omega$  are the frequencies of the *thermal radiation*.

#### The condensate at finite temperature

To show that the type II solution indeed describes accelerated trajectories, let us first redefine the fields  $\phi, \chi$  in terms of coordinates of space and time as follows  $\phi = \alpha a t(\tau)$  and  $\chi = \alpha a (\frac{\lambda}{\mu} - 2)^{1/2} z(\tau)$ such that

$$t(\tau) = \frac{1}{\alpha} \tanh \alpha \tau,$$
  
$$z(\tau) = \frac{1}{\alpha} \operatorname{sech} \alpha \tau,$$

where  $\tau \equiv r$  is identified with (Euclidean) proper time. Recall that  $\alpha = 2\mu a$ .

#### The condensate at finite temperature

Now using the definition of the acceleration  $a_c^{\mu} = \frac{d^2 x(\tau)^{\mu}}{d\tau^2}$  we find that

$$a_{c} \equiv \sqrt{a^{\mu}a_{\mu}} = \alpha - \frac{1}{2}\alpha(\alpha\tau)^{2} + \dots$$
$$a_{c} \simeq \alpha$$
(26)

for very slow velocities  $v = \alpha \tau \ll 1$  (non-relativistic Rindler observer).

Thus, using the Unruh temperature we find that  $\alpha = 2\pi T$ .

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#### The condensate at finite temperature

Since  $\alpha = 2\mu a \rightarrow T$  for type II solution it is natural to assume that for type I solution  $\lambda a \rightarrow T_c$ . This is because the temperature can be interpreted as a parameter of changing the domain wall of type I to type II.

- Temperature:
- Type I: *T* ≥ *T<sub>c</sub>*
- Type II:  $(0 \le T < T_c)$

The condensate should be given by the form

$$<\chi>\simeq T_c\sqrt{1-T/T_c},$$
 (27)

which vanishes for a critical temperature  $T_c$ .

#### The condensate at finite temperature

This condensate can be easily isolated by expanding the scalar solution  $\chi(r)$  around the core of the domain walls of type II in  $r \approx 0$ , in the form

$$\chi(r) = m - \frac{1}{2}m\alpha^2 r^2 + ...,$$
(28)

The condensate is given by the leading term  $<\chi>\simeq m$ .

#### The condensate at finite temperature

Notice we have redefined constants into original type II solution as  $\phi(r) = -a \tanh(\alpha r), \ \chi(r) = m \operatorname{sech}(\alpha r)$ , that is

$$m = a \sqrt{\frac{\lambda}{\mu} - 2}, \qquad \alpha = 2\mu a.$$
 (29)

In order to the type II solution to flow to type I solution the parameters  $m, \alpha$  should go as  $m \to 0$  as  $\alpha \equiv T \to T_c$ . Thus, to obtain the condensate as a function of temperature as we anticipated we use the following definitions.

#### The condensate at finite temperature

Since we have already identified  $\alpha \equiv T$  and  $\lambda a \equiv T_c$ , we can also identify all the parameters in the form

$$\lambda \sim \frac{T_c^{1/2}}{T^{1/2}}, \qquad \mu \sim \frac{1}{2} \frac{T^{1/2}}{T_c^{1/2}}, \qquad a \sim T_c^{1/2} T^{1/2}.$$
 (30)

Now substituting the equation (30) in the equation (29), we find

$$m = \sqrt{2}T_c\sqrt{1 - \frac{T}{T_c}},\tag{31}$$

that is precisely the anticipated form of the condensate  $<\chi>\simeq$   $m=\sqrt{2}T_c\sqrt{1-T/T_c}.$ 

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#### The condensate at finite temperature



Figure: The condensate as a function of the temperature.

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#### The condensate at finite temperature

Since  $\ell = 2\sqrt{2}qa\sqrt{\frac{\lambda}{\mu}-2}$ , we can determine the *effective condensate* seen by the electromagnetic field as

$$<\chi>_{eff} \simeq \ell = 2\sqrt{2}qm \simeq 4qT_c\sqrt{1-T/T_c}$$
 (32)

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#### The condensate at finite temperature



Figure: The effective condensate as a function of the temperature for the charges q = 1, 2, and 3 — curves from bottom to top.

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Type II domain walls - Bloch walls The condensate at finite temperature Conductivity

### Conductivity

Starting from the Ohm's law we can easily obtain the conductivity

$$\sigma_x(x,y) = \frac{J_x}{E_x} = \frac{A'_x(0)}{i\omega A_x(0)},$$
(33)

where we use  $E_x = -\partial_t A_x = i\omega A_x$ , with  $A_x = A_x(x, y, r)e^{-i\omega t}$  and defining the current as  $J_x = A'_x(0)$ .

### Conductivity

We use surface boundary conditions applied to electromagnetic field for an interface at r = 0, for instance. Specially for the magnetic field we find

$$\hat{n} imes \vec{B} = \vec{J}, \quad \text{at } r = 0, \quad (34)$$

where  $\hat{n}$  is normal vector to the surface of the superconductor and  $\vec{J}$  is a surface current.

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### Conductivity

For  $\hat{n} = (0, 0, 1)$  and  $\vec{A} = (A_x, A_y, 0)$  we have that the Eq. (34) is now simply given by

$$-\partial_r A_x(r) = J_x, \tag{35}$$

evaluated at r = 0. This essentially confirms our assumption  $J_x = A'_x(0)$  above.

#### Conductivity

Now, using the solution to the electromagnetic field and expanding around a generic plane  $r \approx \delta$  we can write the explicit form of the conductivity  $\sigma_x = \sigma_y \equiv \sigma$  which is given by

$$\sigma(\omega, \alpha, \ell, \delta) = \frac{\frac{1}{8}i\left(4\omega^2 + 4i\omega\alpha - \ell^2\right)\operatorname{sech}^2(\alpha\,\delta)}{\omega\left(i\omega - \alpha\right)}$$

$$\times \frac{{}_2F_1\left[b1, b2; b3; \frac{1}{2}(1 - \tanh\left(\alpha\,\delta\right))\right]}{{}_2F_1\left[a1, a2; a3; \frac{1}{2}(1 - \tanh\left(\alpha\,\delta\right))\right]}$$

$$+ \tanh\left(\alpha\,\delta\right), \qquad (36)$$

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#### Conductivity

$$b1 = -\frac{1}{2} \frac{2i\omega - 3\alpha + \sqrt{-\ell^2 + \alpha^2}}{\alpha}$$
  

$$b2 = \frac{1}{2} \frac{-2i\omega + 3\alpha + \sqrt{-\ell^2 + \alpha^2}}{\alpha}$$
  

$$b3 = -\frac{i\omega - 2\alpha}{\alpha}.$$
(37)

The temperature was already set  $\alpha \equiv T$ . We now consider the conductivity normalized by *effective condensate*, taking  $\ell \rightarrow q\ell$ , so we set  $\alpha = q^{-1}q\ell$  and  $\omega = \omega_r q\ell$  in  $\sigma$ . We can also write  $\frac{\alpha}{q < \chi >} = q^{-1}$  and  $\frac{\omega}{q < \chi >} = \omega_r$  (reduced frequency). Finally, we can replace it all in (36)-(37).

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#### Conductivity





Fro. 3. Results of measurements of the real part of the normalized conductivity of three thin lead films at  $2^{+}$ K, compared with Matis-Bardeen theory with gap frequency fitted to 22.5 cm<sup>-1</sup>. To reduce the clutter in the figure, only about one fourth as many points are shown as were taken and recorded in Kef. 7. The points shown are selected typical points above the gap and local averages below the gap.

Figure: (*left*) The real part of the conductivity as a function of frequency normalized by condensate. For charges q = 8, 20 and 32, from top to bottom;  $\delta = 0.01$  and  $\ell \simeq <\chi >_{eff} = 4$ . (*right*) The real part of the conductivity as a function of normalized frequency for three superconducting thin films of lead at  $2^{\circ}$ K — Palmer and Tinkham.

#### Conductivity

For  $\omega \to 0$  and  $T \to 0$  the conductivity (36) approaches a delta function  $\delta(\omega)$ . This is because for  $T \to 0$ , we have  $\ell \sim T_c$ . Thus, in this limit,  $\alpha^2 \ll \ell^2$  and  $\omega^2 \ll \ell^2$  implies that the real and imaginary part of the conductivity, up to a factor  $\sim 2\alpha/\ell$  that comes from the hypergeometric functions, can be written as

$$\operatorname{Re}\sigma(\omega) \propto \frac{(\ell/\alpha)^2}{(\omega/\alpha)^2 + 1} \to \delta(\omega),$$
 (38)

Im 
$$\sigma(\omega) \propto \frac{(\ell/\alpha)^2}{(\omega/\alpha)^3 + \omega/\alpha} \to \frac{\ell^2}{\alpha} \frac{1}{\omega}$$
. (39)

#### Conductivity

Now let us consider the conductivity as a function of temperature. Repeating the above analysis for  $\alpha\delta \to \infty$ , the argument of the hypergeometric functions tend to zero of the form  $e^{-2\alpha\delta}$ . In this regime, the ratio of hypergeometric functions in formula conductivity may be approximated by a series of a few terms. Considering up the next to leading term, we found

$$\operatorname{Re}\sigma(\omega,\alpha) \propto \delta(\omega) \left(1 - \frac{1}{8}\frac{\ell^2}{\alpha^2}e^{-2\alpha\delta} + \ldots\right) \simeq \delta(\omega)e^{-\frac{1}{8}\left(\frac{\Delta}{\alpha}\right)^2}, \quad (40)$$

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#### Conductivity

#### where

$$\Delta = \ell e^{-\alpha \delta} \tag{41}$$

precisely defines the binding energy of a Cooper pair, provided that we identify  $\ell = 2\omega_D$  as the Debye temperature and  $\delta \alpha = 1/VN_F$ , being V > 0 the binding potential and  $N_F$  the orbital density in the Fermi level. The binding energy of a Cooper pair can be written as follows (*BCS theory*):

$$\Delta = \frac{2\hbar\omega_d}{e^{\frac{1}{N_FV}} - 1}.$$
(42)

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### Conductivity

Note that the limit  $\delta \alpha \to \infty$  corresponds to  $VN_F \to 0$ , which is the limit of weak coupling and is in agreement with the BCS theory. Moreover, the limit  $\delta \alpha \to 0$  represents  $VN_F \to \infty$  which is the limit of strong coupling or superconductors of high critical temperatures ("High- $T_c$  superconductors").

#### Conductivity

Generalizing the expansion of condensate (28) around a plane  $r \approx \delta$  parallel to the domain walls, we have

$$\chi(\mathbf{r}) = \mathbf{m}\operatorname{sech}\left(\alpha\delta\right) - \mathbf{m}\operatorname{sech}\left(\alpha\delta\right)\operatorname{tanh}\left(\alpha\delta\right)\alpha(\mathbf{r}-\delta) + \dots, \quad (43)$$

redefining condensates we find

• 
$$\langle \chi \rangle \simeq m \operatorname{sech} (\alpha \delta) = \sqrt{2} T_c \sqrt{1 - T/T_c} \operatorname{sech} (\alpha \delta)$$
  
•  $\langle \chi \rangle_{eff} \simeq \ell \operatorname{sech} (\alpha \delta) = 2\sqrt{2} q m \operatorname{sech} (\alpha \delta)$   
•  $\langle \chi \rangle_{eff} \simeq 4q T_c \sqrt{1 - T/T_c} \operatorname{sech} (\alpha \delta).$ 

In the limit  $\alpha\delta \to \infty$ , we have

$$<\chi>_{eff} \simeq 2\ell e^{-\alpha\delta}.$$
 (44)

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#### Conductivity

The equations (41) and (44) now allow to write the important relation

$$\frac{2\Delta}{T_c} = \frac{\langle \chi \rangle_{eff}}{T_c}.$$
(45)

For comparison, we know that superconducting BCS have a typical ratio  $2\Delta \simeq 3.5 T_c$ , while the High- $T_c$  superconductors typically have ratios  $2\Delta \simeq 5 T_c$  to  $2\Delta \simeq 8 T_c$ .

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#### Condensate



Figure: The effective condensate as a function of temperature for charges q = 1, 2, e 3 — curves from bottom to top.

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### Conductivity

Let us relate the shifts  $\delta$  relative to condensate in  $r \approx 0$  for BCS and High- $T_c$  superconductors. Taking the ratio of the binding energies  $\Delta$  defined in (41) and inverting the relationship to write it in terms of  $\delta$ , we have

$$\frac{\delta_{BCS}}{\delta_{HT_c}} = 1 - \frac{1}{\alpha \delta_{HT_c}} \ln \frac{\Delta_{BCS}}{\Delta_{HT_c}}.$$
(46)

As  $\Delta_{BCS} < \Delta_{HT_c}$ , in the limit as  $\alpha \delta_{HT_c} \ll 1$ , as expected for High- $T_c$  superconductors, we have  $\delta_{BCS} \gg \delta_{HT_c}$ .

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#### Resistivity



Figure: (*left*) The real part of the resistivity at low frequencies as a function of temperature. We use  $\delta = 0.40$ , 0.45, and 0.55, from bottom to top;  $T_c = 3$ ,  $\omega = 0.8$  and q = 1. (*right*) The resistivity versus temperature for three superconductor samples of La-Ba-Cu-O with  $T_c = 35^{\circ}K$  — Bednorz and Müller, 1986.

#### Conclusions

- We have identified a relationship between the binding energy of Cooper pairs and condensed effective depending on the temperature and the electrical charge.
- For charges large enough we get a typical ratio of  $High T_c$  superconducting.
- We calculate the optical conductivity and show that in the regime of low temperatures and frequencies we get a zero infinite DC conductivity.

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#### Conclusions

- We conclude that the resistivity as a function of temperature is similar to what happens in  $High T_c$  superconductors.
- The critical temperature tends to be reduced when we move from the condensate via the deviation parameter  $\delta$ .
- As future prospects, we intend to attack the problem by investigating other quantities such as the London penetration length as a function of temperature and effects of anisotropy within the domain walls.

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### Thanks a lot !

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